AN OPTIMIZATION STRATEGY FOR RECONFIGURABLE CONTROL SYSTEMS

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Abstract

An optimization algorithm for the problem of control reconfiguration in response to operating condition changes or abrupt system component failures is presented here. The algorithm utilizes a version of the Broyden-Fletcher-Goldfarb-Shanno (BFGS) optimization method of conjugate directions. The algorithm provides an output feedback controller that not only stabilizes the new/impaired system, when possible, but also preserves much of the dynamics of the original/unimpaired system. The design is such that the closed-loop system is robust with respect to inevitable uncertainties/modelling errors on the state-space matrices of the impaired system. The algorithm is applied to an aircraft longitudinal control system, for which two severe cases of failure are considered, first the loss of an actuator and then the loss of a sensor in addition to the actuator loss.

1 Motivation-Previous work

Reconfigurable control systems are control systems that are characterized by the ability to perform in the presence of drastic changes in the system dynamics due, for example, to abrupt system component (actuator/sensor) failures or rapid changes in the operating conditions. Their task is twofold; first they need to guarantee safe performance (stability), whenever possible, and then recover maximum control performance under impairments. In aircraft flight control systems, for instance in an emergency situation, the first objective is to maintain the aircraft in a stable, flyable state and then try to recover as much of the performance specifications of the unfailed system as possible. Established techniques exist for the cases of anticipated failures/operating condition changes, for which control laws are precomputed, stored and used according to need. However, the interest here is mainly for the cases of unanticipated scenarios, where an automated on-line failure accommodation technique is needed. Here, we are primarily interested in the control reconfiguration part. That is, for the cases of component failures, a failure detection and identification scheme is assumed to provide the dynamics (state-space description) of the impaired system; for the cases of operating condition changes, an on-line modelling technique is assumed to identify the state-space model that corresponds to the new operating conditions. Before we present our control system reconfiguration approach to maintain stability and performance, other techniques that have appeared in the literature will be briefly discussed; furthermore, the needs that our methodology will attempt to accommodate will be identified.

Several approaches for aircraft flight control problems have appeared in the literature. In [13], a design is proposed for an autonomous lateral directional flight control system that utilizes a multiprocessor reconfigurable control and an adaptive learning network for the monitoring of control surface compliance, control law synthesis and system attribute learning. In [20], a gain schedule design procedure is presented. This procedure uses a linear quadratic optimization based simultaneous stabilization algorithm in which the schedule gain is constrained to stabilize a collection of plant models that represent the aircraft in various control failure modes. In [21], non-reconfigured and reconfigured control laws to accommodate three control element failures for a commercial airplane are studied. Note also that an interesting overview of the reconfiguration problem for aircraft flight control systems can be found in [9].

An alternative approach to reconfigurable control is presented in [26], where failures enter the system as uncertainties in system parameters and an H_∞ controller is designed that provides robust stability in the presence of some prescribed failures. In a similar fashion, reconfiguration is incorporated in a general FDI framework in [25]. A different viewpoint is presented in [4], [7], where the emphasis is on the identification of the parameters of the impaired system which then determines the design of the new control laws. In all the papers above, reconfiguration is either just a part of a more general adaptive or FDI/stability robustness scheme or treated as an uncertainty that enters the system or restricted to some specific classes of failures for which a control law may be stored and used upon need. Besides, the requirement of maintaining the dynamics of the original closed-loop system is not explicitly included in the reconfiguration design procedure. In this paper we are interested in an explicit control reconfiguration scheme that will provide a controller for the impaired model so that not only the stability is guaranteed but the performance of the impaired system closely approximates the control performance and specifications of the original system as well. In other words, we need to deal with the case of severe unanticipated failures and design on-line a controller that will maintain as much of the original closed-loop dynamics as possible. This is the way reconfiguration is treated in the papers that follow.

In [18], an approach to the automatic redesign of flight control systems for aircrafts that have suffered one or more control element failures is presented. This approach is based on linear quadratic (LQ) design techniques and attempts to maximize a measure of feedback system performance while satisfying the bandwidth limitation of the control system. This results in reconstructing the nominal forces and moments of the unfailed aircraft as nearly as possible. The proposed scheme maintains closed-loop stability and some robustness due to uncertain system parameters but does not necessarily guarantee the recovery of the closed-loop performance. Another drawback is that all states are assumed available for measurement, that is full state-feedback is considered.

In [6], [22], [23], the *Pseudo-inverse method (PIM)* is used to compute the control law for the impaired system. This method relies on the fact that the new feedback gain based on the pseudo-inverse theory is optimal in the sense that it is the solution of smallest norm for the linear least-squares problem of minimizing the Frobenius norm of the difference matrix between the original and the impaired closed-loop system transition matrices. The main problem, however, is that the stability of the impaired closed-loop system can not be guaranteed. This problem was overcome in [10], where the reconfigurable control problem was formulated as a constrained minimization problem and a *modified pseudo-inverse method (MPIM)* was proposed that guarantees the stability of the closed-loop system. The optimal solution for single-input systems is given in closed form; for multi-input systems the optimality is sacrificed for the sake of stability and a sub-optimal solution is given. However, the approach has two drawbacks. First it assumes full state-feedback, which can be quite unrealistic at times; and it relies on some stability bounds that may give very conservative results; this results in certain limitations of the proposed scheme.

More recently, an approach was presented in [14], where the full-state measurability (state-feedback) is relaxed and the output feedback case is considered. The presented method is based on the eigenstructure assignment approach of [1]. There are, however, two major drawbacks in the proposed technique. First, the stability of only $Max(m, r)$ eigenvalues of the closed-loop system is maintained, where $(m,\ r)$ are the numbers of inputs, outputs respectively. Although a sufficient condition for the stability of the remaining eigenvalues is provided, there is no guarantee that they will remain stable. Note that similar limitations are encountered in another reconfiguration design for some classes of failure scenarions that is based on eigenstructure assignments in [11]. The second drawback of [14] concerns the fact that the proposed methodology relies on the assumption that the input matrix B_f of the impaired model is of full column rank. This is a restrictive assumption, considering the common case of actuator loss which corresponds to zeroing a whole column of the input matrix.

Here, we consider the output feedback case and propose an optimization algorithm which guarantees the stability of all the closed-loop eigenvalues, even in the case of severe failure scenarions, such as the simultaneous loss of an actuator and a sensor. Note that this happens under the assumption that a stabilizing controller does exist for the impaired state-space model. The new stabilizing feedback controller for the impaired system captures as much of the dynamics of the original system as possible, since it is designed to minimize the Frobenius norm of the difference matrix between the original and the impaired closed-loop transition matrices. Another useful feature of the proposed design is that it is robust with respect to modelling errors concerning the state-space matrices of the new/impaired system. In other words, the realistic possibility of imperfect modelling of the impaired system is incorporated in our design and the controller derived by the proposed algorithm is capable of maintaining the closed-loop stability even in the presence of uncertainty in the state-space matrices of the impaired system. In section 2, the problem is formulated, the algorithmic approach is presented, and discussion is carried out concerning features of the technique and its extension to more complex control problems. In section 3, the algorithm is applied to an aircraft longitudinal control system, for which two severe cases of failure are considered, first the loss of an actuator and then the loss of a sensor in addition to the actuator loss. Finally, in section 4, concluding remarks are briefly discussed.

2 Problem Formulation

We consider the linear multivariable continuous system with the state-space description

$$\dot{x}(t) = A\,x(t) + B\,u(t), \qquad y(t) = C\,x(t) \tag{1}$$

where $x \in \Re^n$ is the state vector, $u \in \Re^r$ is the input vector and $y \in \Re^q$ is the output vector. We assume static output feedback of the form

$$u(t) = K\,y(t) = KC\,x(t) \tag{2}$$

The above gain matrix has been selected so that it satisfies specific control specifications or guarantees a certain control performance (transient response characteristics etc). Suppose now that due to system component failures (e.g. actuator or sensor failure/loss) or operating condition changes, the previous state-space representation can no longer model the dynamics of the new/impaired plant, which is now described by

$$\dot{x}(t) = A_f\,x(t) + B_f\,u(t), \qquad y(t) = C_f\,x(t) \tag{3}$$

Our objective is to design fast a new stabilizing output feedback control law

$$u(t) = K_f\,y(t) = K_f C_f\,x(t) \tag{4}$$

so that the new closed-loop system $A_f + B_f K_f C_f$ captures as much of the dynamics of the nominal closed-loop system $A + BKC$ as possible. In other words, we need to find a new control gain matrix that minimizes the Frobenius norm of the difference between the nominal and the new closed-loop system transition matrices. Therefore, the minimizing quantity of our interest is given by

$$
\begin{aligned}
J_{11} &= ||A + BKC - A_f - B_f K_f C_f||_F^2 \\
&= Tr[\,(A + BKC - A_f - B_f K_f C_f)^T (A + BKC - A_f - B_f K_f C_f)\,]
\end{aligned} \tag{5}
$$

where $||A||_F$ and $Tr(A)$ denote the Frobenius norm and the trace of a matrix A respectively. This is clearly an unconstrained minimization problem. We know that given an asymptotically stable matrix A and an arbitrary symmetric positive definite matrix Q, there exists a unique symmetric positive definite matrix P, such that

$$
A^T P + PA + Q = 0 \tag{6}
$$

The new gain matrix K_f needs to be stabilizing, that is it has to make $A_f + B_f K_f C_f$ stable. Therefore, according to above, it suffices to satisfy the following Lyapunov equation

$$
\hat{A}_f^T P + P\hat{A}_f + Q = 0 \tag{7}
$$

where

$$
\hat{A}_f = A_f + B_f K_f C_f \tag{8}
$$

By including (7) in (5), we have a constrained minimization problem. Therefore, the minimizing quantity is given by

$$
J_1 = J_{11} + Tr[\,L_1\,(\hat{A}_f^T P + P\hat{A}_f + Q)\,] \tag{9}
$$

where $L_1 \in \Re^{n \times n}$ is the Lagrange multiplier matrix. In the analysis above, there is the underlying assumption that we know exactly the state-space matrices of the impaired model. This is not usually the case in applications (e.g. flight control examples). When the operating conditions change abruptly or a severe failure, such as an actuator loss, occurs, then we can only approximate the state-space matrices of the impaired system. In cases like that, it is imperative that we design a controller that allows some *stability margin* to the closed-loop system, that is a controller that will stabilize the closed-loop system, even in the presence of uncertainty in some or all the state-space matrices $\{A_f, B_f, C_f\}$ of the closed-loop system $A_f + B_f K_f C_f$. We need the following theorem, which has been proven in [8]

Theorem 2.1 *Consider $\dot{x}(t) = Ax(t)$ where A is a stability matrix; let P, Q be as in (6). Suppose that $A \rightarrow A + \Delta A$, then $\dot{y}(t) = (A + \Delta A)\,y(t)$ remains asymptotically stable if*

$$(\Delta A)\, Q^{-1}\, (\Delta A)^T \;<\; \frac{1}{4}\, P^{-1}\, Q\, P^{-1} \tag{10}$$

or equivalently

$$(\Delta A)^T\, P\, Q^{-1}\, P\, (\Delta A) \;<\; \frac{1}{4}\, Q \tag{11}$$

We can easily see that a sufficient condition for (11) to hold is

$$\begin{aligned} \sigma^2_{max}(\Delta A)\, \sigma^2_{max}(P)\, \sigma_{max}(Q^{-1}) \;&<\; \frac{1}{4}\, \sigma_{min}(Q) \\ \Longrightarrow \quad \sigma_{max}(\Delta A) \;&<\; \frac{1}{2}\, \frac{\sigma_{min}(Q)}{\sigma_{max}(P)} \end{aligned} \tag{12}$$

Returning to our problem, we see that (12) can be used to maintain stability in cases of inevitable uncertainties in the closed-loop system $A_f + B_f K_f C_f$, that is we apply (12) for $A = \hat{A}_f$. Since Q is selected beforehand, it is apparent that in order to maximize the implied stability bound of (12) for inevitable uncertainties in the closed-loop system given above, we need to minimize $\sigma_{max}(P)$. Since $\sigma^2_{max}(A) \leq ||A||^2_F = Tr(A^T A)$ for any matrix A, we choose to minimize

$$J_2 = Tr(P^T P) = Tr(P^2) \tag{13}$$

Therefore, the overall minimizing quantity is finally given by

$$\begin{aligned} J \;&=\; J_1 + J_2 \\ &=\; Tr[\, (\tilde{A}_f - B_f K_f C_f)^T\, (\tilde{A}_f - B_f K_f C_f) \;+\; L_1\, (\hat{A}_f^T P + P \hat{A}_f + Q) \;+\; P^2\,] \end{aligned} \tag{14}$$

where

$$\tilde{A}_f = A + BKC - A_f \tag{15}$$

Next we compute the partial derivatives of the final cost J with respect to all the matrix variables entailed; these partial derivatives are needed for the algorithm that is presented next for the minimization of J_1. In order to compute them, we need the following properties from [2]

$$\frac{\partial}{\partial X} Tr(X^2) = 2X^T \tag{16}$$

$$\frac{\partial}{\partial Y} Tr(A_1 Y B_1) = A_1^T B_1^T \tag{17}$$

$$\frac{\partial}{\partial Y} Tr(A_2 Y^T B_2) = B_2 A_2 \tag{18}$$

$$\frac{\partial}{\partial Y} Tr(A_3 Y B_3 Y^T) = A_3 Y B_3 + A_3^T Y B_3^T \tag{19}$$

for any $X \in \Re^{n\times n}$, $Y \in \Re^{n\times m}$, $A_1 \in \Re^{l\times n}$, $B_1 \in \Re^{m\times l}$, $A_2 \in \Re^{l\times m}$, $B_2 \in \Re^{n\times l}$, $A_3 \in \Re^{n\times n}$, $B_3 \in \Re^{m\times m}$. With these properties, we have

$$\frac{\partial J}{\partial L_1} = \Delta_{L_1} = \hat{A}_f^T P + P\hat{A}_f + Q \tag{20}$$

$$\frac{\partial J}{\partial P} = \Delta_P = \hat{A}_f L_1^T + L_1^T \hat{A}_f^T + 2P \tag{21}$$

$$\frac{\partial J}{\partial K_f} = \Delta_{K_f} = 2B_f^T B_f K_f C_f C_f^T - 2B_f^T \tilde{A}_f C_f^T + B_f^T P(L_1 + L_1^T) C_f^T \tag{22}$$

To minimize (14) we use a version of the Broyden family method of conjugate directions, which is based on the Broyden-Fletcher-Goldfarb-Shanno (BFGS) update rule; details in [3]. Note that a version of this algorithm has been used in [15], [16], [17] for the design of controllers for robust stability and optimal performance of uncertain discrete-time systems. The proposed algorithm is presented next.

Initialization Step Let $\epsilon > 0$ be the termination scalar. Choose an initial stabilizing gain

$$K_f^1 = \begin{pmatrix} (\tau_1^1)^T \\ \vdots \\ (\tau_r^1)^T \end{pmatrix} \tag{23}$$

where $(\tau_l^1)^T, l = 1, .., r$ are the $1 \times q$ rows of K_f^1, which stabilizes (A_f, B_f, C_f), that is $\hat{A}_f$ stable. Also, choose an initial symmetric positive definite matrix D_1. Let

$$y_1 = x_1 = (\, (\tau_1^1)^T \quad \cdots \quad (\tau_r^1)^T \,)^T \tag{24}$$

be a column vector consisting of the transposes of the rows of K_f^1. Also let $k = j = 1$ and go to the *Main Step*.

Main Step

M1. Substitute the gain matrix K_f^j in the gradients of (20)-(21), set them to zero, that is $\Delta_{L1} = 0$, $\Delta_P = 0$, and solve for P, L_1 respectively, in that specific order.

M2. Substitute these parameters in (22) and compute

$$\Delta_{K_f^j} = \begin{pmatrix} (\sigma_1^j)^T \\ \vdots \\ (\sigma_r^j)^T \end{pmatrix} \tag{25}$$

where $(\sigma_l^j)^T, l = 1, .., r$ are the $1 \times q$ rows of $\Delta_{K_f^j}$.

M3. Define $\nabla J(y_j) = (\,(\sigma_1^j)^T \quad \cdots \quad (\sigma_r^j)^T\,)^T$.

If $\|\nabla J(y_j)\| < \epsilon$, *STOP*. The optimal gain is K_f^j. Otherwise, go to M4.

M4. If $j > 1$, update the positive definite matrix D_j as follows:

$$D_j = D_{j-1} + \frac{p_{j-1}p_{j-1}^T}{p_{j-1}^T q_{j-1}}\,[1 + \frac{q_{j-1}^T D_{j-1} q_{j-1}}{p_{j-1}^T q_{j-1}}] - \frac{[D_{j-1}q_{j-1}p_{j-1}^T + p_{j-1}q_{j-1}^T D_{j-1}]}{p_{j-1}^T q_{j-1}} \tag{26}$$

where

$$p_{j-1} = \lambda_{j-1} d_{j-1} = y_j - y_{j-1} \qquad\qquad q_{j-1} = \nabla J(y_j) - \nabla J(y_{j-1}) \tag{27}$$

M5. Define $d_j = -D_j \nabla J(y_j)$.

Let λ_j be an optimal solution to the problem of minimizing $J(y_j + \lambda d_j)$ subject to $\lambda \geq 0$.

Let $y_{j+1} = y_j + \lambda_j d_j = (\,(\tau_1^{j+1})^T \quad \cdots \quad (\tau_r^{j+1})^T\,)^T$, which implies that

$$K_j^{j+1} = \begin{pmatrix} (\tau_1^{j+1})^T \\ \vdots \\ (\tau_r^{j+1})^T \end{pmatrix} \tag{28}$$

where obviously $(\tau_l^{j+1}), l = 1, .., r$ are $q \times 1$ column vectors.

M6. If $j < rq$, replace j by $j + 1$ and repeat the *Main Step.*

Otherwise, if $j = rq$, then let $y_1 = x_{k+1} = y_{rq+1}$, replace k by $k + 1$, let $j = 1$ and repeat the *Main Step.* □□

There are several issues that need to be discussed here.

Remark 2.1 The line search in *(M5)* is restricted to stabilizing gains. Therefore, the selected new gain matrix needs first to stabilize the closed loop matrix (8) of the impaired system and then minimize (14). Note that the line search in *(M5)* of the *Main Step* was performed in our examples by the Fibonacci method; details in [3].

Remark 2.2 Since our algorithm is an indirect version of the BFGS algorithm, as an alternative to the stopping criterion of *(M3)*, we could use another quite practical criterion. Specifically, we may consider monitoring J and stop when we reach a plateau or when we see that J is sufficiently small and the associated bound derived is acceptably large. Additionally, note that for optimization problems similar to the one we study here, alternative methods based on gradient-type and nongradient-type algorithms have been proposed in [12] and [19] respectively.

Remark 2.3 The minimizing quantity of (14) consists of two components, the reconfiguration term (J_1) and the robustness term (J_2). By assigning weights to these terms, we could emphasize the one that is of more interest to us. Specifically, for cases where we are quite uncertain about the state-space matrices of the impaired system, we could assign a large weight to J_2, in order to maximize the stability region within which the perturbations of the closed-loop matrix are allowed to vary without jeopardizing the stability of the closed-loop system. Without loss of generality, we can always consider a weight $\omega_1 = 1$ for J_1, so that the minimizing quantity is given by

$$J_w = J_1 + \omega_2 J_2 \tag{29}$$

Note that the introduction of ω_2 affects only the gradient of (21), where the term 2 P needs to be substituted by 2 $\omega_2 P$.

Remark 2.4 For the algorithm mentioned above, we need an initial stabilizing output gain. If such a gain is not available, then we can use the heuristic approach of [5].

Remark 2.5 Several stability bounds similar to the one given in Theorem 2.1 above can be found in the robust stability literature. Which one is the best (less conservative) is not the issue of interest here. We have chosen the bound of (12) because it is easy to use and suits our analysis.

Remark 2.6 In our analysis design above, we have considered the robustness of the closed-loop system $\hat{A}_f$ assuming possible uncertainties in all the state-space matrices of the impaired system. If we are certain for some of these matrices, then the bound of (12) can be easily modified. For instance, let's consider a common case in reconfigurable systems, where the state and output matrices remain the same $A = A_f$, $C = C_f$ and we only have changes in the input matrix B. Then, the allowable perturbations in B_f are easily given from (12) by

$$\sigma_{max}(\Delta B_f) \; < \; \frac{1}{2} \, \frac{\sigma_{min}(Q)}{\sigma_{max}(P) \; \sigma_{max}(K_f C_f)} \tag{30}$$

It is apparent that by minimizing $\sigma_{max}(K_f C_f)$, we can further enlarge the above stability region. This can easily be done by including its upper bound $Tr[\ (K_f C_f)^T (K_f C_f)\ ]$ in the minimizing quantity J. Note that such an inclusion could enhance the robustness aspect of the proposed design but would affect its reconfiguration aspect, which is of primary concern here. However, for cases where there is a serious uncertainty about the input matrix B_f, the inclusion of the above term is recommended to avoid instability due to imperfect modelling of the impaired system.

Remark 2.7 In addition to the reconfiguration and the robustness objectives studied above, we may wish to attain a specific control performance; in that case we need to include in the minimizing quantity another term that evaluates this control performance. This term is the familiar LQR cost given by

$$J_3' = \int_0^{\infty} (x^T Q_1 x + u^T R_1 u)\ dt \tag{31}$$

where Q_1, R_1 are positive definite matrices of appropriate dimensions. It can easily be shown that the minimization of (31) is equivalent to the minimization of $Tr[\ P_2 x(0) x(0)^T\ ]$, where P_2 is the unique solution of $\hat{A}_f^T P_2 + P_2 \hat{A}_f + Q_1 + C_f^T K_f^T R_1 K_f C_f = 0$. Therefore, similar to before, the minimizing quantity in that case will be

$$\hat{J} = J + Tr[\ P_2\ x(0)\ x(0)^T + L_2\ (\hat{A}_f^T P_2 + P_2 \hat{A}_f + Q_1 + C_f^T K_f^T R_1 K_f C_f)\ ] \tag{32}$$

where L_2 is another Lagrange multiplier matrix. Of course, in addition to the gradients computed in (20)-(22) above, we also need to compute the gradients with respect to P_2 and L_2, which will be used by our algorithm.

Remark 2.8 In the analysis above, we have studied the static output feedback case. When dynamic output feedback is considered, then the formulation given in the appendix of [17] can readily be used. Note that in this formulation, the order of the controller is fixed. In the same respect, when considering the output feedback gain for the impaired system, the assumption is made that the new controller will be of the same order with the dynamic controller of the nominal system.

3 An illustrative example

Consider an aircraft longitudinal control system from [14], whose the linearized dynamic model is given by

$$\begin{aligned}
\begin{pmatrix} \dot{\alpha}(t) \\ \dot{\beta}(t) \\ \dot{\psi}(t) \\ \dot{\theta}(t) \end{pmatrix} &= \begin{pmatrix} -0.0582 & 0.0651 & 0 & -0.171 \\ -0.303 & -0.685 & 1.109 & 0 \\ -0.0715 & -0.658 & -0.947 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \alpha(t) \\ \beta(t) \\ \psi(t) \\ \theta(t) \end{pmatrix} + \begin{pmatrix} 0 & 1 \\ -0.0541 & 0 \\ -1.11 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \eta(t) \\ \tau(t) \end{pmatrix} \\
y(t) &= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \alpha(t) \\ \beta(t) \\ \psi(t) \\ \theta(t) \end{pmatrix} \qquad (33)
\end{aligned}$$

where $\alpha(t)$ and $\beta(t)$ are the forward and vertical speeds, $\psi(t)$ is the pitch rate and $\theta(t)$ is the pitch angle. The control inputs $\eta(t)$ and $\tau(t)$ are the elevator angle and throttle position respectively. When we consider the static output feedback law (2), the controller that assigns the closed-loop eigenvalues at $\{-2, -0.5973, -1.5 \pm 2j\}$ is given by

$$K = \begin{pmatrix} -0.00031 & 4.77004 & 1.70457 \\ -2.01505 & -1.13002 & 0.02904 \end{pmatrix} \qquad (34)$$

Next, we suppose that the system dynamics change due to operating condition variations and at the same time severe failures happen at the actuators or sensors. First, we study the case of actuator loss and then the case of both actuator and sensor losses.

3.1 Actuator loss

The state-space matrices of the impaired model are given below. Note the loss of the second actuator.

$$\begin{aligned}
A_f &= \begin{pmatrix} -0.0582 & 0.10 & 0.0 & -0.171 \\ -0.103 & -0.685 & 1.109 & 0 \\ -0.0715 & -0.658 & 1.98 & 0 \\ 0 & 0 & 1.5 & 0 \end{pmatrix}, \quad B_f = \begin{pmatrix} 0 & 0.0 \\ -0.09 & 0.0 \\ -1.11 & 0.0 \\ 0 & 0.0 \end{pmatrix} \\
C_f &= \begin{pmatrix} 0.9 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.7 \\ 0 & 0 & 1 & 0 \end{pmatrix} \qquad (35)
\end{aligned}$$

model	eigenvalues	gain K (first row)
nominal	$\{-2.0, -0.5973, -1.50 \pm 2.00j\}$	[-0.00031 4.77004 1.70458]
imp ($\omega_2 = 0.01$)	$\{-0.0770, -0.5591, -1.4610 \pm 2.5498j\}$	[0.00680 6.79671 4.31982]
imp ($\omega_2 = 0.1$)	$\{-0.0797, -0.5576, -1.4603 \pm 2.5491j\}$	[-0.06511 6.79624 4.31961]
imp ($\omega_2 = 1$)	$\{-0.0947, -0.5491, -1.4575 \pm 2.5352j\}$	[-0.45799 6.75011 4.32028]
imp ($\omega_2 = 10$)	$\{-0.1371, -0.5194, -1.4840 \pm 2.4060j\}$	[-1.37154 6.31477 4.37946]
imp ($\omega_2 = 50$)	$\{-0.1779, -0.4819, -1.6283 \pm 2.1596j\}$	[-1.92021 5.75013 4.64244]

Table 1: Actuator loss: eigenvalues and feedback gain for the nominal and the impaired models.

impaired model	J_1 (reconfiguration term)	J_2 (robustness term)	bound (general)	bound for ΔB_f	$(\Delta B_f = \kappa_1 B_1)$ $\lvert\kappa_1\rvert <$
$\omega_2 = 0.01$	5.7469	2.3324	0.0668	0.0104	1.04
$\omega_2 = 0.1$	5.7534	2.2092	0.0687	0.0107	1.07
$\omega_2 = 1$	5.9737	1.7149	0.0785	0.0122	1.22
$\omega_2 = 10$	7.8258	1.1772	0.0956	0.0151	1.51
$\omega_2 = 50$	10.3066	1.0538	0.1012	0.0159	1.59

Table 2: Actuator loss: results for different weight factors considered for the impaired model.

Note that the state-space matrices given above correspond to new operating conditions, as given in [14]; in addition, we imposed the loss of one of the actuators. We use the algorithm to compute the stabilizing static output feedback controller K_f that minimizes the Frobenius norm of the difference $A + BKC - A_f - B_f K_f C_f$ denoted by J_1 and maximizes the robustness of the impaired closed-loop system, which corresponds to the minimization of J_2. We assign different weights to the robustness term, so that the minimizing quantity is given by (29).

In Table 1, we give the closed-loop eigenvalues and the first row of the stabilizing output feedback gain for the original and several impaired models. Note that for the impaired models the second row of the controller becomes irrelevant due to the actuator loss. In Table 2, we give the results for the reconfiguration and the robustness terms, when assigning different weights to J_w in (29). We also give the robustness bound, that is the maximum singular value of the variations of the closed-loop system that can be allowed so that the asymptotic stability is maintained. We also restrict these variations in perturbations in the input matrix B and compute the same bound for $\sigma_{max}(\Delta B_f)$. Finally, since by inspecting the input matrices B, B_f we see that the uncertainty is mainly with regard to the element $(2, 1)$, we express the structured perturbation in B_f as

$$\Delta B_f = \kappa_1 \; B_1 = \kappa_1 \begin{pmatrix} 0 & 0 \\ 0.01 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \tag{36}$$

and give the robustness bound for κ_1. Note that we could also restrict the variations in the output matrix C_f and obtain the perturbation bound in the same way. From Table 2, it is quite obvious that for larger ω_2, we enhance the robustness of the closed-loop system, which translates into smaller J_2 and larger perturbation bounds (for ΔB_f or general); at the same time, however, the reconfiguration term J_1 increases, which results in the deterioration of the closeness of the impaired closed-loop system $A_f + B_f K_f C_f$ to the nominal closed-loop system $A + BKC$.

In Fig. 1- 2, we compare the state-responses of the original and the impaired closed-loop systems. For the latter, we have chosen the controller obtained by our algorithm for $\omega_2 = 1$. The initial conditions vector was chosen as $(\,0.1 \quad 0.5 \quad 0.3 \quad 1\,)^T$. The two plots are very close to each other, which implies that despite the severity of the actutator loss, we were able to recover quite successfully the dynamics of the nominal model.

3.2 Actuator/sensor losses

Now, we consider the even more severe case of losing both the second actuator and the first sensor. Therefore, A_f and B_f remain the same as before, whereas the output matrix changes to

$$\tilde{C}_f = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.7 \\ 0 & 0 & 1 & 0 \end{pmatrix} \tag{37}$$

Note that these losses make the second row and the $(1,1)$ element of the first row of the controller gain irrelevant. Therefore, we try to recover the behavior of the nominal plant based upon the optimal selection of only 2 elements of the output feedback gain, namely elements $(1,2)$ and $(1,3)$. In Tables 3-4, we give the same results as before for the present case. Note that only the cases of $\omega_2 \leq 10$ are included, since for $\omega_2 > 10$ no significant changes were observed in the results. This is not surprising, since even from the results provided in Tables 3-4, we see that the parameters of interest did not change significantly even when we increased ω_2 from 0.01 to 10.

Comparing with the results of the previous subsection, with only actuator loss, we see that the sensor loss, in addition to the actuator loss, affects the robustness of the closed-loop system.

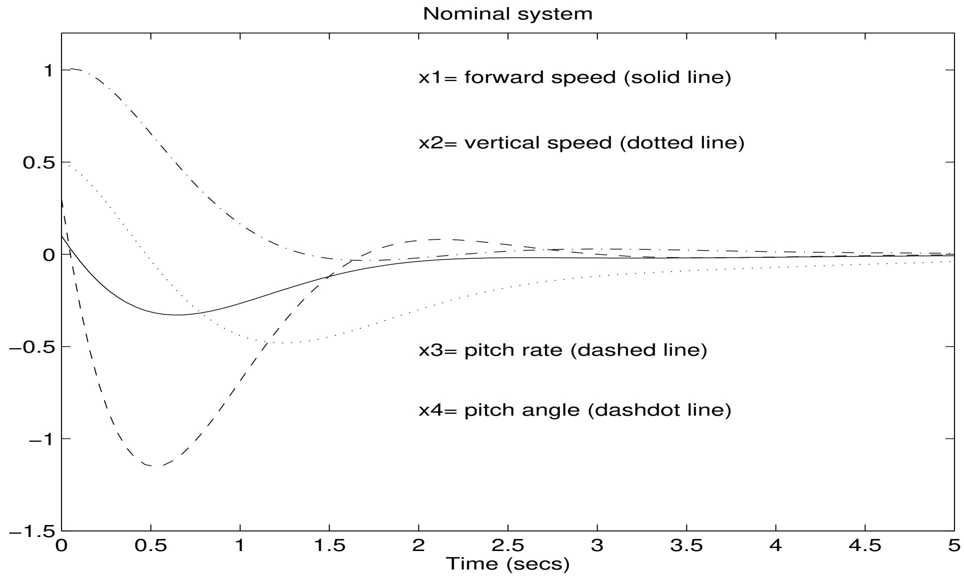

Figure 1: Nominal system: state trajectories for the closed-loop system.

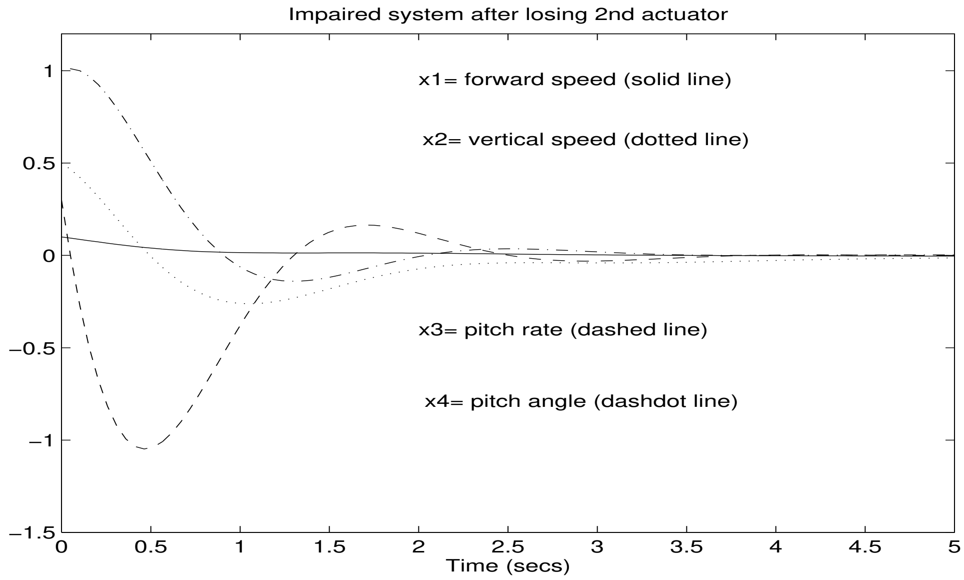

Figure 2: Actuator loss: state trajectories for the impaired closed-loop system.

model	eigenvalues	gain K (first row)
nominal	$\{-2.0, -0.5973, -1.50 \pm 2.00j\}$	[-0.00031 4.77004 1.70458]
imp ($\omega_2 = 0.01$)	$\{-0.0773, -0.5589, -1.4610 \pm 2.5497j\}$	[-0.00031 6.79669 4.31982]
imp ($\omega_2 = 1$)	$\{-0.0773, -0.5590, -1.4594 \pm 2.5527j\}$	[-0.00031 6.80620 4.31707]
imp ($\omega_2 = 10$)	$\{-0.0772, -0.5598, -1.4488 \pm 2.5762j\}$	[-0.00031 6.88535 4.29861]

Table 3: Actuator/sensor losses: eigenvalues and feedback gain for the original and the impaired models.

impaired model	J_1 (reconfiguration term)	J_2 (robustness term)	bound (general)
$\omega_2 = 0.01$	5.7471	2.3202	0.0670
$\omega_2 = 1$	5.7472	2.3200	0.0670
$\omega_2 = 10$	5.7525	2.3190	0.0670

Table 4: Actuator/sensor losses: different weight factors for the impaired model.

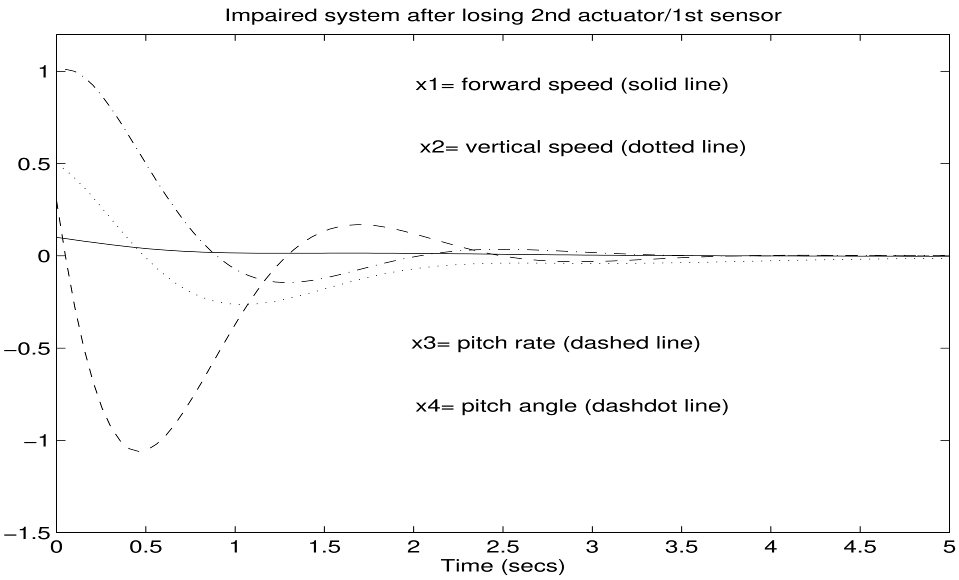

Figure 3: Actuator/sensor losses: State trajectories for the impaired closed-loop system.

example of Gao *et al* (1991)	present algor. $\omega_2 = 1$	present algor. $\omega_2 = 0.1$	example of Jiang (1994)	present algor. $\omega_2 = 1$	present algor. $\omega_2 = 0.1$
3.8525	2.7038	2.4225	2.4627	0.6392	0.6392

Table 5: Comparing $||A + BKC - A_f - B_f K_f C_f||_F$ for literature examples.

Specifically, because of the $(1, 1)$ term of the output controller being obsolete, we can not affect the location of the closed-loop poles. Therefore, unlike the actuator case, we can not remove the pole at -0.0773, see Table 3, no matter how much we increase ω_2 in the minimizing quantity. Compare with the case of actuator loss, where we were able to enhance the robustness of the closed-loop system, by assigning a large weight to the robustness term J_2, which resulted in removing the problematic pole from -0.0770 to -0.1779.

Comparing the reconfiguration terms J_1 of Tables 2 and 4, we see that the loss of the first sensor did not affect at all the reconfiguration aspect of our design. This can also be seen in Fig. 3, where we give the state-responses of the impaired closed-loop system (for $\omega_2 = 1$) after the loss of both the actuator and the sensor for the same initial vector as before. This plot is quite similar to the one of Fig. 2, which shows that despite the loss of the sensor, in addition to the actuator loss, our scheme was capable of recovering the dynamics of the original system. This was not the case, however, when we considered the loss of the second sensor instead of the first. In that case, we obtained $J_1 = 37.3052$ which is not even close to what we obtained before.

Finally note that in Table 5, we compare the Frobenius norm of the difference between the original and the impaired closed-loop transition matrices $||A + BKC - A_f - B_f K_f C_f||_F$ for the controllers derived in the examples of [10] and [14] and the ones derived by the proposed algorithm here for the cases of $\omega_2 = 0.1$ and $\omega_2 = 1$. It is obvious that the present algorithm, in addition to maintaining closed-loop stability even for the output-feedback case, is more successful in preserving the characteristics of the original system compared to the techniques presented in the papers above.

Note that for all the simulations mentioned above, our algorithm proved to be quite fast. The algorithm, written in MATLAB code, was terminated, that is the stopping criterion of step *(M3)* for $\epsilon = 0.01$ was satisfied in just several iterations of the algorithm; this took less than 10-15 seconds on a Sun SPARCStation 20.

4 Conclusions

The problem of control reconfiguration due to operating condition changes or abrupt system component failures has been studied here. An optimization algorithm has been presented that provides an output feedback controller that not only stabilizes the new/impaired system but also preserves much of the dynamics of the original/unfailed system. The design is such that the closed-loop system is robust with respect to uncertainties/modelling errors in the state-space model of the impaired system.

Although the interest here is in continuous-time systems, a similar approach can be applied to the discrete-time case, for which robust stability theorems for unstructured/structured perturbations from the literature can readily be used in the place of theorem 2.1 used for continuous systems here. Results concerning eigenstructure assignment, [24], can be included in our design so that additional restrictions/specifications can be introduced in the minimizing quantity with respect to the eigenspace where the closed-loop eigenvectors are desired/needed to vary. In that respect, the requirement of closeness to the original closed-loop system can be specialized to the case where the closeness to the eigenstructure of the original system becomes the main objective of the reconfiguration design.

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