ISIS Technical Report Draft: Event-Triggered/Self-Triggered Real-Time Scheduling For Stabilization Of Passive/Dissipative Systems

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ISIS Technical Report Draft: Event-Triggered/Self-Triggered Real-Time Scheduling For Stabilization Of Passive/Dissipative Systems

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I. Introduction

The majority of feedback control laws are nowadays implemented on digital platforms since microprocessors offer many advantages of running real-time operating systems. This creates the possibility of sharing the computational resources among control and other kinds of applications thus reducing the deployment costs of complex control systems[9]. Moreover, in distributed environment, control systems are often implemented through a shared communication media where controllers, sensors and actuators exchange data. However, since the executions of the controllers are traditionally implemented in a periodic fashion where the inter-execution time T is a constant units of time which is selected based on a worst-case scenario to guarantee the performance of the actuator for all possible operation points, thus the control task is executed at the same rate regardless of the states of the plant and leads to inefficient implementations in terms of processor usage or available communication bandwidth.

To overcome the drawbacks of the periodic paradigm, several researchers suggested the idea of event-triggered control, see [3]-[4] and [6]. In event-triggered real-time scheduling algorithm, the control tasks are executed whenever a certain error becomes large when compared with the states' norm of the plant (so the triggering condition is based on the full-state of the plant). The event-triggered technique reduces resource usage and provides a high degree of robustness (since the plant is measured continuously). Unfortunately, in many case it requires dedicated hardware to monitor the plant permanently otherwise one might run the risk of consuming the processor time.

Self-triggered real-time scheduling strategy is studied in [5],[7],[8],[9] and it takes the advantage of the event-triggered technique without resorting to extra hardware. The key idea of self-triggered control is to compute the next instants of time at which the control action is to be recomputed based on the current or last state measurements of the plant. A first attempt to explore self-triggered paradigm for linear systems was developed in [5], by discretizing the plant, and in [2] for linear $\mathcal{H}_\infty$ controllers. A study on self-triggered scheduling for nonlinear dynamic systems is shown in [7] and [9], where a simple self-trigger condition based on the norm of the current states is proposed by exploiting the properties of the trajectories of homogeneous control systems.

In some cases, we can stabilize the control systems directly through negative output feedback if some observability or detectability conditions are satisfied, i.e., passive systems and dissipative systems, and for these systems, it is often not necessary to design stabilizing control laws based on the measurement of the full states. In this report, we derive the event-triggered and self-triggered real-time scheduling strategy for stabilization of passive/dissipative systems based on output feedback. In our work, the system under study needs to satisfy certain passive or dissipative equalities, and it has to be zero-state detectable, and notice that this can often be achieved if we select a proper output. The event-triggering and self-triggering conditions derived in the current work can be applied to both linear or nonlinear passive/dissipative systems. This might be the first work to study self-triggering scheduling for linear and nonlinear dynamic systems under the same framework, and also this might be the first work to propose event-triggering and self-triggering real-time scheduling strategy based on output feedback stabilization control action.

The rest of this report is organized as follows. We first set the notions and introduce some background on passive/dissipative systems in section II; the problem statement is made in section III; the main results of event-triggered real-time scheduling strategy are provided in section IV and followed by the examples provided in section V; an alternative event-triggered real-time scheduling strategy is shown in section VI, and based on this alternative strategy, we derive our proposed self-triggered real-time scheduling strategy which is given in section VII; also we provide examples to illustrate our self-triggering strategy in section VIII. Finally, conclusion is made in section IX and discussion on possible future work is also included.

II. Notations and Background Material

To set the background and notation for what follows, we need to introduce some basic concepts on passive and dissipative systems.

Consider the following nonlinear system:

$$H : \begin{cases} \dot{x} = f(x,u) \\ y = h(x,u) \end{cases} \tag{1}$$

where $x \in X \subset \mathbb{R}^n$, $u \in U \subset \mathbb{R}^m$ and $y \in Y \subset \mathbb{R}^m$ are the state, input and output variables, respectively, and X, U and Y are the state, input and output spaces, respectively. The representation $x(t) = \phi(t, t_0, x_0, u)$ is used to denote the state at time t reached from the initial state x_0 at t_0.

Definition 1(Supply Rate [1]). *The supply rate $\omega(t) = \omega(u(t), y(t))$ is a real valued function defined on $U \times Y$, such that for any $u(t) \in U$ and $x_0 \in X$ and $y(t) = h(\phi(t, t_0, x_0, u))$, $\omega(t)$ satisfies*

$$\int_{t_0}^{t_1} |\omega(\tau)| d\tau < \infty \tag{2}$$

Definition 2(Dissipative System [1]). *System H with supply rate $\omega(t)$ is said to be dissipative if there exists a nonnegative real function $V(x) : X \to \mathbb{R}^+$, called the storage function, such that, for all $t_1 \geq t_0 \geq 0$, $x_0 \in X$ and $u \in U$,*

$$V(x_1) - V(x_0) \leq \int_{t_0}^{t_1} \omega(\tau) d\tau \tag{3}$$

where $x_1 = \phi(t_1, t_0, x_0, u)$ and $\mathbb{R}^+$ is a set of nonnegative real numbers.

Definition 3(Passive System [1]). *The dynamic system given in (1) is said to be **passive** if there exists a $\mathcal{C}^1$ storage function $V(x) \geq 0$ such that*

$$\dot{V} = \frac{\partial V(x)}{\partial x} f(x(t), u(t)) \leq -S(x) + u(t)^T y(t) \ \forall t \tag{4}$$

*for some positive semi-definite function $S(x)$. We say it is **strictly passive** if $S(x) > 0$.*

Definition 4(Excess/Shortage of Passivity [2]). *System H is said to be:*

- *Input Feed-forward Passive (IFP) if it is dissipative with respect to supply rate $\omega(u, y) = u^T y - \nu u^T u$ for some $\nu \in \mathbb{R}$, denoted as IFP(ν).*
- *Output Feedback Passive (OFP) if it is dissipative with respect to the supply rate $\omega(u, y) = u^T y - \rho y^T y$ for some $\rho \in \mathbb{R}$, denoted as OFP(ρ).*
- *Input Feed-forward Output Feedback Passive (IF-OFP) if it is dissipative with respect to the supply rate $\omega(u, y) = u^T y - \rho y^T y - \nu u^T u$ for some $\rho \in \mathbb{R}$ and $\nu \in \mathbb{R}$, denoted as IF-OFP(ν, ρ).*

A positive ν or ρ means that the system has an excess of passivity; otherwise, the system is lack of passivity.

Definition 5(Zero-State Observability and Detectability [2]). *Consider the system H with zero input, that is $\dot{x} = f(x, 0)$, $y = h(x, 0)$, and let $Z \subset \mathbb{R}^n$ be its largest positively invariant set contained in $\{x \in \mathbb{R}^n | y = h(x, 0) = 0\}$. We say H is zero-state detectable(ZSD) if $x = 0$ is asymptotically stable conditionally to Z. if $Z = \{0\}$, we say that H is zero-state observable (ZSO).*

III. Problem Statement

We consider a nonlinear control system H given by

$$H : \begin{cases} \dot{x} = f(x, u) \\ y = h(x) \end{cases} \tag{5}$$

where $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}^m$ is the control input and $y \in \mathbb{R}^m$ is the output. We assume that H is a passive system, which means that there exists a nonnegative storage function $V(x) : \mathbb{R}^n \to \mathbb{R}^+$, such that

$$\dot{V}(x) \leq u^T y. \tag{6}$$

We know that if H is zero-state detectable(ZSD), then under the feedback control law

$$u = -Ky \tag{7}$$

where $K > 0$ could be a scalar or an $m \times m$ positive definite matrix, the origin of H is asymptotically stable.

In real time, the implementation of the feedback control law (7) on an embedded processor is typically done by sampling the output y at time instants

$$t_0,\ t_1,\ t_2,\ t_3,\ t_4,\ \ldots,$$

computing the control action $u(t_i) = -Ky(t_i)$ and updating the actuator at time instants

$$t_0 + \triangle,\ t_1 + \triangle,\ t_2 + \triangle,\ t_3 + \triangle,\ t_4 + \triangle,\ \ldots,$$

where $\triangle \geq 0$ represents the time required to read the output from the sensor, compute the control action and update the actuators. This means that a sequence of measurements

$$y(t_0),\ y(t_1),\ y(t_2),\ y(t_3),\ y(t_4),\ \ldots,$$

corresponds to a sequence of actuation updates

$$u(t_0 + \triangle),\ u(t_1 + \triangle),\ u(t_2 + \triangle),\ u(t_3 + \triangle),\ u(t_4 + \triangle),\ \ldots,$$

as shown in Figure 1.

Between actuator updates, the control law u is held constant according to

$$t \in [t_i + \triangle, t_{i+1} + \triangle] \Rightarrow u(t) = u(t_i + \triangle). \tag{8}$$

Furthermore, the sequence of times $t_0, t_1, t_2, t_3, \ldots$ is typically periodic, which means that $t_{i+1} - t_i = T$, where $T > 0$ is the sampling period; in this case, we can regard the execution of the output feedback

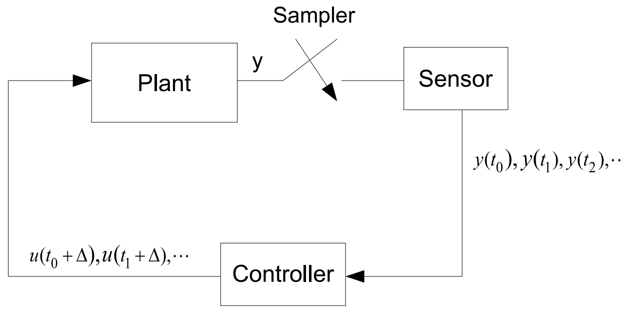

Fig. 1: Implementation of the feedback law in real time

control law (7) as being "time-triggered". In this report, we first consider "event-triggered"executions where the sequence $t_0, t_1, t_2, t_3, \ldots$ of execution times is neither periodic nor specified in advance but rather implicitly defined by an execution rule based on the output of the plant.

To introduce this execution rule, we define the measurement error at the actuator to be

$$t \in [t_i + \triangle, t_{i+1} + \triangle] \Rightarrow e(t) = y(t) - y(t_i), \tag{9}$$

so

$$\dot{x} = f(x, -Ky(t_i)) = f(x, -K(y(t) - e(t))),\ t \in [t_i + \triangle, t_{i+1} + \triangle]. \tag{10}$$

Let us first consider the hypothetical case $\triangle = 0$. Since system H is passive, with (6),(7) and (9) we can obtain

$$\begin{aligned} \dot{V}(x) &\leq u^T y = -K(y-e)^T y = -Ky^T y + Ke^T y \\ &\leq K\|e\|_2 \|y\|_2 - K\|y\|_2^2 \end{aligned} \tag{11}$$

so if

$$\|e\|_2 \leq \|y\|_2, \tag{12}$$

then we will have $\dot{V}(x) \leq 0$, and asymptotical stability of the origin follows from the assumption that the system H is ZSD. Inequality (12) can be enforced by executing the control task when

$$\|e\|_2 = \sigma \|y\|_2,\ with\ 0 < \sigma < 1. \tag{13}$$

Since $\triangle = 0$ implies that if the control task is executed at time t_i, we will have $e(t) = y(t_i) - y(t_i) = 0$ and $\|e(t_i)\|_2 = 0$ thus enforcing the inequality (12). When $\triangle > 0$, the control task needs to be executed before the equality (13) is satisfied in order to account for the delay $\triangle$ between measuring the output and updating the actuators.

Although the output feedback control (7) and the execution rule (13) guarantee that global asymptotical stability will be achieved as long as the control system H is passive and ZSD, there are three important questions as mentioned in [3], which are needed to be answered in order to assess the feasibility of this scheduling policy:

1) Since the execution times are implicitly defined, can we guarantee that they will not become arbitrarily close and result in an accumulation-point?
2) In the absence of accumulation points, can we compute an estimate of the time elapsed between consecutive executions of the control task?
3) How can we use the execution rule (13) when there are more tasks competing for processor time and still guarantee that no deadlines are missed?

IV. Event-Triggered Real-Time Scheduling For Stabilization Of Passive/Dissipative Systems

Theorem 1. *Consider the control system given by*

$$H : \begin{cases} \dot{x} = f(x, u) \\ y = h(x) \end{cases} \tag{14}$$

where $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}^m$ is the control input and $y \in \mathbb{R}^m$ is the output; H is a passive system satisfying the passive inequality given by

$$\dot{V}(x) \leq u^T y \tag{15}$$

with $V(x) \geq 0$. If the following assumptions are satisfied

1) *$f : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$ is Lipschitz continuous on compacts;*
2) *$h : \mathbb{R}^n \to \mathbb{R}^m$ is a static nonlinear function of x which belongs to a sector $[\alpha, \beta]$ such that $\alpha x^T x \leq x^T h(x) \leq \beta x^T x$, where $\alpha\beta > 0$;*
3) *$\|\frac{\partial h(x)}{\partial x}\|_2 \leq \gamma$, where $0 < \gamma < \infty$;*
4) *system H is ZSD;*

then for any compact set $S \subseteq \mathbb{R}^n$ containing the origin, there exists an $\varepsilon > 0$, such that for all response time $\triangle \in [0, \varepsilon]$, there exists a time $\tau \in \mathbb{R}^+$ such that for any initial condition in S the inter-execution times $\{t_{i+1} - t_i\}_{i \in \mathbb{N}}$ implicitly defined by the execution rule (13) based on the control action $u(t) = -Ky(t_i)(K > 0)$ are lower bounded by τ, that is $t_{i+1} - t_i \geq \tau$ for any $i \in \mathbb{N}$.

Proof. From Lipschitz continuity on compacts of $f(x,u)$, we can conclude that $f(x,-K(y-e))$ is also Lipschitz continuous on compacts, that is

$$\begin{aligned} \|f(x,-K(y-e))-f(0,0)\|_2 &\le L\|(x,(y-e))\|_2 \le L\|x\|_2 + L\|y-e\|_2 \\ &\le L\|x\|_2 + L\|y\|_2 + L\|e\|_2. \end{aligned} \tag{16}$$

Let us bound the inter-execution times by looking at the dynamics of $\frac{\|e\|_2}{\|y\|_2}$,

$$\begin{aligned} \frac{d\|e\|_2}{dt\|y\|_2} &= \frac{d(e^Te)^{\frac{1}{2}}}{dt(y^Ty)^{\frac{1}{2}}} \\ &= \frac{(e^Te)^{-\frac{1}{2}}e^T\dot{e}(y^Ty)^{\frac{1}{2}} - (y^Ty)^{-\frac{1}{2}}y^T\dot{y}(e^Te)^{\frac{1}{2}}}{y^Ty} \\ &= \frac{e^T\dot{e}}{\|e\|_2\|y\|_2} - \frac{y^T\dot{y}}{\|y\|_2\|y\|_2}\frac{\|e\|_2}{\|y\|_2} \end{aligned} \tag{17}$$

Since $e(t) = y(t) - y(t_i)$ and $y(t_i)$ is constant for $t \in [t_i + \triangle, t_{i+1} + \triangle), \forall i$, we have $\dot{e}(t) = \dot{y}(t)$. So for $t \in [t_i + \triangle, t_{i+1} + \triangle)$, we can obtain,

$$\begin{aligned} \frac{d\|e\|_2}{dt\|y\|_2} &= \frac{e^T\dot{y}}{\|e\|_2\|y\|_2} - \frac{y^T\dot{y}}{\|y\|_2\|y\|_2}\frac{\|e\|_2}{\|y\|_2} \\ &\le \frac{\|e\|_2\|\dot{y}\|_2}{\|e\|_2\|y\|_2} + \frac{\|y\|_2\|\dot{y}\|_2\|e\|_2}{\|y\|_2\|y\|_2\|y\|_2} \\ &= (1 + \frac{\|e\|_2}{\|y\|_2})\frac{\|\dot{y}\|_2}{\|y\|_2} \end{aligned} \tag{18}$$

since

$$\begin{aligned} \|\dot{y}\|_2 = \|\frac{\partial h(x)}{x}\dot{x}\|_2 &\le \|\frac{\partial h(x)}{x}\|_2\|\dot{x}\|_2 \\ &\le \gamma L\big(\|x\|_2 + \|y\|_2 + \|e\|_2\big), \end{aligned} \tag{19}$$

and moreover, since $y = h(x)$ is static nonlinearity belongs to a sector $[\alpha, \beta]$ such that $\alpha x^T x \le x^T h(x) \le \beta x^T x$, where $\alpha\beta > 0$, we can show that

$$\frac{\|x\|_2}{\|y\|_2} \le \max\{\frac{1}{|\alpha|}, \frac{1}{|\beta|}\} = \zeta. \tag{20}$$

(Notice that $\|x\|_2$ and $\|y\|_2$ are never zero since the closed-loop system converges to zero asymptotically and thus they never reache zero in finite time).

Now (18) will be bounded by

$$\begin{aligned} \frac{d\|e\|_2}{dt\|y\|_2} &\le \big(1 + \frac{\|e\|_2}{\|y\|_2}\big)\frac{\|\dot{y}\|_2}{\|y\|_2} \\ &\le \gamma L\big(1 + \frac{\|e\|_2}{\|y\|_2}\big)\big(\frac{\|x\|_2}{\|y\|_2} + \frac{\|e\|_2}{\|y\|_2} + 1\big) \\ &\le \gamma L\big(1 + \frac{\|e\|_2}{\|y\|_2}\big)\big(1 + \zeta + \frac{\|e\|_2}{\|y\|_2}\big), \end{aligned} \tag{21}$$

if we denote $\frac{\|e\|_2}{\|y\|_2}$ by p, we will have

$$\dot{p} \leq \gamma L(1+p)(1+\zeta+p) \tag{22}$$

consider the differential equation given by

$$\dot{\phi} = \gamma L(1+\phi)(1+\zeta+\phi), \tag{23}$$

and let $\phi(t,\phi_0)$ be the solution defined at time t with the initial condition ϕ_0. We can see that (23) is a non-negative system, because for any initial condition $\phi_0 \geq 0$, we have $\dot{\phi} > 0$ and thus $\phi(t,\phi_0) > 0, \forall t$. So we can conclude that for any initial condition $p_0 = \phi_0 \geq 0$, we can have $p(t) \leq \phi(t,\phi_0)$.

Now let us first assume that $\triangle = 0$. The inter-execution times are bounded by the time it takes for ϕ to evolve from 0 to σ, where σ is defined in (13). So an estimate of the inter-execution time could be obtained by the solution $\tau \in \mathbb{R}^+$ of $\phi(\tau,0) = \sigma$. The solution to (23) with $\sigma_0 = 0$ at any time t is given by

$$\phi(t,0) = \frac{1-e^{\gamma L\zeta t}}{\frac{1}{1+\zeta}e^{\gamma L\zeta t}-1}, \tag{24}$$

let $\phi(\tau,0) = \sigma$, we can get

$$\tau = -\big(\ln\frac{1}{1+\zeta} - \ln\frac{1+\sigma}{1+\zeta+\sigma}\big)\frac{1}{\gamma L\zeta}, \tag{25}$$

and it is easy to verify that $\tau > 0$ for any $0 < \sigma < 1$.

For $\triangle > 0$, we need a more detailed analysis. First, pick σ' which satisfies $0 < \sigma < \sigma' < 1$, and let $\varepsilon_1 \in \mathbb{R}^+$ satisfy $\phi(\varepsilon_1,\sigma) = \sigma'$. Notice that ε_1 always exists since ϕ is continuous and $\dot{\phi} > 0, \forall t$ if $\phi_0 \geq 0$. Then by executing the control task at time t_*, which is denoted by $\|e(t_*)\|_2 = \sigma\|y(t_*)\|_2$, we can guarantee that for $t \in [t_*, t_*+\varepsilon_1]$, we have $\|e(t)\|_2 \leq \sigma'\|y(t)\|_2$, and since $0 < \sigma' < 1$, asymptotic stability of the origin is still guaranteed if system H is ZSD. The inter-execution times are now bounded by $\triangle + \tau$ where τ is the time it takes for ϕ to evolve from $\frac{\|e(t_i+\triangle)\|_2}{\|y(t_i+\triangle)\|_2} = \frac{\|y(t_i+\triangle)-y(t_i)\|_2}{\|y(t_i+\triangle)\|_2}$ to σ. We need to pick $\triangle$ small enough so that $\frac{\|e(t_i+\triangle)\|_2}{\|y(t_i+\triangle)\|_2} < \sigma$. It follows from the continuity of $\frac{\|y(t_i+\triangle)-y(t_i)\|_2}{\|y(t_i+\triangle)\|_2}$ with respect to $\triangle$ that there exists $\varepsilon_2 > 0$ such that for any $0 \leq \triangle \leq \varepsilon_2$, we have $\frac{\|y(t_i+\triangle)-y(t_i)\|_2}{\|y(t_i+\triangle)\|_2} < \sigma$. The proof is completed by choosing $\varepsilon = \min\{\varepsilon_1,\varepsilon_2\}$. ■

Remark 1: From the above analysis, we can see that the lower bound of τ is nontrivial for any $\sigma > 0$, which means that the inter-execution times $\{t_{i+1}-t_i\}_{i\in\mathbb{N}}$ can never be zero. ■

Remark 2: For linear system case, consider a linear passive system given by:

$$H : \begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases} \tag{26}$$

then we have

$$\begin{aligned}\|\dot{y}\|_2 &= \|C\dot{x}\|_2 = \|C[Ax+Bu]\|_2 \\ &= \|CAx - CBK(y-e)\|_2 = \|CAx - CBKy + CBKe\|_2,\end{aligned} \tag{27}$$

$$\begin{aligned}\Rightarrow \frac{\|\dot{y}\|_2}{\|y\|_2} &= \frac{\|CAx - CBKy + CBKe\|_2}{\|y\|_2} \\ &\leq \frac{\|CAx\|_2}{\|y\|_2} + \frac{\|CBKy\|_2}{\|y\|_2} + \frac{\|CBKe\|_2}{\|y\|_2} \\ &\leq \frac{\|CAx\|_2}{\|y\|_2} + \frac{\|CBK\|_2\|y\|_2}{\|y\|_2} + \frac{\|CBK\|_2\|e\|_2}{\|y\|_2},\end{aligned} \tag{28}$$

since $\frac{\|CAx\|_2}{\|y\|_2} = \big(\frac{x^TA^TC^TCAx}{x^TC^TCx}\big)^{\frac{1}{2}}$, if $\big(\frac{x^TA^TC^TCAx}{x^TC^TCx}\big)^{\frac{1}{2}}$ is well bounded, such that

$$\big(\frac{x^TA^TC^TCAx}{x^TC^TCx}\big)^{\frac{1}{2}} \leq \alpha \tag{29}$$

where $0 < \alpha < \infty$, then we have

$$\begin{aligned}\frac{\|\dot{y}\|_2}{\|y\|_2} &\leq \alpha + \frac{\|CBK\|_2\|y\|_2}{\|y\|_2} + \frac{\|CBK\|_2\|e\|_2}{\|y\|_2} \\ &\leq \|C\|_2\|BK\|_2\big(\frac{\alpha}{\|C\|_2\|BK\|_2} + 1 + \frac{\|e\|_2}{\|y\|_2}\big),\end{aligned} \tag{30}$$

in this case (21) becomes

$$\begin{aligned}\frac{d\|e\|_2}{dt\|y\|_2} &\leq \big(1 + \frac{\|e\|_2}{\|y\|_2}\big)\frac{\|\dot{y}\|_2}{\|y\|_2} \\ &\leq \|C\|_2\|BK\|_2\big(1 + \frac{\|e\|_2}{\|y\|_2}\big)\big(1 + \frac{\alpha}{\|C\|_2\|BK\|_2} + \frac{\|e\|_2}{\|y\|_2}\big).\end{aligned} \tag{31}$$

As shown in the proof of Theorem 1, the evolution of $\frac{\|e\|_2}{\|y\|_2}$ is bounded by the solution of (23), and here we have

$$\gamma = \|C\|_2,\ L = \|BK\|_2,\ \zeta = \frac{\alpha}{\|C\|_2\|BK\|_2}, \tag{32}$$

so again, we can get non-trivial inter-execution times following the analysis shown in Theorem 1. ■

Remark 3: Assumption 2) in Theorem 1 is rather conservative since we restrict output $y = h(x)$ of system H to belong to a bounded sector. But most of the time, this condition can be relaxed as long as

$$\frac{\|\dot{y}\|_2}{\|y\|_2} \leq C_1(C_2 + \frac{\|e\|_2}{\|y\|_2}) \tag{33}$$

is satisfied for some constant C_1, C_2, where $0 < C_1 < \infty$ and $0 \leq C_2 < \infty$, and we will see this in the examples provided in the next section. ■

Remark 4: If we assume that $e(t_0) = 0$, then by choosing $0 < \sigma_{t_i+\triangle} < \sigma < \sigma' < 1$, we can let

$$\varepsilon_1 = \frac{1}{\zeta\gamma L}\ln\big[\frac{(1+\sigma')(1+\zeta+\sigma)}{(1+\sigma'+\zeta)(1+\sigma)}\big], \tag{34}$$

$$\varepsilon_2 = \frac{1}{\zeta\gamma L}\ln\big[\frac{(1+\sigma_{t_i+\triangle})(1+\zeta)}{1+\sigma_{t_i+\triangle}+\zeta}\big], \tag{35}$$

$$\tau = \frac{1}{\zeta\gamma L}\ln\big[\frac{(1+\sigma)(1+\zeta+\sigma_{t_i+\triangle})}{(1+\sigma+\zeta)(1+\sigma_{t_i+\triangle})}\big], \tag{36}$$

where ε_1 is the time it takes for ϕ to evolve from σ to $\sigma^{'}$, ε_2 is the time it takes for ϕ to evolve from 0 to $\sigma_{t_i+\triangle}$, and τ is the time it takes for ϕ to evolve from $\sigma_{t_i+\triangle}$ to σ, ϕ is the solution of (23). Then an estimate for the upper bound of the delay $\triangle$ is $\varepsilon = \min\{\varepsilon_1, \varepsilon_2\}$, and $\triangle \in [0, \varepsilon]$. We need to choose the value of $\sigma_{t_i+\triangle}, \sigma, \sigma^{'}$ properly so that we can get a longer inter-execution time $\tau + \triangle$ (which means we can update the output feedback control action less frequently) while the performance of the control systems(settling time, overshoot, etc.) can still be satisfied. ■

Remark 5: It can be shown that similar real-time scheduling approach can be applied to stabilization of dissipative systems. Again consider the system H given by (5), and we assume that H satisfies the dissipative inequality given by

$$\dot{V}(x) \leq u^T y + \rho y^T y \tag{37}$$

with $V(x) \geq 0$ and $\rho > 0$, ρ is the smallest positive constant such that the above dissipative inequality holds. In this case, system H is non-passive and unstable, and by applying negative output feedback $u = -Ky$, where $K > \rho > 0$, we can stabilize the dissipative system if it is ZSD, and it is easy to show that the execution threshold is determined by:

$$\|e\|_2 \leq \|K^{-1}(K - \rho I_{m\times m})\|_2 \|y\|_2, \tag{38}$$

we need to choose the feedback gain $K > \rho I_{m\times m}$ and the execution signal σ in (13) to satisfy $0 < \sigma \leq \|K^{-1}(K - \rho I_{m\times m})\|_2$. ■

V. Examples For Event-Triggered Scheduling

Example 1. Consider the passive linear system given by

$$\begin{aligned}\dot{x}_1(t) &= -5x_1(t) - x_2(t)\\ \dot{x}_2(t) &= -x_2(t) + u(t)\\ y(t) &= x_2(t)\end{aligned} \tag{39}$$

we can see that the system is stable and ZSD. If we choose the storage function $V(x) = \frac{1}{2}x_2^2$, we can verify that the system is passive. And with

$$A = \begin{bmatrix} -5 & -1 \\ 0 & -1 \end{bmatrix},\ B = \begin{bmatrix} 0 \\ 1 \end{bmatrix},\ C = \begin{bmatrix} 0 & 1 \end{bmatrix}, \tag{40}$$

we can see that $\frac{\|CAx\|_2}{\|y\|_2} = \big(\frac{x^T A^T C^T CAx}{x^T C^T Cx}\big)^{\frac{1}{2}} = 1$, so in this case (27) becomes

$$\begin{aligned}\frac{d\|e\|_2}{dt\|y\|_2} &\leq \big(1 + \frac{\|e\|_2}{\|y\|_2}\big)\frac{\|\dot{y}\|_2}{\|y\|_2}\\ &\leq \|C\|_2\|BK\|_2\big(1 + \frac{\|e\|_2}{\|y\|_2}\big)\big(1 + \frac{1}{\|C\|_2\|BK\|_2} + \frac{\|e\|_2}{\|y\|_2}\big).\end{aligned} \tag{41}$$

By choosing $K = 2$, $\triangle = 0s$, $\sigma = 0.9$, we get an estimate of the lower bound of the inter-execution time based on (25), and we get $\tau = 0.1351s$. The total number of times we need to update the actuator in $2s$ is 12. The simulation results compared with "time-triggered"execution (a periodic sampling time $0.001s$ and no response delay of the actuator) are shown in Fig2.-Fig5. If we apply the execution law (13) with $\sigma = 0.8$ and $\sigma^{'} = 0.9$ and $\triangle = 0.0095s$, the simulation results are shown in Fig6.-Fig.9. The total number of times we need to update the actuator in $2s$ is 13. So compared with "time-triggered"execution where we need to update the sensor 2000 times in $2s$, the "event-triggered "execution significantly reduce the frequency we need to update the actuator.

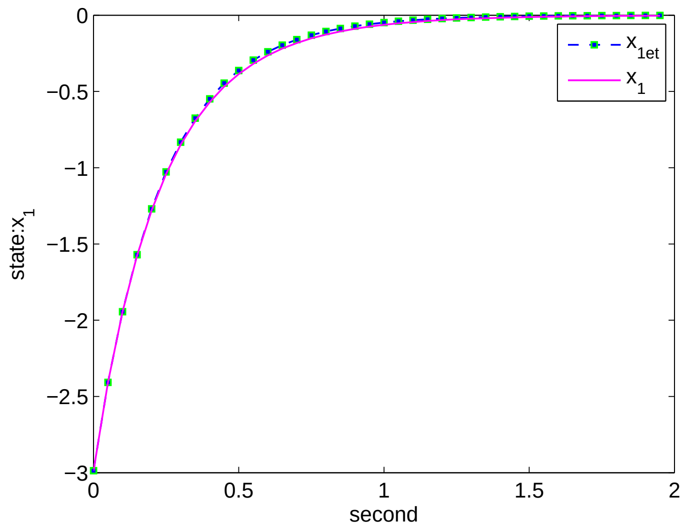

Fig. 2: x_{1et} represents the evolution of state x_1 with execution signal $\sigma = 0.9$ and no execution delay of the actuator $\triangle = 0s$; x_1 represents the evolution of state x_1 with periodic sampling $0.001s$ and no execution delay.

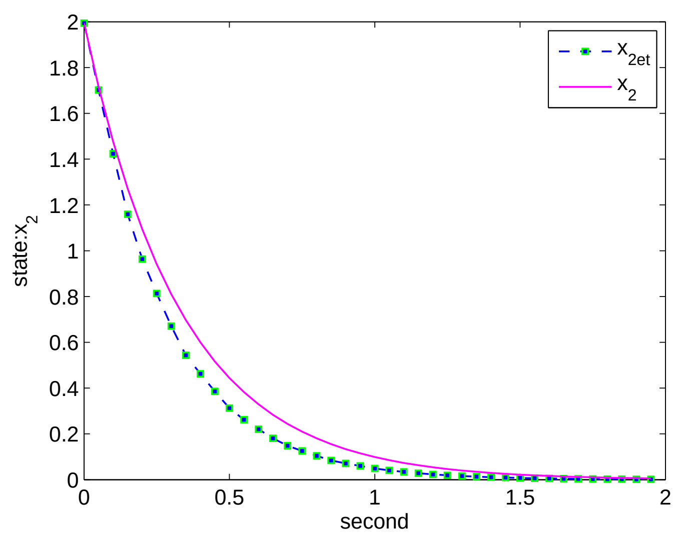

Fig. 3: x_{2et} represents the evolution of state x_2 with execution signal $\sigma = 0.9$ and no execution delay of the actuator $\triangle = 0s$; x_2 represents the evolution of state x_2 with periodic sampling $0.001s$ and no execution delay.

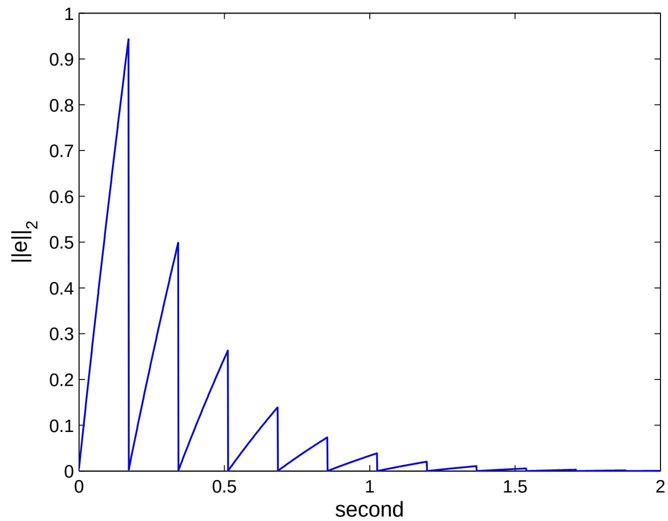

Fig. 4: evolution of the norm of error($\|e(t)\|_2$) between the actual output of the plant and the sampling value

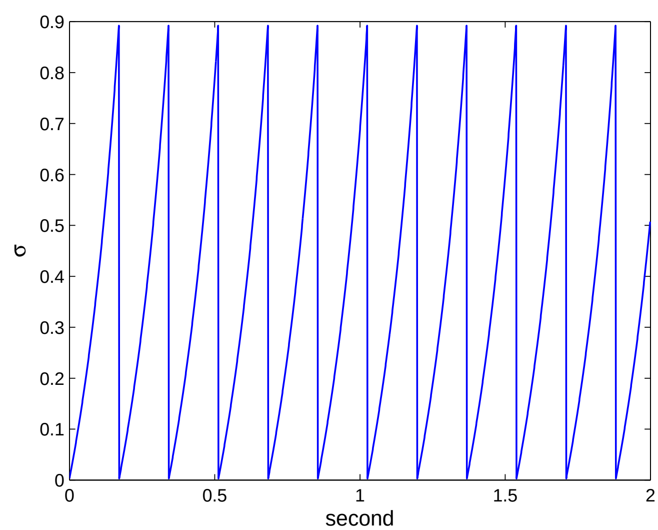

Fig. 5: evolution of $\sigma = \frac{\|e\|_2}{\|y\|_2}$

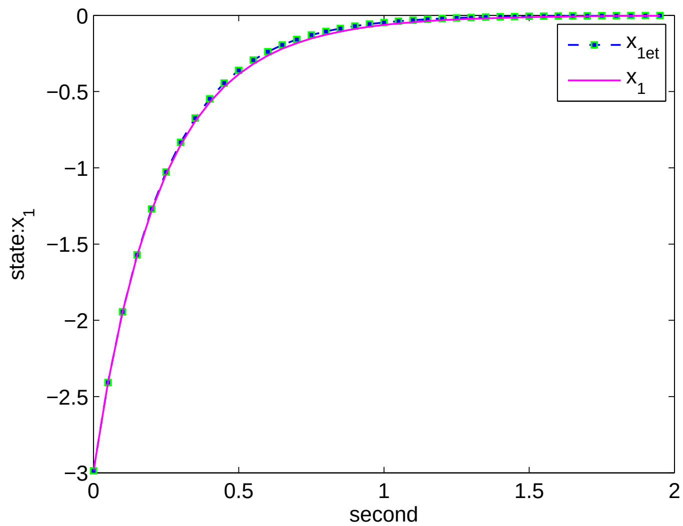

Fig. 6: x_{1et} represents the evolution of state x_1 with execution signal $\sigma = 0.8$ and $\sigma^{'} = 0.9$ and execution delay of the actuator $\triangle = 0.0095s$; x_1 represents the evolution of state x_1 with periodic sampling $0.001s$ and no execution delay.

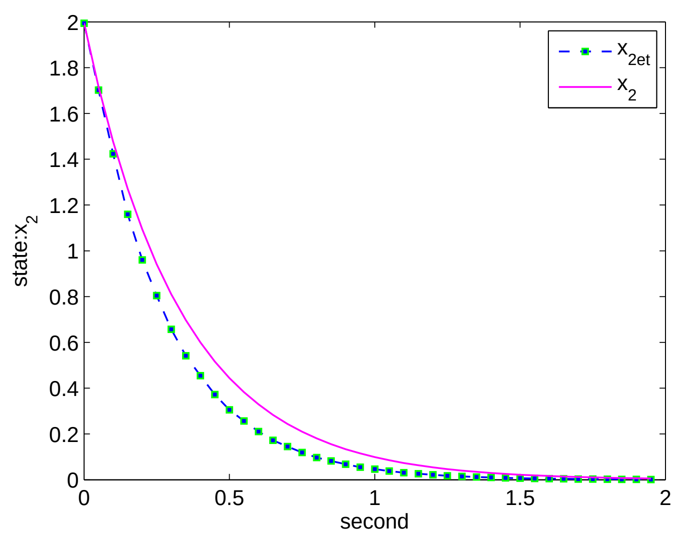

Fig. 7: x_{2et} represents the evolution of state x_2 with execution signal $\sigma = 0.8$ and $\sigma^{'} = 0.9$ and execution delay of the actuator $\triangle = 0.0095s$; x_2 represents the evolution of state x_2 with periodic sampling $0.001s$ and no execution delay.

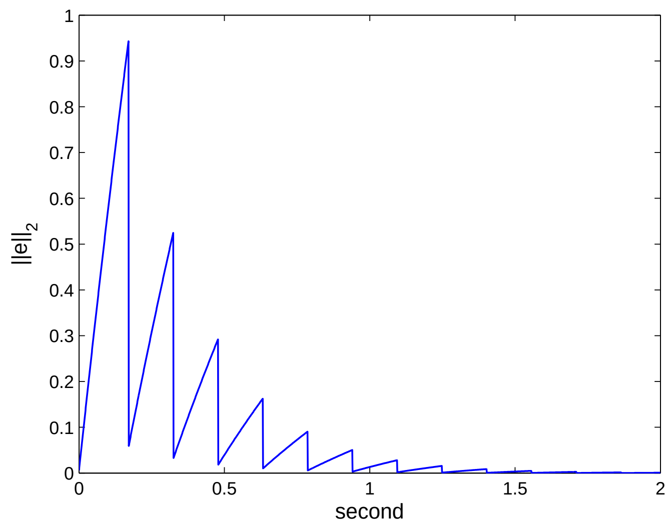

Fig. 8: evolution of the norm of error($\|e(t)\|_2$) between the actual output of the plant and the sampling value

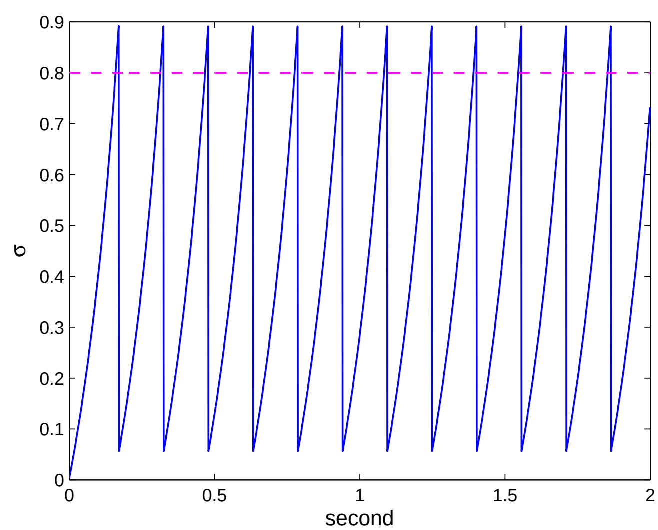

Fig. 9: evolution of $\sigma = \frac{\|e\|_2}{\|y\|_2}$

Example 2. Consider the dissipative system given by

$$\begin{aligned}\dot{x}_1(t) &= -3x_1^3(t) + x_1(t)x_2(t)\\ \dot{x}_2(t) &= 3x_2(t) + 2u(t)\\ y(t) &= x_2(t)\end{aligned} \tag{42}$$

we can see that the system is ZSD but unstable. If we choose the storage function $V(x) = \frac{1}{4}x_2^2$, we can get

$$\dot{V}(x) = u(t)y(t) + 1.5y^2(t), \tag{43}$$

and in this case $\rho = 1.5$. Since the output $y = h(x)$ is only a linear function of x_2, so y does not belong to a bounded sector defined by $x = [x_1^T, x_2^T]^T$ such that $\alpha x^T x \leq x^T h(x) \leq \beta x^T x$ as required in Assumption 2) in Theorem 1. However, we have

$$\begin{aligned}\frac{\|\dot{y}\|_2}{\|y\|_2} = \frac{\|\dot{x}_2\|_2}{\|y\|_2} &= \frac{\|3x_2 - 3K(y-e)\|_2}{\|y\|_2}\\ &\leq 3\frac{\|x_2\|_2}{\|y\|_2} + 3K + 3K\frac{\|e\|_2}{\|y\|_2} = 3 + 3K + 3K\frac{\|e\|_2}{\|y\|_2},\end{aligned} \tag{44}$$

so we can still bound $\frac{\|e\|_2}{\|y\|_2}$ by

$$\begin{aligned}\frac{d\|e\|_2}{dt\|y\|_2} &\leq \big(1 + \frac{\|e\|_2}{\|y\|_2}\big)\frac{\|\dot{y}\|_2}{\|y\|_2}\\ &\leq \big(1 + \frac{\|e\|_2}{\|y\|_2}\big)\big(3 + 3K + 3K\frac{\|e\|_2}{\|y\|_2}\big)\\ &= 3K\big(1 + \frac{\|e\|_2}{\|y\|_2}\big)\big(1 + \frac{1}{K} + \frac{\|e\|_2}{\|y\|_2}\big).\end{aligned} \tag{45}$$

Choose $K = 3$ so that $K > \rho$, $K^{-1}(K - \rho) = 0.5$; choose $\sigma = 0.4$ and set $\triangle = 0s$, the estimate lower bound of τ is $0134s$, the total number of times we need to update the actuator in $2s$ is 25. The simulation results compared with "time-triggered "execution(a periodic sampling time $0.001s$ and no execution delay of the actuator) are shown in Fig10.-Fig13. If we apply the execution law (13) with $\sigma = 0.3$, $\sigma' = 0.4$, $\triangle = 0.0028s$, the simulation results are shown in Fig14.-Fig.17, the total number of times we need to update the actuator in $2s$ is 34. So again, compared with "time-triggered"execution where we need to update the sensor 2000 times in $2s$, the "event-triggered "execution significantly reduce the frequency we need to update the actuator.

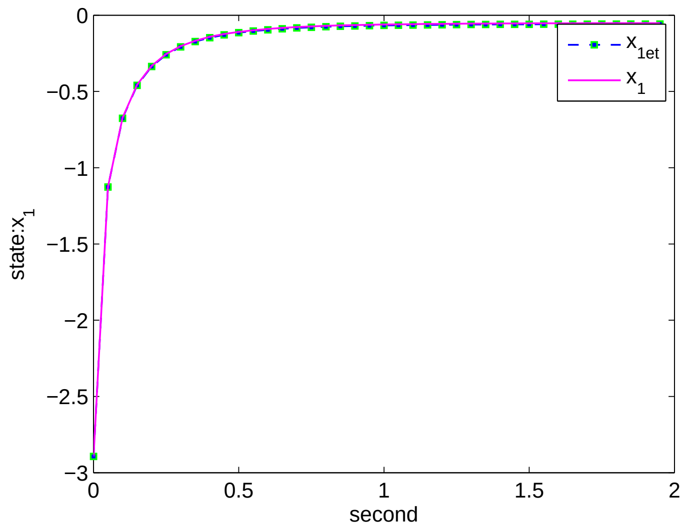

Fig. 10: x_{1et} represents the evolution of state x_1 with execution signal $\sigma = 0.4$ and no execution delay of the actuator $\triangle = 0s$; x_1 represents the evolution of state x_1 with periodic sampling $0.001s$ and no execution delay.

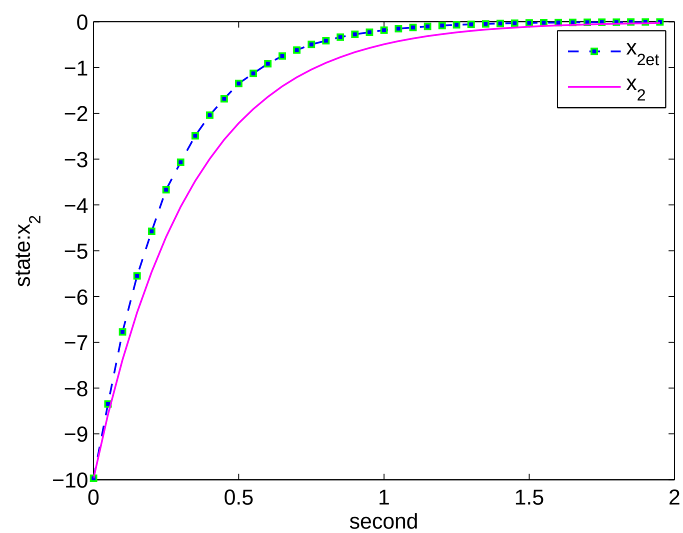

Fig. 11: x_{2et} represents the evolution of state x_1 with execution signal $\sigma = 0.4$ and no execution delay of the actuator $\triangle = 0s$; x_2 represents the evolution of state x_1 with periodic sampling $0.001s$ and no execution delay.

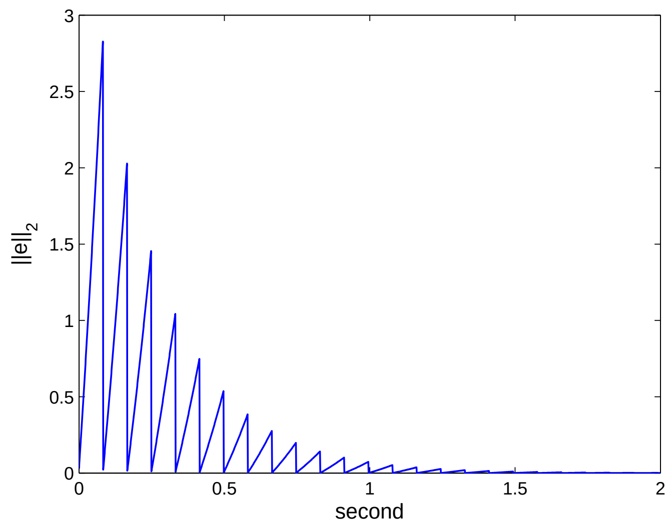

Fig. 12: evolution of the norm of error($\|e(t)\|_2$) between the actual output of the plant and the sampling value

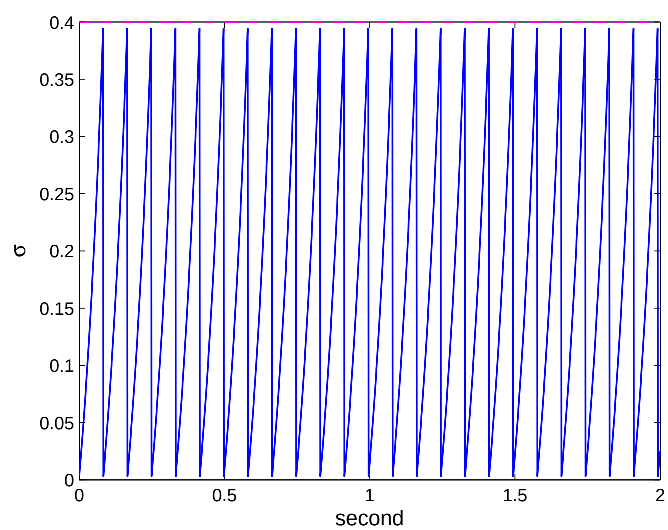

Fig. 13: evolution of $\sigma = \frac{\|e\|_2}{\|y\|_2}$

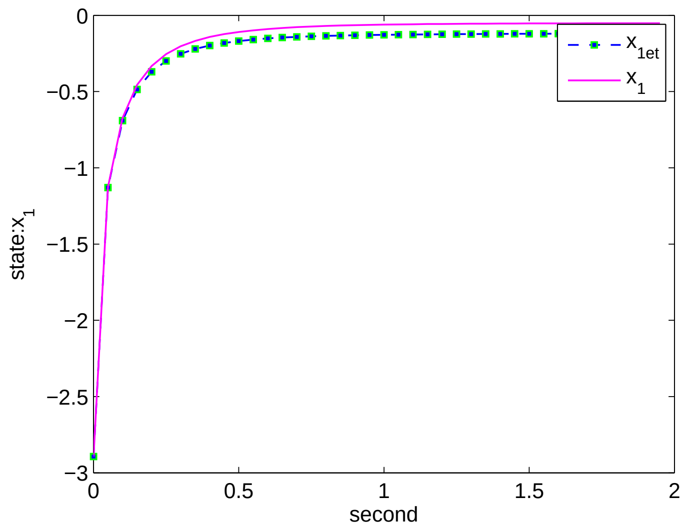

Fig. 14: x_{1et} represents the evolution of state x_1 with execution signal $\sigma = 0.3$ and $\sigma^{'} = 0.4$ and execution delay of the actuator $\triangle = 0.0028s$; x_1 represents the evolution of state x_1 with periodic sampling $0.001s$ and no execution delay.

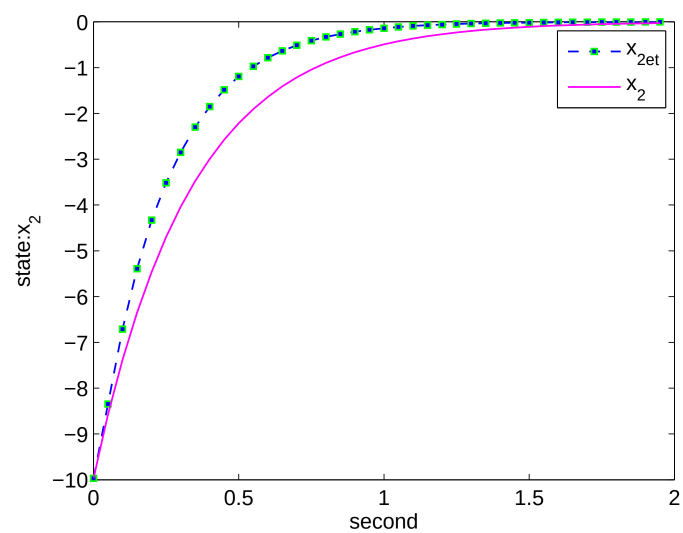

Fig. 15: x_{2et} represents the evolution of state x_2 with execution signal $\sigma = 0.3$ and $\sigma^{'} = 0.4$ and execution delay of the actuator $\triangle = 0.0028s$; x_2 represents the evolution of state x_1 with periodic sampling $0.001s$ and no execution delay.

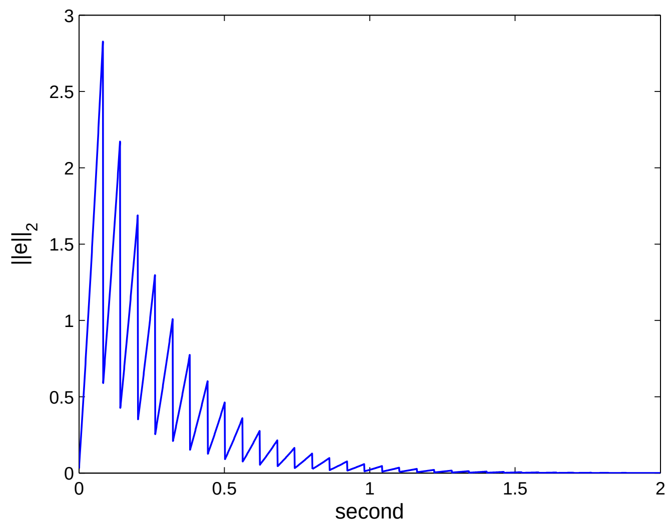

Fig. 16: evolution of the norm of error($\|e(t)\|_2$) between the actual output of the plant and the sampling value

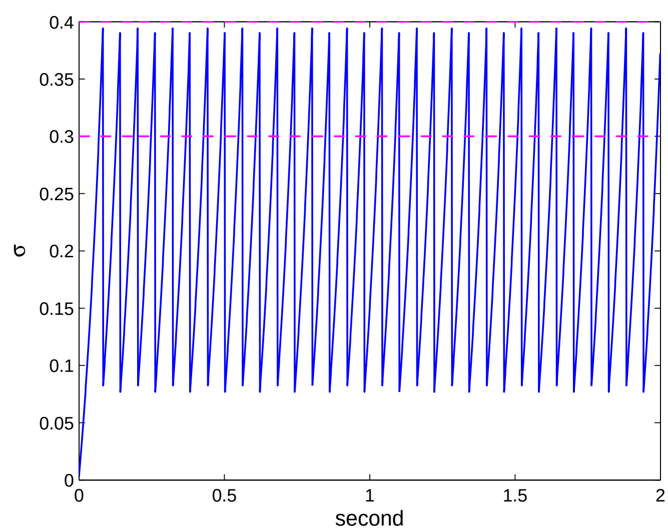

Fig. 17: evolution of $\sigma = \frac{\|e\|_2}{\|y\|_2}$

VI. An Alternative Event-Triggered Real Time Scheduling Strategy

In this section, we show an alternative event-triggered real time scheduling strategy in addition to the strategy discussed in the previous section.

In the previous event-triggered strategy, the threshold for updating feedback control action of the actuator is determined by

$$\|e(t)\|_2 \leq \|y(t)\|_2, \ \forall t \in [t_i + \triangle, t_{i+1} + \triangle). \tag{46}$$

Since $e(t) = y(t) - y(t_i)$, we have

$$\|e(t)\|_2 \geq \|y(t) - y(t_i)\|_2 \geq \|y(t_i)\|_2 - \|y(t)\|_2, \tag{47}$$

it is easy to find that a sufficient condition for (46) to be true is

$$\|e(t)\|_2 \leq \frac{1}{2}\|y(t_i)\|_2, \tag{48}$$

so an alternative execution law could be

$$\|e(t)\|_2 = \hat{\sigma}\|y(t_i)\|_2, \ where \ 0 < \hat{\sigma} < \frac{1}{2}. \tag{49}$$

Theorem 2. *Consider the control system given by*

$$H : \begin{cases} \dot{x} = f(x, u) \\ y = h(x) \end{cases} \tag{50}$$

where $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}^m$ is the control input and $y \in \mathbb{R}^m$ is the output; H is a passive system satisfying the passive inequality given by

$$\dot{V}(x) \leq u^T y \tag{51}$$

with $V(x) \geq 0$. If the following assumptions are satisfied

1) *$f : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$ is Lipschitz continuous on compacts;*
2) *$h : \mathbb{R}^n \to \mathbb{R}^m$ is a static nonlinear function of x which belongs to a sector $[\alpha, \beta]$ such that $\alpha x^T x \leq x^T h(x) \leq \beta x^T x$, where $\alpha\beta > 0$;*
3) *$\|\frac{\partial h(x)}{\partial x}\|_2 \leq \gamma$, where $0 < \gamma < \infty$;*
4) *system H is ZSD;*

then for any compact set $S \subseteq \mathbb{R}^n$ containing the origin, there exists an $\varepsilon > 0$, such that for all response time $\triangle \in [0, \varepsilon]$, there exists a time $\tau \in \mathbb{R}^+$ such that for any initial condition in S the inter-execution times $\{t_{i+1} - t_i\}_{i \in \mathbb{N}}$ implicitly defined by the execution rule (49) based on the control action

$u(t) = -Ky(t_i)(K > 0)$ *are lower bounded by* τ*, that is* $t_{i+1} - t_i \geq \tau$ *for any* $i \in \mathbb{N}$.

Proof. Since for $t \in [t_i + \triangle, t_{i+1} + \triangle]$ we have

$$\begin{aligned} \frac{d}{dt}\frac{\|e(t)\|_2}{\|y(t_i)\|_2} &\leq \frac{1}{\|y(t_i)\|_2}\|\dot{e}(t)\|_2 = \frac{1}{\|y(t_i)\|_2}\|\dot{y}(t)\|_2 \\ &= \frac{1}{\|y(t_i)\|_2}\|\frac{\partial y}{\partial x}\dot{x}\|_2 \leq \frac{1}{\|y(t_i)\|_2}\|\frac{\partial y}{\partial x}\|_2\|\dot{x}\|_2 \\ &\leq \frac{\gamma L}{\|y(t_i)\|_2}(\|x(t)\|_2 + \|y(t)\|_2 + \|e(t)\|_2), \end{aligned} \tag{52}$$

in view of (20), we have $\|x\|_2 \leq \zeta\|y\|_2$, where $\zeta = \max\{\frac{1}{|\alpha|}, \frac{1}{|\beta|}\}$, so

$$\frac{d}{dt}\frac{\|e(t)\|_2}{\|y(t_i)\|_2} \leq \frac{\gamma L}{\|y(t_i)\|_2}(\zeta\|y(t)\|_2 + \|y(t)\|_2 + \|e(t)\|_2). \tag{53}$$

Since $\|y(t)\|_2 = \|y(t_i) + e(t)\|_2 \leq \|y(t_i)\|_2 + \|e(t)\|_2$, we can obtain

$$\begin{aligned} \frac{d}{dt}\frac{\|e(t)\|_2}{\|y(t_i)\|_2} &\leq \frac{\gamma L}{\|y(t_i)\|_2}\big[(\zeta+2)\|e(t)\|_2 + (\zeta+1)\|y(t_i)\|_2\big] \\ &= \gamma L\big[(\zeta+2)\frac{\|e(t)\|_2}{\|y(t_i)\|_2} + (\zeta+1)\big]. \end{aligned} \tag{54}$$

Let $p = \frac{\|e(t)\|_2}{\|y(t_i)\|_2}$, then we have

$$\dot{p} \leq \gamma L(\zeta+2)p + \gamma L(\zeta+1). \tag{55}$$

Now, let us first consider the case $\triangle = 0$, and one can show that $p(t)$ is bounded by

$$p(t) \leq \frac{\zeta+1}{\zeta+2}\big[e^{\gamma L(\zeta+2)(t-t_i)} - 1\big],\ \forall t \in [t_i, t_{i+1}]. \tag{56}$$

At $t = t_{i+1}$ assume that we have $\frac{\|e(t)\|_2}{\|y(t_i)\|_2} = p(t_{i+1}) = \hat{\sigma}$, so we have

$$\hat{\sigma} \leq \frac{\zeta+1}{\zeta+2}\big[e^{\gamma L(\zeta+2)(t_{i+1}-t_i)} - 1\big],\ t = t_{i+1} \tag{57}$$

the inter-execution times $\{t_{i+1} - t_i\}_{i\in\mathbb{N}}$ are bounded by

$$\tau \geq \frac{1}{\gamma L(\zeta+2)}\ln(1 + \frac{2+\zeta}{1+\zeta}\hat{\sigma}), \tag{58}$$

in this case, τ is the lower bound of the time for $p(t)$ to evolve from 0 to $\hat{\sigma}$, and we can see that for any nontrivial $\hat{\sigma}$, $\tau > 0$.

For $\triangle > 0$, the analysis is the same as in the proof of Theorem 1, and thus is omitted here. ■

VII. Self-Triggered Real-Time Scheduling For Stabilization Of Passive/Dissipative Systems

An event-triggered implementation based on the execution rule (13) or (49) would require testing the execution rule frequently. Unless this testing process is implemented in hardware, one might run the risk of consuming the processor time. To overcome this drawback, we propose a self-triggered real-time scheduling strategy, where the current output measurement $y(t_i)$ and the last output measurement $y(t_{i-1})$ are used to determine the next sampling time t_{i+1} and the execution delay of the actuator $\triangle_{i+1}$. In this regard, we do not need to test the execution rule as often as we did for the "event-triggered"case.

Theorem 3. *Consider the passive system given in (5), if the following conditions are satisfied*

1) $f : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$ *is Lipschitz continuous on compacts;*
2) $h : \mathbb{R}^n \to \mathbb{R}^m$ *is a static nonlinear function of* x *which belongs to a sector* $[\alpha, \beta]$ *such that* $\alpha x^T x \leq x^T h(x) \leq \beta x^T x$*, where* $\alpha\beta > 0$*;*
3) $\|\frac{\partial h(x)}{\partial x}\|_2 \leq \gamma$*, where* $0 < \gamma < \infty$*;*
4) *system* H *is ZSD;*

then under the following scheduling strategy, the passive system under the control action $u(t) = -Ky(t_i)$ *is asymptotically stable:*

- $t_0 = t_0 + \triangle_0$*;*
- $t_1 = t_0 + \tau_0$*,* $\tau_0 = \frac{1}{\gamma L(2+\zeta)} \ln\big(1 + \frac{2+\zeta}{1+\zeta}\hat{\sigma}\big)$*;*
- $t_{i+1} = t_i + \triangle_i + \tau$*,* $i = 1, 2, \ldots$*;*

where t_i *is the time at which the sensor gets the measurement of the output* $y(t_i)$*, and* $\zeta = \max\{\frac{1}{|\alpha|}, \frac{1}{|\beta|}\}$*;* L *is the Lipschitz constant of* $f(x, u)$*; choose* $\tilde{\sigma}, \hat{\sigma}, \hat{\sigma}'$ *as constants such that* $0 < \tilde{\sigma} < \hat{\sigma} < \hat{\sigma}' < 0.5$ *and*

$$\tau = \frac{1}{\gamma L(2+\zeta)} \ln\big(\frac{\frac{1+\zeta}{2+\zeta} + \hat{\sigma}}{\frac{1+\zeta}{2+\zeta} + \tilde{\sigma}}\big), \tag{59}$$

$$\triangle_i = \min\big\{\frac{1}{(2+\zeta)\gamma L} \ln\big[\frac{(2+\zeta)\tilde{\sigma}\|y(t_i)\|_2}{(1+\zeta)\|y(t_i)\|_2 + \|y(t_i) - y(t_{i-1})\|_2} + 1\big], \frac{1}{\gamma L(2+\zeta)}\big[\ln\frac{\frac{1+\zeta}{2+\zeta} + \hat{\sigma}'}{\frac{1+\zeta}{2+\zeta} + \hat{\sigma}}\big]\big\}, \tag{60}$$

where $\triangle_i$ is the estimated admissible execution delay of the actuator after the sensor gets the ith measurement of the system's output.

Proof. If we consider non-zero execution delay of the actuator, the evolution of $\frac{\|e(t)\|_2}{\|y(t_i)\|_2}$ appears as shown in Figure 2. Let $e(t) = y(t) - y(t_i)$ for $t \in [t_i + \triangle_i, t_{i+1} + \triangle_i]$ denote the error of the measurement at the actuator, and let $\tilde{e}(t) = y(t) - y(t_i)$, for $t \in [t_i, t_{i+1}]$ denote the error of the measurement at the

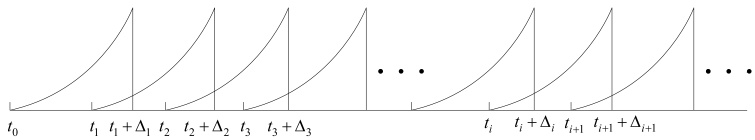

Fig. 18: triggering signal

sensor.

For $t \in [t_i, t_i + \triangle_i]$, we have $e(t) = y(t) - y(t_{i-1})$ and $\tilde{e}(t) = y(t) - y(t_i)$. Moreover, since

$$\begin{aligned}
\frac{d}{dt}\|\tilde{e}(t)\|_2 &\le \|\dot{\tilde{e}}(t)\|_2 = \|\dot{y}(t)\|_2 = \|\frac{\partial y}{\partial x}\dot{x}\|_2 \\
&\le \|\frac{\partial y}{\partial x}\|_2\|\dot{x}\|_2 \le \gamma L[\|x(t)\|_2 + \|y(t)\|_2 + \|e(t)\|_2] \le \gamma L[(1+\zeta)\|y(t)\|_2 + \|e(t)\|_2] \\
&= \gamma L[(1+\zeta)\|\tilde{e}(t) + y(t_i)\|_2 + \|e(t)\|_2] \\
&= \gamma L[(1+\zeta)\|\tilde{e}(t) + y(t_i)\|_2 + \|\tilde{e}(t) + y(t_i) - y(t_{i-1})\|_2] \\
&\le \gamma L[(2+\zeta)\|\tilde{e}(t)\|_2 + (1+\zeta)\|y(t_i)\|_2 + \|y(t_i) - y(t_{i-1})\|_2]
\end{aligned} \tag{61}$$

so the evolution of $\|\tilde{e}(t)\|_2$ during the time $[t_i, t_i + \triangle_i]$ is bounded by the solution of

$$\dot{\phi}(t) = \gamma L[(2+\zeta)\phi(t) + (1+\zeta)\|y(t_i)\|_2 + \|y(t_i) - y(t_{i-1})\|_2], \tag{62}$$

with $\phi(t_i) = \|y(t_i) - y(t_i)\|_2 = 0$, the solution to (62) is given by

$$\phi(t) = \frac{(1+\zeta)\|y(t_i)\|_2 + \|y(t_i) - y(t_{i-1})\|_2}{2+\zeta}[e^{(2+\zeta)\gamma L(t-t_i)} - 1]. \tag{63}$$

From Theorem 2, we know that an alternative execution rule for stabilization of the passive plant is $\|e(t)\|_2 \le \tilde{\sigma}\|y(t_i)\|_2$, where $0 < \tilde{\sigma} < 0.5$. So let $\phi(t_i + \triangle_i) = \tilde{\sigma}\|y(t_i)\|_2$, we can get an estimate of the $\triangle_i$, if we denote it by ε_i^-, then ε_i^- is given by

$$\varepsilon_i^- = \frac{1}{(2+\zeta)\gamma L}\ln\big[\frac{(2+\zeta)\tilde{\sigma}\|y(t_i)\|_2}{(1+\zeta)\|y(t_i)\|_2 + \|y(t_i) - y(t_{i-1})\|_2} + 1\big], \tag{64}$$

and notice that $\varepsilon_i^- > 0$ for any $\tilde{\sigma} > 0$ and $\|y(t_i)\|_2$ cannot goes to zero in a finite time because by applying output feedback to the passive system (5), the output $y(t)$ goes to zero asymptotically.

Assume that the actuator updates the control action at $t = t_i + \triangle_i$, and choose a $\hat{\sigma}$ such that $0 < \tilde{\sigma} < \hat{\sigma} < 0.5$, then for $t \in [t_i + \triangle_i, t_{i+1}]$, we have

$$e(t) = \tilde{e}(t) = y(t) - y(t_i), \tag{65}$$

and we can obtain

$$\begin{aligned}\frac{d}{dt}\|e(t)\|_2 &\leq \|\dot{e}(t)\|_2 = \|\dot{y}(t)\|_2 = \|\frac{\partial y}{\partial x}\dot{x}\| \\ &\leq \|\frac{\partial y}{\partial x}\|_2\|\dot{x}\|_2 \leq \gamma L[\|x(t)\|_2 + \|y(t)\|_2 + \|e(t)\|_2] \\ &\leq \gamma L[(1+\zeta)\|y\|_2 + \|e\|_2] \\ &\leq \gamma L[(1+\zeta)\|e(t) + y(t_i)\|_2 + \|e\|_2] \\ &\leq \gamma L[(2+\zeta)\|e(t)\|_2 + (1+\zeta)\|y(t_i)\|_2],\end{aligned} \tag{66}$$

the evolution of $\|e(t)\|_2$ during $[t_i + \triangle_i, t_{i+1}]$ is bounded by the solution of

$$\dot{\phi}(t) = \gamma L[(2+\zeta)\phi(t) + (1+\zeta)\|y(t_i)\|_2] \tag{67}$$

with $\phi(t_i + \triangle_i) = \|y(t_i + \triangle_i) - y(t_i)\|_2$ and $\phi(t_i + \triangle_i) = \tilde{\sigma}\|y(t_i)\|_2$, we can get the solution to (67) which is given by

$$\phi(t) = \frac{(1+\zeta)\|y(t_i)\|_2 + (2+\zeta)\|y(t_i + \triangle_i) - y(t_i)\|_2}{2+\zeta}e^{\gamma L(2+\zeta)(t-t_i-\triangle_i)} - \frac{(1+\zeta)\|y(t_i)\|_2}{2+\zeta} \tag{68}$$

and assume at $t = t_{i+1}$, we have $\phi(t_{i+1}) = \hat{\sigma}\|y(t_{i+1})\|_2$, then an estimate of the time it takes for $\frac{\|e(t)\|_2}{\|y(t_i)\|_2}$ to evolve from $\tilde{\sigma}$ to $\hat{\sigma}$ is given by

$$\tau = \frac{1}{\gamma L(2+\zeta)} \ln\big(\frac{\frac{1+\zeta}{2+\zeta} + \hat{\sigma}}{\frac{1+\zeta}{2+\zeta} + \tilde{\sigma}}\big), \tag{69}$$

and notice that for any $\hat{\sigma} > \tilde{\sigma} > 0$, we have $\tau > 0$.

Assume at $t = t_{i+1} + \triangle_{i+1}$, we have $\phi(t_{i+1} + \triangle_{i+1}) = \hat{\sigma}'\|y(t_i)\|_2$, where $0.5 > \hat{\sigma}' > \hat{\sigma} > \tilde{\sigma} > 0$. Since $e(t) = y(t) - y(t_i)$ for $t \in [t_{i+1}, t_{i+1} + \triangle_{i+1}]$, we can still get an estimate of the time it takes for $\frac{\|e(t)\|_2}{\|y(t_i)\|_2}$ to evolve from $\hat{\sigma}$ to $\hat{\sigma}'$ based on (67). If we denote it by ε_i^+, then we can get

$$\varepsilon_i^+ = \frac{1}{\gamma L(2+\zeta)} \ln\big(\frac{\frac{1+\zeta}{2+\zeta} + \hat{\sigma}'}{\frac{1+\zeta}{2+\zeta} + \tilde{\sigma}}\big). \tag{70}$$

An estimate of $\triangle_i$ (notice that $\triangle_i$ is the execution delay of the actuator after the sensor gets the measurement $y(t_i)$) is given by

$$\triangle_i = \min\{\varepsilon_i^-, \varepsilon_i^+\}, \tag{71}$$

and the corresponding estimate of the time for the sensor to get the next new measurement is given by

$$t_{i+1} = t_i + \triangle_i + \tau. \tag{72}$$

Since we choose $0.5 > \hat{\sigma}' > \hat{\sigma} > \tilde{\sigma} > 0$, then based on the scheduling strategy as claimed in Theorem 3, we can guarantee that for each execution interval $[t_i + \triangle_i, t_{i+1} + \triangle_{i+1}], i = 1, 2, \ldots,$ we have

$\|e(t)\|_2 \leq 0.5\|y(t_i)\|_2$, and thus we can guarantee that the passive system under the control action $u(t) = -Ky(t_i)$ is asymptotically stable. ■

Remark 6: For self-triggered scheduling strategy as discussed in Theorem 3, we only need the measurements of $y(t_i)$ and $y(t_{i-1})$ to estimate the next sampling time, so compared with the event-triggered scheduling strategy which requires testing the execution rule frequently (or in another words, we need to measure the output frequently), self-triggered scheduling strategy has less demand on the sampling frequency of the sensor. ■

Remark 7: It can be shown that similar self-triggered real-time scheduling approach can be applied to stabilization of dissipative systems. Again consider the system H given by (5), and we assume that H satisfies the dissipative inequality given by

$$\dot{V}(x) \leq u^T y + \rho y^T y \tag{73}$$

with $V(x) \geq 0$ and $\rho > 0$, ρ is the smallest positive constant such that the above dissipative inequality holds. In this case, system H is non-passive and unstable, and by applying negative output feedback $u = -Ky$, where $K > \rho > 0$, we can stabilize the dissipative system if it is ZSD. in Remark 5 we have shown that to stabilize the dissipative system, an admissible execution law can be determined by:

$$\|e(t)\|_2 \leq \|K^{-1}(K - \rho I_{m\times m})\|_2 \|y(t)\|_2, \; for \; t \in [t_i + \triangle_i, t_{i+1} + \triangle_{i+1}]. \tag{74}$$

An sufficient condition for (74) to hold is given by

$$\|e(t)\|_2 \leq \frac{K-\rho}{2K-\rho}\|y(t_i)\|_2, \; for \; t \in [t_i + \triangle_i, t_{i+1} + \triangle_{i+1}]. \tag{75}$$

Then the scheduling strategy for stabilization of the dissipative system is the same as proposed in Theorem 3 but we need to choose $0 < \tilde{\sigma} < \hat{\sigma} < \hat{\sigma}' < \frac{K-\rho}{2K-\rho}\|y(t_i)\|_2$. ■

VIII. Examples For Self-Triggered Scheduling

Example 3. We consider the same system as discussed in Example 1. By choosing $\tilde{\sigma} = 0.01$, $\hat{\sigma} = 0.45$, $\hat{\sigma}^{'} = 0.46$, we get the simulation results as shown in Fig.19-Fig.22. Minimum admissible delay $\triangle$ obtained from the simulation is $0.0015s$, the corresponding inter-execution time $\tau + \triangle$ is $0.0850s$. The sensor gets 25 measurements of system's output in $2s$ and the actuator updates the control action 24 times in $2s$.

Example 4. We consider the same system as discussed in Example 2. In this case, since $\rho = 1.5$, by choosing $K = 10$, we have $(K - \rho)/(2K - \rho) = 0.4595$. Choose $\tilde{\sigma} = 0.05$, $\hat{\sigma} = 0.4$, $\hat{\sigma}^{'} = 0.45$, we get the simulation results as shown in Fig.23-Fig.25. Minimum admissible delay $\triangle$ obtained from the simulation is $0.0004082s$, the corresponding inter-execution time $\tau + \triangle$ is $0.0038s$, and the sensor gets 520 measurements of system's output in $2s$ while the actuator updates the control action 519 times in $2s$.

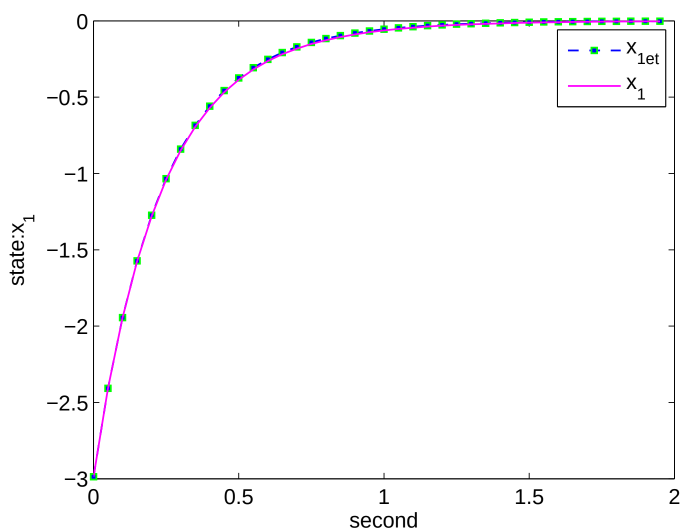

Fig. 19: x_{1et} represents the evolution of state x_1 with execution delay of the actuator $\triangle = 0.0015s$ based on the self-triggered scheduling strategy; x_1 represents the evolution of state x_1 with "time-triggered"execution (periodic sampling time $0.001s$ and no execution delay).

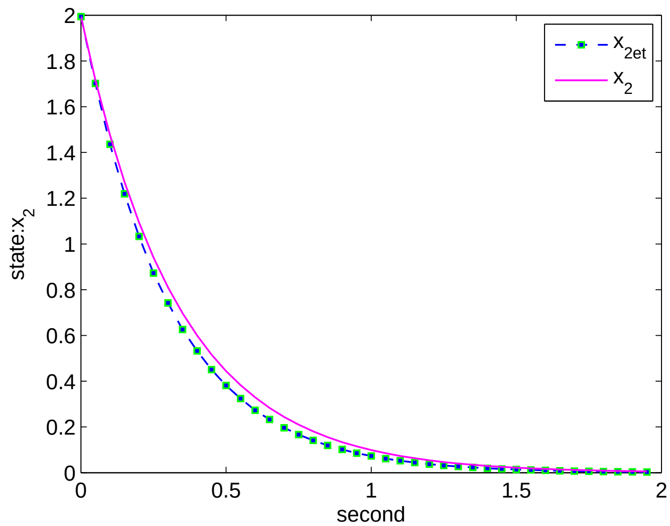

Fig. 20: x_{2et} represents the evolution of state x_1 with execution delay of the actuator $\triangle = 0.0015s$ based on the self-triggered scheduling strategy; x_2 represents the evolution of state x_2 with "time-triggered"execution (periodic sampling time $0.001s$ and no execution delay).

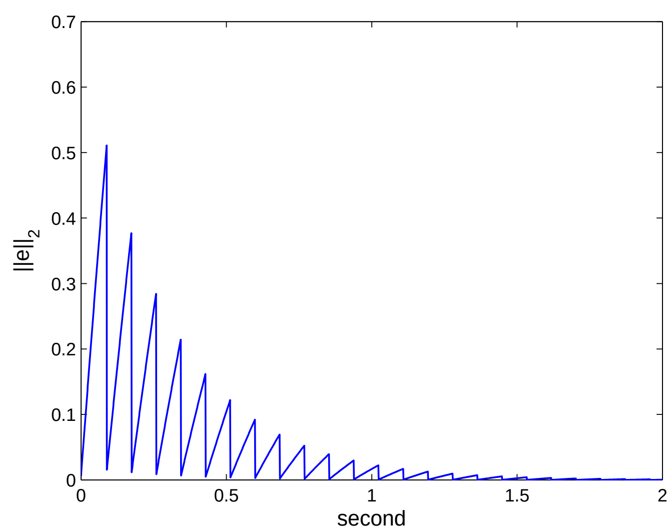

Fig. 21: evolution of the norm of error($\|e(t)\|_2$) between the actual output of the plant and the sampling value

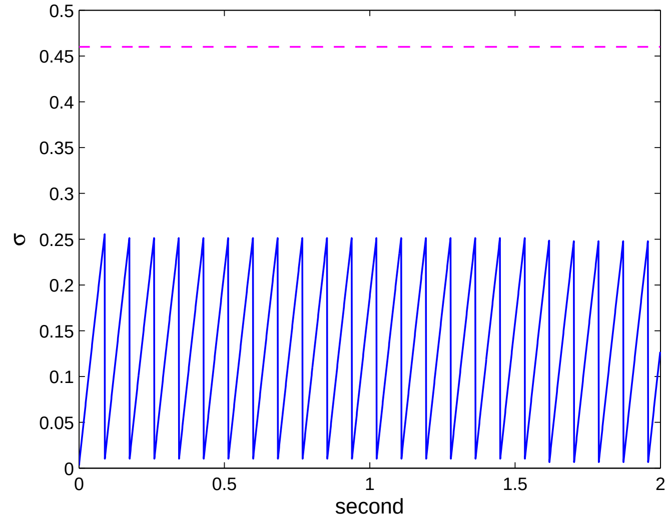

Fig. 22: evolution of $\sigma = \frac{\|e\|_2}{\|y\|_2}$

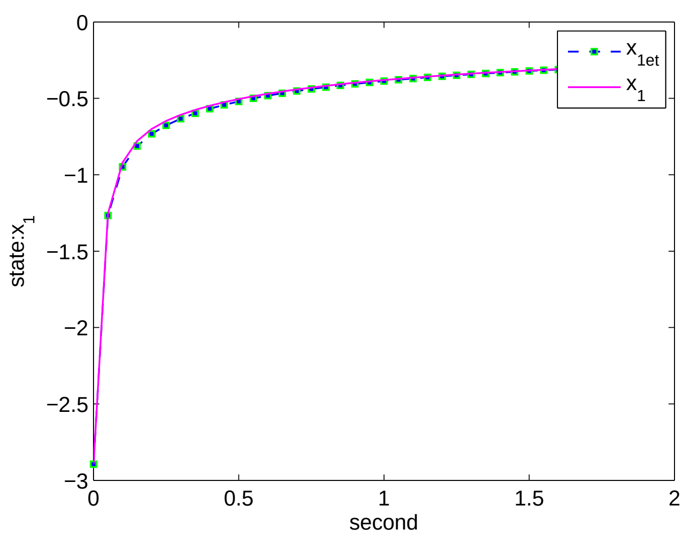

Fig. 23: x_{1et} represents the evolution of state x_1 with execution delay of the actuator $\triangle = 0.0004082s$ based on the self-triggered scheduling strategy; x_1 represents the evolution of state x_1 with "time-triggered"execution (periodic sampling time $0.001s$ and no execution delay).

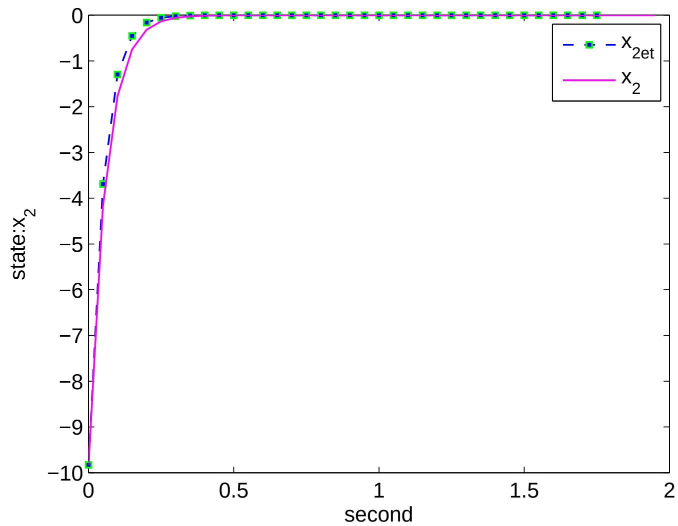

Fig. 24: x_{2et} represents the evolution of state x_2 with execution delay of the actuator $\triangle = 0.0004082s$ based on the self-triggered scheduling strategy; x_1 represents the evolution of state x_1 with “time-triggered”execution (periodic sampling time $0.001s$ and no execution delay).

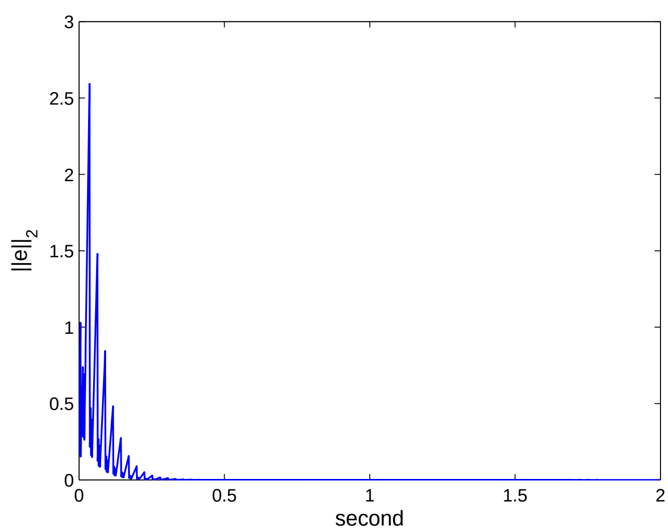

Fig. 25: evolution of the norm of error($\|e(t)\|_2$) between the actual output of the plant and the sampling value

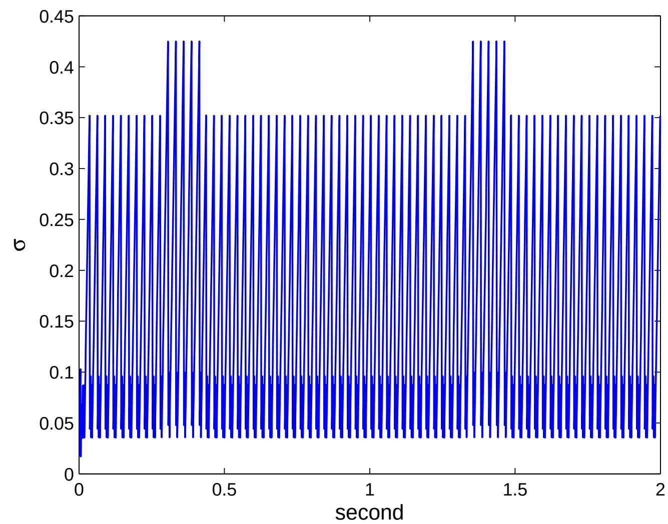

Fig. 26: evolution of $\sigma = \frac{\|e\|_2}{\|y\|_2}$

IX. Conclusion And Future Work

In this report, we derive the event-triggered and self-triggered real-time scheduling strategy for stabilization of passive/dissipative systems based on output feedback. We assume that the systems under study satisfied certain passive/dissipative inequalities and the systems are zero-state detectable. In the event-triggered scheduling strategy, the control tasks are executed whenever a certain error becomes large when compared with the current output's norm of the plant, this strategy would require a dedicate effort of hardware to monitor the output of the plant, but compared with the full-state measurement and the state feedback control action suggested in [6], output feedback control action usually demands less information on system's dynamics; in the self-triggered scheduling strategy, we propose an estimate of the next instant of time at which the control action is updated based on the current and the last measurement of the output of the plant; in between updates of the controller the control signal is held constant for both the event-triggered scheduling strategy and the self-triggered scheduling strategy, and the execution of the control action is in a aperiodic fashion. Since the triggering condition for both event-trigger and self-trigger is based on certain degree of instability of the system, processor or communication media usage is more efficient compared with the traditional periodic implementation of the control action. We have also shown this significant improvement by the examples provided in section V and section VIII. The proposed event-triggered/self-triggered real-time scheduling strategy can be applied to both linear and nonlinear dynamic systems in the current work. For the self-triggered scheduling strategy, the intervals of time in which no attention is devoted to the plant pose a possible future work on the robustness analysis of the proposed self-triggered implementations.

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