Multiple Antenna Transmission With Channel State Information: A Low-Rate Feedback Approach

Shirish Nagaraj and Yih-Fang Huang

Abstract—This paper presents a novel multiple antenna transmission scheme that depends on channel state information. The proposed approach uses low-rate feedback to adaptively estimate the downlink fading channel at the base-station. The optimal transmission scheme given channel state information can in turn be determined from these channel estimates. It is shown that the channel estimation requires selective feedback of the adaptive filter prediction errors, that can be computed at the mobile by receiving a beamformed pilot signal. To this end, a randomized channel inversion pilot beamforming scheme is developed that makes possible this feedback of prediction errors possible. Further, this method is shown to be relatively insensitive to feedback quantization effects for a given average feedback rate. Simulation results indicate that the feedback scheme provides significant closed-loop beamforming gains with low bit rate feedback even for large number of transmit antennas.

Index Terms—Adaptive channel estimation, feedback communications, multiple antennas.

I. INTRODUCTION

FEEDBACK of channel state information (CSI) or calculated transmit weights from mobiles to the base-station is one of the ways that spatial pre-filtering over nonreciprocal channels can be achieved. Maximal gains from Multiple Input Multiple Output (MIMO) systems are obtained by transmitting optimally to the different eigen-modes of the instantaneous channel covariance matrix, which requires CSI at the transmitter. Frequency-division-duplex (FDD) systems require the mobile to feedback CSI since the carrier-frequencies are different on the uplink and downlink. There has been interest in incorporating feedback of side information from the mobile to the base-station, especially for certain 3G systems such as UMTS [1], [2]. When the CSI is used to transmit the signal in a single eigen-mode of the channel, the transmission is referred to as closed-loop beamforming or *maximal ratio transmission*.

However, feedback usually causes a serious degradation to uplink information capacity. An alternative to this is to infer certain average downlink channel parameters from uplink measurements and use that for transmit beamforming on the downlink; see, e.g., [3]. This requires the antennas to be closely spaced

so that open-loop beamforming gains can be achieved by using spatial correlation properties. The use of other purely open-loop transmit diversity schemes, e.g., space-time codes, has also been of great interest.

This paper considers the problem of downlink transmission, like closed-loop beamforming, based on CSI feedback from the mobile. A novel approach to feedback-based channel estimation and downlink transmission, that results in a solution with significantly reduced uplink overhead, is proposed. The next section develops the system model including a model for the received signal at any particular mobile (after preliminary base-band processing). Section III is devoted to the development of the proposed *selective* feedback mechanism and adaptive channel estimation. Simulation results are presented to evaluate the performance of this scheme. Further analysis of the uplink feedback quantization requirements for the proposed scheme is also presented.

II. SYSTEM MODEL

A multi-path fading wireless channel with M transmit antennas and N receive antennas is considered here. For simplicity, we restrict attention to beamformed MIMO systems with maximal ratio receive combining performed across L multiple paths and the N receive antennas. Since in this context, receive antennas and multiple paths essentially act as additional uncorrelated sources of receive diversity, we refer to the system as having $P = NL$ effective receive paths. A transmit beamformer for each user is a linear spatial filter with M taps. The $M \times 1$ -vector channel, as seen by the k^{th} user for certain multi-path and receive antenna, is a complex Gaussian random vector, denoted by $\mathbf{a}_{k,p}$. Here, $p = 1, 2, \dots, P$, denotes the index of the effective received path. Assuming that there are a total of K users in the system, the information bit streams of the k^{th} user is denoted by $\{b_k(i) \in \{+1, -1\}\}_i$, $k \in \{1, 2, \dots, K\}$. The sent signal for that user (linearly modulated) takes the form

$$\mathbf{x}_k(t) = \sum_i [b_k(i)\phi_k(t - iT_s)\mathbf{v}_k(i) + m_k(i)\psi_k(t - iT_s)\mathbf{w}_k(i)] \quad (1)$$

where T_s denotes the signaling time for each bit. The data bits, $b_k(i)$, are modulated with the signaling waveform $\phi_k(\cdot)$, whereas the pilot bits, $m_k(i)$, are modulated with a waveform $\psi_k(\cdot)$ that is orthogonal to $\phi_k(\cdot)$. Here, $\phi_k(\cdot)$ and $\psi_k(\cdot)$ are a set of orthonormal square-integrable complex-valued functions. In the context of DS-CDMA, they represent the spreading code waveforms for the data and pilot, respectively. If the pilot were to be time-multiplexed with the data, then the functions would be the same spreading code, restricted to the time-duration

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S. Nagaraj is with the Open Innovations Laboratory, Lucent Technologies-Bell Laboratories, Whippany, NJ 07981 USA (e-mail: snagaraj@lucent.com).

Y.-F. Huang is with the Department of Electrical Engineering, University of Notre Dame, Notre Dame, IN 46556 USA (e-mail: huang@nd.edu).

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of the data and the pilot symbols. Thus, this formulation is generic and allows the pilot to be multiplexed with the data signal either by choosing disjoint time/frequency intervals or by orthogonal codes sharing the same time and frequency. The vectors $\mathbf{v}_k(i)$ and $\mathbf{w}_k(i)$ are, respectively, the beamformers for the data and pilot signals, and are a set of K complex-valued M -dimensional vectors. The overall sent signal is given by $\mathbf{x}(t) = \sum_{k=1}^K \mathbf{x}_k(t); \forall t$.

The received pilot signal is code-matched filtered with $\psi_k^*(t)$ at the p^{th} receive path (which corresponds to a combination of a receive antenna and multipath index). The equivalent received signal, for the i^{th} transmitted pilot symbol at the p^{th} receive path takes the form

$$d_{k,p}(i) = \mathbf{a}_{k,p}^T \mathbf{w}_k(i) m_k(i) + n_{k,p}(i) \quad \forall i, p = 1, 2, \dots, P \quad (2)$$

where $n_{k,p}(i)$ is an independent and identically distributed (*i.i.d.*) Gaussian random sequence with variance σ_p^2 . The interference term $n_{k,p}(i)$ consists of outer-cell interference, thermal noise and in-cell interference (generated due to multipath propagation). It can be seen that $d_{k,p}(i)$ is the output of a linear system with an input given by $\mathbf{x}_k(i) = \mathbf{w}_k(i) m_k(i)$. Similarly, the data signal is demodulated by matched-filtering with $\phi_k^*(\cdot)$.

An optimal beamformer in the case of single path channels would be of the form $\kappa(\mathbf{a}_k) \mathbf{a}_k^*$, where $\kappa(\mathbf{a}_k)$ is a (possibly channel-dependent, hence time-varying) constant so as to ensure an average power constraint is met. Optimal power allocation strategies that maximize ergodic mutual information for a fixed average transmit power are discussed in [4], [5]. In multipath/multiple receive antenna systems that employ beamforming transmission, one could choose the beamformer as the principle eigenvector of $\sum_p \mathbf{a}_{k,p} \mathbf{a}_{k,p}^H$. Generic MIMO transmission schemes, using the channel knowledge at the transmitter, e.g., [2], [6], are feasible if an efficient scheme for obtaining the instantaneous channel state information at the base-station were possible. A method that allows for such transmitter knowledge is described in the next section.

III. LOW-RATE FEEDBACK MECHANISM

Various feedback schemes have been proposed in the literature; see, e.g., [2], [7]. Most of the feedback schemes require the mobile to perform computationally intensive operations such as channel estimation/prediction. Further, the number of feedback signals grows with the number of antennas. The proposed feedback concept is motivated by the requirement that the feedback information should be simple to compute and entail low overhead on the uplink channel.

The principles of the feedback mechanism can be outlined as follows: The channel vectors for each effective path can be estimated adaptively (at the base-station) while the prediction errors corresponding to those different paths are computed at the mobile from the beamformed pilot measurements and fed back to the base-station. As such, the downlink vector channels can be identified at the base-station using the available input-desired output pairs, denoted by $\{d_{k,p}(i), \mathbf{x}_k(i)\} \quad \forall p$. In general, computation of this prediction error requires the mobile to know the transmit beamforming weight vector. The proposed beamforming scheme is devised in such a manner that the mobile

can actually compute the prediction errors without explicitly knowing the instantaneous transmit beamforming weight vector used by the pilot.

A. Pilot Beamformer Design

Assume that the adaptive channel estimator at the base-station has a sequence of *a priori* channel estimates given by $\hat{\mathbf{a}}_{k,p}(i-1)$ for the user k 's p^{th} path at time instant i . The mobile receives the signal $d_{k,p}(i)$ which is then used to generate its feedback information. Given $d_{k,p}(i)$, the prediction error of an adaptive algorithm is

$$\delta_{k,p}(i) = d_{k,p}(i) - \hat{\mathbf{a}}_{k,p}^T(i-1) \mathbf{w}_k(i) m_k(i) \quad \forall p.$$

It is desirable to be able to compute and feedback the prediction error since there is lower power in the prediction error as compared to $d_{k,p}(i)$. The pilot sequence $\{m_k(i)\}$, which is needed for the above computation, is known to the mobile. However, in order to compute the prediction errors, the mobile also needs to know the inner product between the beamforming vector $\mathbf{w}_k(i)$ and the previous channel estimates $\hat{\mathbf{a}}_{k,p}(i-1)$.

A method to circumvent this problem is to choose a pilot beamformer such that it presents a constant projection along the different *a priori* channel estimates, $\mathbf{a}_{k,p}(i-1)$. Note that for any adaptive algorithm to converge, it is important that the sequence of input vectors $\mathbf{w}_k(i)$ be *persistently exciting* [8]. Intuitively, this translates to a requirement that the adaptive filter be given input vectors in all directions of the M -dimensional space, such that all the modes of the unknown channel vector are excited. A simple design criteria would be to generate a random unit norm vector, $\mathbf{u}_k(i)$ at each instance i , and find the "closest" beamforming vector $\mathbf{w}_k(i)$ to it such that the beamformer has a known projection along the different *a priori* channel estimates. This approach can be cast in the form of a constrained optimization problem as: For a given random vector $\mathbf{u}_k(i)$ such that $\|\mathbf{u}_k(i)\|^2 = 1$

$$\begin{aligned} &\text{Minimize } J(\mathbf{w}_k(i)) = \|\mathbf{w}_k(i) - \mathbf{u}_k(i)\|^2 \\ &\text{subject to } \hat{\mathbf{a}}_{k,p}^T(i-1) \mathbf{w}_k(i) = c_p \quad \forall p = 1, 2, \dots, P. \end{aligned} \quad (3)$$

Define the matrix of channel estimates by $\hat{\mathbf{A}}_k(i-1) = [\hat{\mathbf{a}}_{k,1}(i-1), \dots, \hat{\mathbf{a}}_{k,P}(i-1)]$ and the vector of constraints by $\mathbf{c} = [c_1, c_2, \dots, c_P]^T$. The value of $\mathbf{c}$ is assumed to be known before hand to both the base-station and to the mobile. By defining $\mathbf{R}_a = (\hat{\mathbf{A}}_k^T(i-1) \hat{\mathbf{A}}_k(i-1))^{-1}$, the solution to the above problem can be obtained simply as

$$\mathbf{w}_k(i) = \left[\mathbf{I} - \hat{\mathbf{A}}_k^*(i-1) \mathbf{R}_a \hat{\mathbf{A}}_k^T(i-1) \right] \mathbf{u}_k(i) + \hat{\mathbf{A}}_k^*(i-1) \mathbf{R}_a \mathbf{c} \quad (4)$$

where $\mathbf{I}$ denotes an $M \times M$ identity matrix. We refer to this pilot beamforming design as a *randomized channel inversion beamformer*. The pilot beamformer does not, in general, have unit power, but the $\mathcal{L}_2$ -norm design criterion is such that it penalizes any deviation from the unit power random vector $\mathbf{u}_k(i)$. Further, if we choose $\mathbf{c} = \mathbf{0}$, then we can always divide the beamformer with its norm to result in a unit norm pilot beamforming vector, while meeting all the constraints. In this case, it is easy to see that the beamformer projects $\mathbf{u}_k(i)$ onto the

null-space of $\hat{\mathbf{A}}_k(i-1)$. The above channel inversion beamformer requires $M > P$. If $M \leq P$, subgroups of paths can be defined and the beamformer then would be designed for one subgroup at a time. In this case, the feedback delay would increase by a factor of the number of subgroups, since only the prediction errors corresponding to one subgroup at a time can be fed back. This beamforming strategy ensures that the prediction error is given by

$$\delta_{k,p}(i) = d_{k,p}(i) - c_p m_k(i),$$

which can be computed very easily by the mobile since both $m_k(i)$ and the projections c_p are known. The details of the adaptive channel estimator, that results in the mobile requiring only selective feedback of $\delta_{k,p}(i)$, are discussed next.

B. Adaptive Channel Estimation

There are many approaches to adaptive filtering, most of which focus on minimizing an $\mathcal{L}_2$ norm of the error sequence [9]. On the other hand, set-membership filtering [8], [10], estimates the filter with a bounded $\mathcal{L}_\infty$ criterion. The filter is designed such that the worst-case difference between the filter output and desired output is within a certain pre-specified error bound for most inputs. Adaptive algorithms, such as those in [8], [10] can adaptively estimate the SMF filter coefficients. It is known that one of the most attractive features of these algorithms is their inherent *selective update* capability, whereby the algorithms do not need to update the estimates all the time. For the specific example of vector channel estimation here, we employ the generalized BEACON algorithm [10] to estimate $\mathbf{a}_{k,p}$ as follows:

$$\begin{aligned} \hat{\mathbf{a}}_{k,p}(i) &= \hat{\mathbf{a}}_{k,p}(i-1) + \nu(i) \mathbf{P}(i) \mathbf{x}_k^*(i) \delta_{k,p}(i) \\ \mathbf{P}^{-1}(i) &= \mathbf{P}^{-1}(i-1) + \nu(i) \mathbf{x}_k^*(i) \mathbf{x}_k^T(i) \\ \nu(i) &= \begin{cases} \frac{1}{G_k(i)} \left(\frac{|\delta_{k,p}(i)|}{\gamma_k} - 1 \right) & \text{if } |\delta_{k,p}(i)| > \gamma_k \\ 0 & \text{else} \end{cases} \quad (5) \end{aligned}$$

where $G_k(i) = \mathbf{x}_k^T(i) \mathbf{P}(i-1) \mathbf{x}_k(i)$ and $\hat{\mathbf{a}}_{k,p}(i)$ is an estimate of $\mathbf{a}_{k,p}$ at time i . The value of γ_k is decided by the mobile based on its average SNR and signaled to the base-station at the time of setting up the communication link. Conceptually, other SMF algorithms [11], that feature selective updating, can also be used for channel estimation in this framework.

When adaptive SMF algorithms are used in the base-station for channel estimation, there would not be any update of the channel estimate when $|\delta_{k,p}(i)|$ is less than the error bound γ_k (since, in that case, $\nu(i) = 0$). This selective update feature can be used to a greater advantage by reducing the amount of feedback required. The mobile compares $|\delta_{k,p}(i)|$ with γ_k and decides not to feed anything back for that path if $|\delta_{k,p}(i)| < \gamma_k$. Otherwise, the mobile feeds back the complex value $(|\delta_{k,p}(i)| - \gamma_k)e^{j\angle\delta_{k,p}}$ to the base-station for each of the paths that satisfy this condition. This is the essence of *selective feedback* that results in a gain in the uplink capacity. Infrequent feedback is a natural outcome of the SMF problem formulation and the best performance is achieved in the presence of selective feedback, since that is the nature of the algorithm update recursions. For a more detailed quantitative analysis on these savings, see [12]. The proposed scheme is thus seen to be efficient on three different levels: 1) *reduction in number of complex-valued feed-*

Fig. 1. Bit error rate performance of beamforming with BEACON and WRLS channel estimation algorithms and prediction error feedback. Selective update fraction for BEACON was 15%.

Fig. 2. Bit error rate performance of beamforming with quantized BEACON and a modified version of closed-loop Mode 1 transmission with feedback delay.

back coefficients, 2) *simple computation of prediction errors at the mobile*, and 3) *selective feedback on the uplink*.

To examine the performance of this feedback scheme, a 4-element transmit array system ($M = 4$) was simulated on a slowly time-varying single path channel. The average SNR per antenna was varied and the bit-error rate of the data sequence computed. We assumed that the data is beamformed with the normalized conjugate of the channel estimate of the previous instant and the pilot is beamformed according to (4) with $c = 0$. The performance of BEACON was examined and compared to the performance of the conventional weighted recursive least-squares (WRLS) algorithm, with a forgetting factor of 0.9. The value of γ_k was chosen to be equal to the square root of twice the noise variance seen by the pilot sequence. It can be seen from Fig. 1 that BEACON performs comparably to the WRLS, while updating only 15% of the time, thus reducing the feedback information substantially. This results in significant savings on the uplink bandwidth/power requirements for feedback.

The important thing to note is that the proposed system only requires that the pilot be beamformed with the randomized channel inversion rule. The data transmission can be any scheme that is optimized for the particular channel conditions and type of traffic. In voice systems, this could be just beamforming as shown in the results of Fig. 1. For packet data systems in rich multipath environments, one could perform optimal pre-equalization since all the multipath channels are estimated at the base-station in the proposed approach. For MIMO systems, space-time codes can be used that utilize channel knowledge at the transmitter to optimally weight and send multiple streams of data over the different eigen-modes of the channel [6].

C. Feedback Delay and Quantization Requirements

The mobile feeds back the value of the prediction error via an uplink channel. We have assumed that there is a one symbol delay in the reception of the feedback. However, this feedback mechanism can also work with larger delays. In that case, there will be a lag in the prediction error and thus the channel estimate will be a delayed version of the current channel by a time equal to the feedback delay. If we have a feedback delay of D symbols, then the channel estimate would get updated once every D symbols. Thus the beamformer also would be updated at this rate. For moderate values of D and Doppler, the feedback delay would be much less than the coherence time of the channel. Thus, delay is not very critical when we consider low Doppler channels. In higher Doppler scenarios, one can predict the channel up to a few symbol times using the delayed data received on the feedback channel. However, this approach is not considered in detail in this work.

Another source of imperfection arises out of quantization effects. Typical feedback mechanisms require continuous feedback of information. Thus, the CSI information is appropriately quantized in order to meet the average feedback bit rate allocated by the system. However, the scheme presented here is by nature a bursty feedback channel. We take this burstiness into account while computing the number of bits for the quantization of the prediction errors for each path. Specifically, we show the gain achievable by this feedback mechanism to be inversely proportional to the probability of update.

Assume that the probability of update for the BEACON channel estimation algorithm is q . Since all the channels for the different paths are assumed to be uncorrelated, we assume that the updating events for the channel estimators are independent and identically distributed. Let the average feedback bit rate supportable be R bits per second. Further, let the number of quantization bits allocated for the prediction error of each path be n . In this case, the average feedback bit rate of the selectively updating system is given by

$$\begin{aligned} R &= \sum_{p=1}^P E[\text{Feedback Rate}|p \text{ updates}] \Pr[p \text{ updates}] \\ &= \sum_{p=1}^P p n {}^P C_p q^p (1-q)^{P-p} = nPq. \end{aligned} \quad (6)$$

Thus, the number of bits one can allocate per prediction error value is $(R/P)/q$, leading to a gain proportional to the inverse of the update probability in comparison to the continuous feedback case. For example, if one were to use WRLS for channel estimation as opposed to BEACON in this scheme, the number of bits would be R/P since $q = 1$.

In Fig. 2, the performance of the prediction error feedback and BEACON based channel estimation scheme with $M = 4$ is compared to a conventional approach, referred to as Modified Mode 1. This conventional approach extends the idea of closed-loop transmit diversity Mode 1 of UMTS (which is applicable only for two transmit antennas). In considering a system with four antennas, the Modified Mode 1 algorithm requires the mobile compute the best beamformer based on its channel estimate, and quantize the relative phase difference between the weights to a QPSK constellation, resulting in 6 bits of feedback information per feedback channel use. For this uplink feedback rate, the quantization requirement for the selective feedback scheme is computed using (6). The update probability of BEACON in this experiment was found to be 0.2. Thus if the prediction error is quantized to 30 bits, it will lead to the same average feedback rate. We assume a feedback delay of $D = 10$ symbols in this simulation. The ideal beamforming performance corresponds to beamforming with the noiseless, but delayed channel knowledge. It can be seen from the figure that the proposed approach exhibits significant improvements over the conventional scheme.

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