DECENTRALIZED SET-MEMBERSHIP ADAPTIVE ESTIMATION FOR CLUSTERED SENSOR NETWORKS

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ABSTRACT

This paper proposes a clustering approach to parameter estimation in distributed sensor networks. The proposed approach is an alternative to the conventional centralized and decentralized approaches. This is made possible by the unique adaptive estimation architecture, U-SHAPE, stemming from set-membership adaptive filtering. At the expense of a slightly degraded mean-square error performance (comparing to the least-squares approach), the proposed approach offers improved data processing flexibility in a distributed sensor network, reduced signal processing hardware and reduced communication bandwidth and power requirements.

***Index Terms*—** Sensor Network Signal Processing, Distributed Estimation, Set-Membership Filtering.

1. INTRODUCTION

Signal processing for distributed sensor networks has been an active area of research recently, see, e.g., [1–5]. In many practical problems, it is desired to estimate an unknown common parameter vector that characterizes the received signal at each sensor [6]. This estimation problem is typically solved either by a centralized approach or a decentralized approach. This paper considers this problem with a clustering approach in which spatially distributed sensors are grouped in clusters, which may consist of sensors distributed in geometric proximity or sensors with similar characteristics.

Comparing to the conventional centralized estimation and decentralized estimation, the proposed clustering approach provides a good compromise between the two. It reduces the amount of data that the estimator at the fusion center needs to process when compared to the centralized approach; it reduces the number of estimators and the communication requirement, e.g., power and bandwidth, between the fusion center and the local (cluster) estimators when compared to the decentralized approach. This approach is also more flexible and makes more effective use of the diversity (e.g., spatial diversity) offered by all sensors. In practice, the sensors located in close proximity usually collect data with similar characteristics and render some redundancy. Thus it is more appealing to process them within a cluster using one estimator. In essence, the clustering approach would require less processing power, communication bandwidth, and transmit power.

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Highlighting our approach to this clustered distributed estimation problem is a novel adaptive estimation architecture, termed U-SHAPE [7, 8], which features selective updates of parameter estimates and reduced hardware processors, and which is uniquely suited for the proposed clustered distributed estimation problem. The U-SHAPE architecture is an outcome of a novel adaptive filtering paradigm, namely, set-membership adaptive filtering (SMAF), see, e.g., [7–10]. To solve the proposed clustered distributed estimation problem, this paper also derives extensions of two conventional Optimal Bounding Ellipsoid (OBE) algorithms to accommodate for the multi-dimensional measured signal vector. These newly derived OBE algorithms offer optimal ways to combine the parameter estimates rendered by different clusters. Simulation results have shown that these extended OBE algorithms implemented with U-SHAPE architecture yield performance comparable to that of RLS, while offering signal processing complexity reduction, reduced bandwidth and power in communications and additional flexibility.

This paper is organized as follows: The next section presents a brief overview of SMAF and the derivation of an extended OBE algorithm. Section 3 presents the clustered solution with a notion of optimality defined in the framework of OBE. Simulation results are given in Section 4 while Section 5 concludes this paper.

2. SMAF AND PROBLEM FORMULATION

All SMAF algorithms are derived from an error-bound specification whose value is defined according to applications, see, e.g., [7, 8]. A general formulation that governs the input-output data relationship for the clustered sensors scenario considered here is given by

$$\begin{aligned}\mathbf{y}_k &= \begin{bmatrix}\mathbf{x}_1^T(k)\\ \mathbf{x}_2^T(k)\\ \vdots\\ \mathbf{x}_M^T(k)\end{bmatrix}\begin{bmatrix}w_1(k)\\ w_2(k)\\ \vdots\\ w_M(k)\end{bmatrix} + \begin{bmatrix}n_1(k)\\ n_2(k)\\ \vdots\\ n_M(k)\end{bmatrix} \\ &= \mathbf{X}_k\mathbf{w}_k + \mathbf{n}_k\end{aligned} \tag{1}$$

where $\mathbf{X}_k \in \mathbb{R}^{M\times N}$ is the input data matrix to the M network nodes (or sensors) that estimate a common global parameter vector $\mathbf{w}_k \in \mathbb{R}^{N\times 1}$. In the case of a time-invariant parameter, $\mathbf{w}_k = \mathbf{w}$.

In the SMAF framework, at a time instant k, the received data pair $\{\mathbf{y}_k, \mathbf{X}_k\}$ defines the constraint set $\mathcal{H}_k$

$$\mathcal{H}_k = \{\mathbf{w} \in \mathbb{R}^N : \|\mathbf{y}_k - \mathbf{X}_k\mathbf{w}\|^2 \leq \gamma^2\}. \tag{2}$$

where γ is a designer specified estimation error bound. Given a sequence of data pairs, $\{\mathbf{y}_k, \mathbf{X}_k\}, k = 1, 2, \cdots, K$, if the parameter vector to be estimated remains constant, it must lie inside the intersection of all the constraint sets, namely,

$$\mathbf{w} \in \Omega_K \triangleq \cap_{k=1}^{K} \mathcal{H}_k \tag{3}$$

The Ω_K in the above equation is termed the *exact membership set*. Every point in the exact membership set is a legitimate estimate for $\mathbf{w}$ as it is consistent with the presumed model and the received data. Note that $\Omega_K \subseteq \Omega_L$, for any $K \geq L$.

One of the important goals for any SMAF algorithm is to obtain an effective analytical description of the exact membership set Ω_K. In practice, however, it is usually more convenient to find some analytically tractable outer bounding sets for Ω_K. For example, the Optimum Bounding Ellipsoid (OBE) algorithms [7, 9] use ellipsoids as such outer bounding sets. The OBE algorithms can be regarded as one of the weighted RLS (WRLS) with forgetting factor whose weighting factor is data-dependent (thus time-varying). Another important difference between OBE and WRLS algorithms is that, at each recursion, the OBE algorithm renders a set of estimates. Each point in the bounding ellipsoid is considered a *feasible solution* to the underlying estimation problem.

A key feature of all recursive SMAF algorithms is a *sparse* data-dependent update of parameter estimates. Specifically, these algorithms update parameter estimates only when the received data contain sufficient new information (namely, *innovation*) to warrant an update of the estimate. This results in a modular adaptive filtering architecture that is comprised of two modules, an information evaluator (IE), which decides whether an update of the parameter estimate is needed; and an updating processor (UDP), which calculates the new parameter estimate. Taking advantage of the sparse updates of SMAF algorithms, the updators can be shared among a number of channels, resulting in U-SHAPE (Updator-SHared Adaptive Parameter Estimation) [7, 8]. For the problem considered here, each cluster has one IE that collects the data from all sensors within the same cluster and decides if an update of the parameter estimate is needed. If an update is needed, the data is passed down to a UDP. For a sensor network that consists of M_c clusters, the proposed U-SHAPE has M_c IE's and M_u updators, where $M_u < M_c$. In this particular scenario, data from each cluster of sensors will result in a parameter estimate which, most likely, differ from other clusters'. These estimates need to be processed collectively by a post-processor that combines all the parameter estimates from all clusters to reach a consensus parameter estimate. Including the post-processor, this expanded U-SHAPE architecture is henceforth referred to as EU-SHAPE, Fig. 1.

Extending the conventional OBE algorithms and using the formulation of (1), this section derives two OBE algorithms for vector measurements. Let E_{k-1} be the optimum bounding ellipsoid at time instant $k-1$,

$$E_{k-1} = \{\mathbf{w} \in \mathbb{R}^N : \|\mathbf{w} - \mathbf{w}_{k-1}\|^2_{\mathbf{P}_{k-1}^{-1}} \leq 1\} \tag{4}$$

where $\mathbf{w}_{k-1}$ is the center of the ellipsoid and $\mathbf{P}_{k-1}$ is a positive semi-definite matrix that characterizes the size (namely, the semi-axes) of the ellipsoid. An ellipsoid E_k that contains the intersection of E_{k-1} and $\mathcal{H}_k$ is obtained by a linear combination of (2) and (4), specifically,

$$\begin{aligned} E_k &= \{\mathbf{w} \in \mathbb{R}^N : \|\mathbf{w} - \mathbf{w}_k\|^2_{\mathbf{P}_k^{-1}} \leq 1\} \\ &= \{\mathbf{w} \in \mathbb{R}^N : \alpha_k \|\mathbf{w} - \mathbf{w}_{k-1}\|^2_{\mathbf{P}_{k-1}^{-1}} \\ &\quad + \beta_k \|\mathbf{y}_k - \mathbf{X}_k \mathbf{w}\|^2 \leq 1 + \lambda_k \gamma^2\} \end{aligned} \tag{5}$$

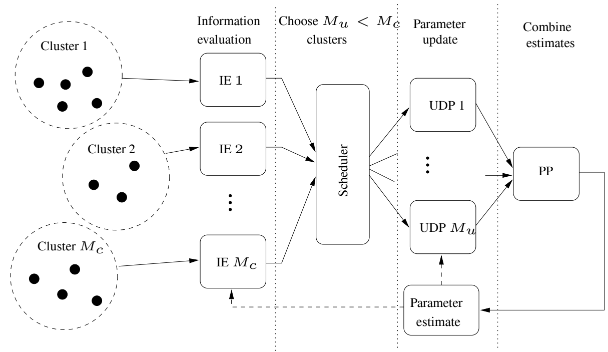

Fig. 1. Clustered estimation with EU-SHAPE.

where α_k and β_k are two coupled variables. In this paper we only consider two possibilities, namely, $(\alpha_k, \beta_k) = (1-\lambda_k, \lambda_k)$ with $\lambda_k \in [0, 1]$, or $(\alpha_k, \beta_k) = (1, \lambda_k)$ with $\lambda_k \in [0, \infty)$, where λ_k is the parameter to be optimized. The former choice leads to an extension of the so-called DH-OBE algorithm [9], referred to here as MIDH-OBE, which offers better MSE estimation performance. The latter corresponds to an extension of the BEACON algorithm [11], termed MI-BEACON, which provides a better trade off between performance and complexity. Expressions for $\mathbf{w}_k$, $\mathbf{P}_k^{-1}$, λ_k, σ_k^2 can be obtained in a similar manner as in [12].

$$\begin{aligned} \mathbf{w}_k &= \mathbf{w}_{k-1} + \lambda_k \mathbf{P}_k \mathbf{X}_k^T \mathbf{e}_k \\ \mathbf{e}_k &= \mathbf{y}_k - \mathbf{X}_k \mathbf{w}_{k-1} \\ \mathbf{P}_k^{-1} &= \alpha_k \mathbf{P}_{k-1}^{-1} + \beta_k \mathbf{X}_k^T \mathbf{X}_k \\ \sigma_k^2 &= \alpha_k \sigma_{k-1}^2 + \beta_k \gamma^2 - \alpha_k \beta_k \mathbf{e}_k^T \mathbf{Q}_k^{-1} \mathbf{e}_k \\ \mathbf{Q}_k &= \alpha_k \mathbf{I} + \beta_k \mathbf{X}_k \mathbf{P}_{k-1} \mathbf{X}_k^T \end{aligned} \tag{6}$$

The MIDH-OBE and MI-BEACON algorithms are complete after determining their corresponding the time-varying factors λ_k. Employing the singular-value decomposition (SVD) of $\mathbf{Q}_k$, σ_k^2 can be rewritten as

$$\begin{aligned} \sigma_k^2 &= \alpha_k \sigma_{k-1}^2 + \beta_k \gamma^2 - \alpha_k \beta_k \sum_i \frac{[\mathbf{e}_k^T \mathbf{v}_i(k)]^2}{\alpha_k + \beta_k \rho_i} \\ &\leq \alpha_k \sigma_{k-1}^2 + \beta_k \gamma^2 - \alpha_k \beta_k \frac{\|\mathbf{e}_k\|^2}{\alpha_k + \beta_k \rho_{\max}} \end{aligned} \tag{7}$$

where $\rho_{\max} = \|\mathbf{X}_k \mathbf{P}_{k-1} \mathbf{X}_k^T\|_2$, i.e., the maximum singular value of $\mathbf{X}_k \mathbf{P}_{k-1} \mathbf{X}_k^T$. To avoid the computation of the maximum singular value, we could use $\rho_{\max} \leq \text{trace}[\mathbf{X}_k \mathbf{P}_{k-1}^{-1} \mathbf{X}_k^T]$. Minimizing the last expression (which upper bounds σ_k^2) yields for the MI-BEACON

<u>MI-BEACON</u>:

$$(\alpha_k, \beta_k) = (1, \lambda_k)$$

$$\lambda_k = \begin{cases} \frac{1}{\rho_{\max}} \left[\frac{\|\mathbf{e}_k\|}{\gamma} - 1 \right] & \text{if } \|\mathbf{e}_k\| > \gamma \\ 0 & \text{otherwise.} \end{cases} \tag{8}$$

and, for the MIDH-OBE,

$$\boxed{\begin{aligned}&\underline{\text{MIDH-OBE}}:\\&(\alpha_k,\beta_k)=(1-\lambda_k,\lambda_k)\\&\lambda_k=\begin{cases}0 & \text{if } \|\mathbf{e}_k\|\le\gamma\\ \min\{\xi,\ \lambda^*\} & \text{if } \lambda^*>0 \text{ and } \|\mathbf{e}_k\|>\gamma\\ \xi\in(0,1)\ \text{(const.)} & \text{otherwise.}\end{cases}\end{aligned}} \tag{9}$$

where ξ is a predefined constant (typically $\xi = 0.5$) and λ^* is the positive (real-valued) root of the following quadratic equation

$$\begin{aligned}f(\lambda)&=(\rho_{\max}-1)\tau_k\lambda^2+2\tau_k\lambda+g_k\\ g_k&=(\gamma^2-\sigma_{k-1}^2)/\|\mathbf{e}_k\|^2,\ \tau_k=(\rho_{\max}-1)g_k+1\\ \Rightarrow\lambda^*&=-\frac{1}{\rho_{\max}-1}+\sqrt{\frac{1}{(\rho_{\max}-1)^2}-\frac{g_k-1}{(\rho_{\max}-1)\tau_k}}.\end{aligned} \tag{10}$$

3. CLUSTERED DECENTRALIZED SOLUTION

Given a network of M sensors grouped into M_c clusters, the sensored data can be formulated similarly to (1) as

$$\mathbf{y}_k=\begin{bmatrix}\mathbf{y}_{1,k}\\ \mathbf{y}_{2,k}\\ \vdots\\ \mathbf{y}_{M_c,k}\end{bmatrix}=\begin{bmatrix}\mathbf{X}_{1,k}\\ \mathbf{X}_{2,k}\\ \vdots\\ \mathbf{X}_{M_c,k}\end{bmatrix}\mathbf{w}+\begin{bmatrix}\mathbf{n}_{1,k}\\ \mathbf{n}_{2,k}\\ \vdots\\ \mathbf{n}_{M_c,k}\end{bmatrix} \tag{11}$$

Clustering the data into M_c groups can, e.g., be motivated by local proximity which results in spatial correlation and similar background noise statistics. For each of the M_c clusters, the MIDH-OBE or the MI-BEACON algorithms in (6)–(9) can be employed to estimate the unknown parameter vector $\mathbf{w}$. With these OBE algorithms, we can employ the EU-SHAPE architecture (see, Fig. 1) which would, in general, require $M_u < M_c$ updators for parameter estimates.

We now consider the special case of two updators. Each updator generates an ellipsoid, say E_{Ui} $(i = 1, 2)$, that contains feasible parameter estimates. To obtain a consensus estimate, $\mathbf{w}^*$, we need to find an ellipsoid E^* that tightly outer bounds the intersection of E_{U1} and E_{U2}. The resulting bounding ellipsoid and its center $\mathbf{w}^*$, which is taken as the point estimate, are given by

$$\begin{aligned}\mathbf{w}_k^*&=\mathbf{P}_k^*[\alpha_k'\mathbf{P}_{U1,k}^{-1}\mathbf{w}_{U1,k}+\beta_k'\mathbf{P}_{U2,k}^{-1}\mathbf{w}_{U2,k}]\\ &=\mathbf{w}_{k-1}^*+\mathbf{P}_k^*[\alpha_k'\lambda_{U1,k}\mathbf{P}_{U1,k}\mathbf{X}_{U1,k}^T\mathbf{e}_{U1,k}\\ &\quad+\beta_k'\lambda_{U2,k}\mathbf{P}_{U2,k}\mathbf{X}_{U2,k}^T\mathbf{e}_{U2,k}]\\ \mathbf{e}_{Ui,k}&=\mathbf{y}_{Ui,k}-\mathbf{X}_{Ui,k}\mathbf{w}_{k-1}^*,\ \ i=1,2\\ \mathbf{P}_k^{*-1}&=\alpha_k'\mathbf{P}_{U1,k}^{-1}+\beta_k'\mathbf{P}_{U2,k}^{-1}\\ \sigma_k^2&=\alpha_k'\sigma_{U1}^2+\beta_k'\sigma_{U2}^2\\ &\quad-\alpha_k'\beta_k'\Delta\mathbf{w}_k^T\left[\beta_k'\mathbf{P}_{U1,k}+\alpha_k'\mathbf{P}_{U2,k}\right]^{-1}\Delta\mathbf{w}_k\\ \Delta\mathbf{w}_k&=\mathbf{w}_{U1,k}-\mathbf{w}_{U1,k}\end{aligned} \tag{12}$$

where, as in the previous section, $(\alpha_k', \beta_k') = (1 - \lambda_k', \lambda_k')$ for the case of the MIDH-OBE algorithm, and $(\alpha_k', \beta_k') = (1, \lambda_k')$ for the MI-BEACON implementation. The equations for the optimal λ_k' are similar to those obtained in the previous section. For the MIDH-OBE algorithm we get

$$\begin{aligned}&(\alpha_k',\beta_k')=(1-\lambda_k',\lambda_k')\\ &\lambda_k'=\begin{cases}\min\{\xi,\ \lambda^*\} & \text{if } \lambda^*>0\\ \xi\in(0,1)\ \text{(const.)} & \text{otherwise.}\end{cases}\end{aligned} \tag{13}$$

where λ^* is the same positive (real-valued) root as in (10) related to the quadratic equation below

$$\begin{aligned}f(\lambda)&=(\rho_{\max}'-1)\tau_k'\lambda^2+2\tau_k'\lambda+g_k'\\ g_k'&=(\sigma_{U2}^2-\sigma_{U1}^2)/\|\Delta\tilde{\mathbf{w}}_k\|^2,\ \tau_k'=(\rho_{\max}'-1)g_k'+1\\ \|\Delta\tilde{\mathbf{w}}_k\|^2&=\Delta\mathbf{w}_k^T\mathbf{P}_{U1}^{-1}\Delta\mathbf{w}_k,\ \rho_{\max}'=\|\mathbf{P}_{U1}^{-1}\mathbf{P}_{U2}\|_2\end{aligned} \tag{14}$$

If complexity is a concern, simulation experience shows that choosing $\lambda_k' = \xi$ (i.e., a fixed constant) gives comparable results.

In the implementation of EU-SHAPE, one also needs to address the issues of contention resolution and scheduling of updates. Such issues have been addressed for U-SHAPE in [7]. Due to space limitation, those issues of EU-SHAPE are not addressed here. However, briefly, one can use the *a priori* estimation error, namely, $\|\mathbf{e}_{i,k}\|^2 = \|\mathbf{y}_{i,k} - \mathbf{X}_{i,k}\mathbf{w}_k\|^2$, of each cluster as a measure of scheduling priority. Note that $\|\mathbf{e}_{i,k}\|^2$ is a known quantity for it is used in IE.

4. SIMULATIONS

In this section, we examine the performance of the MIDH-OBE and MI-BEACON algorithms, (6)–(9), implemented with EU-SHAPE for the clustered adaptive estimation problem. We also compare the results obtained to those of a standard WRLS algorithm which is considered the fastest converging algorithm. The WRLS solution used a forgetting factor $\lambda = 0.99$, and it reaches the consensus estimate by simply averaging the parameter and covariance estimates rendered by all clusters. The error bound for the MIDH-OBE and MI-BEACON algorithms is set by $\gamma = \sqrt{3M_c\sigma_n^2}$.

The environment consists of $M = 30$ sensors and $M_c = 10$ clusters. Each sensor estimates a common parameter vector $\mathbf{w}_o$, which is randomly generated here, with $N = 20$ coefficients. The input of sensor i $(i = 1, \ldots, M)$ is taken as colored noise generated according to

$$x_i(k)=\eta_i x(k-1)+\vartheta_i w_i(k),\ i=1,\ \ldots,\ M \tag{15}$$

where $w_i(k)$ is a zero-mean Gaussian noise sequence, $\eta_i \in [0,\ 1)$ is chosen randomly and $\vartheta_i = \sqrt{1-\eta_i^2}$. The SNR for each sensor was set to 30dB. The spatial correlation between sensors in one cluster is defined by the following correlation matrix

$$\mathbf{R}_c=\begin{bmatrix}1 & \varsigma & \cdots & \varsigma^{M/M_c-1}\\ \varsigma & 1 & \cdots & \varsigma^{M/M_c-2}\\ \vdots & \cdots & \ddots & \vdots\\ \varsigma^{M/M_c-1} & \varsigma^{M/M_c-2} & \cdots & 1\end{bmatrix}. \tag{16}$$

where we have chosen $\varsigma = 0.9$.

Fig. 2 shows the MSE versus iterations for the WRLS algorithm and the MIDH-OBE algorithm that employs EU-SHAPE. The EU-SHAPE was implemented with either $M_u = 1$ or $M_u = 2$ updators. We see that the MIDH-OBE algorithm shows comparable results to those of the WRLS in a clustered environment for the case when only $M_u = 2$ updators are employed. For $M_u = 1$, the convergence speed is slightly decreased. However, we stress that the EU-SHAPE architecture generally reduces the maximum updating complexity to only about 10% for $M_u = 1$ and about 20% for $M_u = 2$ of that observed with the WRLS implementation. This is because the WRLS implementation requires updates to take place for all 10 clusters. In addition, the sparse updates of the MIDH-OBE algorithm will further reduce the overall complexity. In 400 iterations, the total number of times an update took place in the EU-SHAPE was 221 for $M_u = 1$, and 324 for $M_u = 2$. This should be compared with a total of 4000 updates ($400 \times M_c$) required by the WRLS algorithm. In other words, the overall complexity of the clustered MIDH-OBE algorithm with $M_u = 1$ is in this example only 6% (8% for $M_u = 2$) of that of the WRLS implementation.

Fig. 3 shows the results obtained with the MI-BEACON algorithm for the same setup as described above. We see that the MI-BEACON converges a little bit slower. However, the number of updates is also reduced. In 400 iterations, the total number of times an update took place in the EU-SHAPE was 170 for $M_u = 1$, and 253 for $M_u = 2$. As with the conventional BEACON [11], increasing γ will reduce the number of updates even further at the expense of an increased steady-state MSE (not shown here).

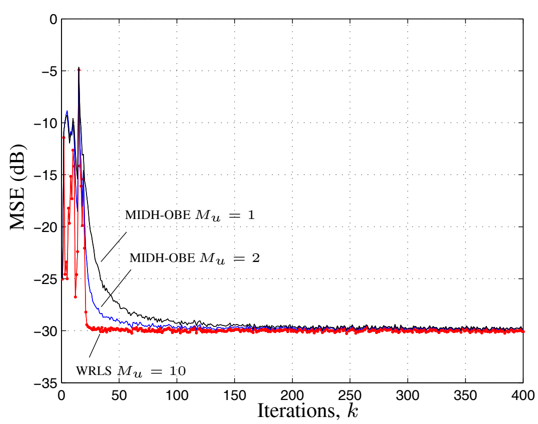

Fig. 2. MSE versus iteration for the MIDH-OBE algorithm.

5. CONCLUSIONS

This paper proposes an distributed adaptive estimation architecture with clustered sensors. Comparing to the conventional centralized or decentralized estimation methods, the proposed approach offers additional flexibility and makes more effective use of diversity in processing data received from individual sensors. In addition, it reduces the signal processing and communication requirements. Simulation results show that such improvements can be achieved without much compromise in the mean-square error performance.

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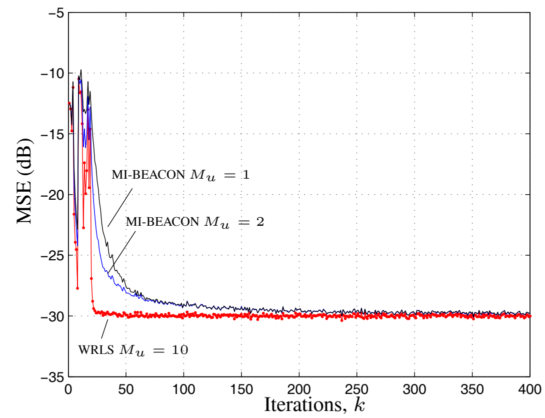

Fig. 3. MSE versus iteration for the MI-BEACON algorithm.

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