AN OPPORTUNISTIC DOWNLINK MIMO-OFDM SCHEME

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ABSTRACT

This paper presents an opportunistic multiple-input multiple-output (MIMO) scheme that achieves increased throughput of multi-user downlink transmission in frequency-selective channels. Developed for OFDM systems, the main idea of the proposed scheme is to find the optimal subset of users for each subchannel and optimal transmit beamforming vector which would maximize the sum-rate. Assuming perfectly known channel information, an optimal solution is derived and a suboptimal solution with much lower complexity that provides comparable results to the optimal solution is also presented. The proposed scheme outperforms the existing FDMA-based and SDMA-based schemes significantly in terms of sum-rate by efficiently exploiting the time-varying nature of radio channels and multi-user diversity.

1. INTRODUCTION

Motivated by the desire to increase the sum-rate for broadcast systems, this paper presents an opportunistic multi-user MIMO (multiple-input multiple-output)-OFDM (orthogonal frequency division multiplexing) scheme. The proposed scheme makes the optimum user assignment for each subchannel, resulting in significant gain in sum-rate. Multi-user MIMO broadcast systems [1] have attracted increasing attention due to the potential gain in sum-capacity. In high data-rate transmission, signals in a multi-user system are corrupted by both co-channel interference (CCI) from other users and intersymbol interference (ISI) due to multipath dispersion. Multi-user transmit beamforming in MIMO systems has been investigated in [2, 3, 4, 5]. In recent years, OFDM technique has received considerable interest, due to its capability of overcoming severe ISI with low complexity.

Practical multicarrier OFDM schemes have been investigated for multi-user MIMO systems, see, e.g., [6, 7]. Most of those schemes fall into two categories: frequency division multiple access (FDMA)-based scheme [6] and space division multiple access (SDMA)-based scheme [7]. In the FDMA-based schemes, each subchannel is assigned to only one user. On the other hand, in the SDMA-based schemes, each subchannel is shared by all users. In practice, like in a time-varying radio environment, FDMA-based schemes are not efficient, while SDMA-based schemes can not guarantee better results, either. To maximize the sum-rate, one should find the optimum subset of users sharing each subchannel based on the channel fluctuations, which is the objective of the proposed scheme. User selection and scheduling algorithms in MIMO systems are found in [8] and the references therein. The approaches for user and power allocation in OFDM systems (without MIMO) have been investigated by, e.g., [9]. The proposed scheme in this paper explores a two-dimensional (user and frequency) resource allocation: dynamic subchannel allocation and dynamic assignment of users occupying each subchannel. Assuming perfect channel information is available at the BTS (base transceiver station), optimal user-channel allocation (i.e., optimal subset of users for each sub-broadcast channel) and transmit beamforming vectors are derived by maximizing the sum-rate subject to the transmit power constraint and CCI cancellation. Significant gain in sum-rate provided by the optimal solution is shown through simulation employing realistic channel models. A reduced complexity suboptimal solution is then presented, which provides nearly optimal performance. The performance of multi-user MIMO-OFDM schemes employing adaptive modulation is also examined. Some user selection algorithms are also studied in [10, 11] for downlink multi-user MIMO system. In contrast to these algorithms where the number of users in the optimal user subset is fixed, the proposed algorithm in this paper finds, for each subchannel, the optimal user number as well as the optimal user subset. This would provide more gain in sum-rate.

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2. EXISTING MULTI-USER MIMO-OFDM SCHEMES

Consider a multi-user MIMO-OFDM system with K users for broadcast channels. Assume there are M_t BTS antennas and, at each user, there are M_r MS antennas. The discrete impulse channel response between the m_t-th BTS antenna and the m_r-th MS antenna of user k is expressed as: $h^{(k)}_{m_t,m_r}(n) = \sum_{l=0}^{L} h^{(k)}_{m_t,m_r,l}\delta(n-l)$. The channel coefficients $h^{(k)}_{m_t,m_r,l}$ are modeled as complex zero-mean Gaussian random variables. Without loss of generality, $\{h^{(k)}_{m_t,m_r,l}\}$ are assumed to be uncorrelated for all k, m_t, m_r and l, and the channel coefficients are invariant within one OFDM data block.

Through N-point OFDM modulation, a frequency-selective broadcast channel is decomposed into N parallel flat broadcast channels. Each subchannel is a flat broadcast channel with M_t BTS antennas and K users. For simplicity of discussion, we focus on the case with M_r = 1. Transmit beamforming is used at the BTS for each subcarrier and each user. So the signal model for subcarrier n is expressed as

$$Y_{k,n} = \sum_{l=1}^{K} \mathbf{W}_n^{(l)^H} \mathbf{H}_n^{(k)} S_{l,n} + V_{k,n},\ k = 1, \ldots, K \qquad (1)$$

where $Y_{k,n}$ and $S_{k,n}$ are, respectively, the received and transmitted signals of user k at the n-th subchannel. $\mathbf{W}_n^{(k)}$ is the beamforming vector and $\mathbf{H}_n^{(k)}$ is the frequency channel response for the n-th frequency tone. Perfect channel information is assumed at the BTS.

2.1. FDMA-Based Scheme

In an FDMA-based scheme, each subchannel is used by only one user and different users occupy different subsets of subchannels. The CCI suppression is achieved by having different users occupy different subchannels. To maximize the sum-rate, the best user (i.e., with largest $||\mathbf{H}_n^{(k)}||^2$) is chosen to occupy each subchannel. The beamforming vectors for subchannel n with power constraint P_n are

$$\mathbf{W}_n^{(k)} = \begin{cases} \frac{\mathbf{H}_n^{(k_o(n))*}}{||\mathbf{H}_n^{(k_o)}||} & k = k_o(n) \\ \mathbf{0} & k \neq k_o(n) \end{cases} \quad (2)$$

where $k_o(n) = \arg\max_{k=1,\ldots,K} ||\mathbf{H}_n^{(k)}||^2$. Thus the sum-rate of an FDMA-based scheme is given by

$$R_{\text{FDMA}} = \sum_{n=0}^{N-1} \max_{k=1,\ldots,K} \log_2\left(1 + \frac{P_n||\mathbf{H}_n^{(k)}||^2}{\sigma_v^2}\right). \quad (3)$$

Low complexity is an attractive feature of FDMA-based schemes. However, the sum-rate improvement is generally not significant, especially in the higher SNR region. This is because it is not efficient to allow each subchannel to be used by only one user and information theory suggests that it is necessary to share some subcarriers among different users to achieve the capacity of broadcast channels.

2.2. SDMA-Based Scheme

In an SDMA-based scheme, each OFDM subchannel is shared simultaneously by all K users. Thus any user could cause CCI to other users. One way to remove CCI is to explore the orthogonality of different users' channels. As such, each user's beamforming vector is found to be in the null space of all the other users' channel subspace, i.e.,

$$\max |{\mathbf{W}_n^{(k)}}^H \mathbf{H}_n^{(k)}|, \ {\mathbf{W}_n^{(k)}}^H \mathbf{H}_n^{(l)} = 0 (\forall l \neq k), \quad (4)$$

and the solution is

$$\mathbf{W}_n^{(k)} = \frac{(\mathbf{I} - {\hat{H}_n^{(k)}}^H (\hat{H}_n^{(k)} {\hat{H}_n^{(k)}}^H)^{-1} \hat{H}_n^{(k)})\mathbf{H}_n^{(k)}}{\sqrt{{\mathbf{H}_n^{(k)}}^H (\mathbf{I} - {\hat{H}_n^{(k)}}^H (\hat{H}_n^{(k)} {\hat{H}_n^{(k)}}^H)^{-1} \hat{H}_n^{(k)})\mathbf{H}_n^{(k)}}} \quad (5)$$

where $\hat{H}_n^{(k)} \triangleq [\mathbf{H}_n^{(1)}, \ldots, \mathbf{H}_n^{(k-1)}, \mathbf{H}_n^{(k+1)}, \ldots, \mathbf{H}_n^{(K)}]^H$. $K \leq M_t$ is needed by the SDMA-based scheme to cancel the CCI completely. With equal power allocated to all users, the sum-rate of SDMA-based scheme would be

$$R_{\text{SDMA}} = \sum_{n=0}^{N-1} \sum_{k=1}^{K} \log_2\left(1 + \frac{P_n|{\mathbf{W}_n^{(k)}}^H \mathbf{H}_n^{(k)}|^2}{K\sigma_v^2}\right). \quad (6)$$

An alternative power allocation scheme may be employed to improve the sum-rate over equal power allocation. However, according to our simulation experience, the gain in sum-rate is not significant, especially when the SNR is large.

3. OPPORTUNISTIC MULTI-USER MIMO-OFDM SCHEME

The objective here is to maximize the sum-rate of the multi-user system. To achieve this, each user is assigned a flag parameter $\alpha_{k,n}$ in each subchannel, which takes the value of either 0 or 1. Specifically, $\alpha_{k,n} = 1$ implies that subchannel n is assigned to user k, while $\alpha_{k,n} = 0$ implies that user k does not have access to subchannel n. With a chosen $\alpha_{k,n}$ and beamforming vector $\mathbf{W}_n^{(k)}$, the received signal of user k at the n-th subchannel is given by

$$Y_{k,n} = \alpha_{k,n} {\mathbf{W}_n^{(k)}}^H \mathbf{H}_n^{(k)} S_{k,n} + \sum_{l=1,\neq k}^{K} \alpha_{l,n} {\mathbf{W}_n^{(l)}}^H \mathbf{H}_n^{(k)} S_{l,n} + V_{k,n} \quad (7)$$

where $\alpha_{k,n} {\mathbf{W}_n^{(k)}}^H \mathbf{H}_n^{(k)} S_{k,n}$ is the desired signal, while the CCI contributed by all the other users is $\sum_{l=1,\neq k}^{K} \alpha_{l,n} {\mathbf{W}_n^{(l)}}^H \mathbf{H}_n^{(k)} S_{l,n}$.

The flag parameter $\alpha_{k,n}$ and beamforming vector $\mathbf{W}_n^{(k)}$ are found such that the sum-rate is maximized with a total transmit power constraint P_n and that the CCI is totally canceled, i.e.,

$$\{\alpha_{k,n}, \mathbf{W}_n^{(k)}\}_{k=1,n=0}^{K,N-1} = \arg \max_{\alpha_{k,n} \in \{0,1\}, \mathbf{W}_n^{(k)} \in \mathbb{C}^{M_t \times 1}} \sum_{n=0}^{N-1} \sum_{k=1}^{K} \log_2\left(1 + \frac{\alpha_{k,n} P_n |{\mathbf{W}_n^{(k)}}^H \mathbf{H}_n^{(k)}|^2}{\sum_{k=1}^{K} \alpha_{k,n} \sigma_v^2}\right) \quad (8)$$

subject to

$$\alpha_{l,n} \mathbf{W}_n^{(k)} \mathbf{H}_n^{(l)} = 0, \ 1 \leq l, k \leq K, l \neq k, 0 \leq n \leq N-1 \quad (9)$$

and

$$||\mathbf{W}_n^{(k)}||^2 \leq 1, \ \forall 1 \leq k \leq K, 0 \leq n \leq N-1. \quad (10)$$

To guarantee that the CCI is totally canceled for all the cases, assume $K \leq M_t$. With some choice of $\Lambda = \{\alpha_{k,n}\}_{1 \leq k \leq K, 0 \leq n \leq N-1}$, the solution of $\mathbf{W}_n^{(k)}$ to the problem (8)∼(10) is shown to be

$$\mathbf{W}_n^{(k)}(\Lambda) = \begin{cases} \mathbf{0} & \text{if } \alpha_{k,n} = 0 \\ \mathbf{f}_n^{(k)} / ||\mathbf{f}_n^{(k)}|| & \text{if } \alpha_{k,n} = 1 \end{cases} \quad (11)$$

where

$$\mathbf{f}_n^{(k)} = (\mathbf{I} - {\tilde{H}_n^{(k)}}^H (\tilde{H}_n^{(k)} {\tilde{H}_n^{(k)}}^H)^{-1}) \mathbf{H}_n^{(k)} \quad (12)$$

and

$$\tilde{H}_n^{(k)} \triangleq [\alpha_{1,n} \mathbf{H}_n^{(k)}, \ldots, \alpha_{k-1,n} \mathbf{H}_n^{(k-1)}, \ldots, \alpha_{k+1,n} \mathbf{H}_n^{(k+1)}, \ldots, \alpha_{K,n} \mathbf{H}_n^{(K)}]^H. \quad (13)$$

To obtain the optimal solutions of $\alpha_{k,n}$, one needs to perform an exhaustive search through all subchannels. There are $2^K - 1$ possible values for $\{\alpha_{1,n}, \ldots, \alpha_{K,n}\}$. For each possible value, we first calculate the corresponding beamforming vector through (11)∼(13). The value of $\{\alpha_{1,n}, \ldots, \alpha_{K,n}\}$ and the corresponding $\{\mathbf{W}_n^{(k)}(\Lambda)\}_{k=1}^{K}$ which result in the largest R constitute the optimal solution, where

$$R = \sum_{n=0}^{N-1} \sum_{k=1}^{K} \log_2\left(1 + \frac{\alpha_{k,n} P_n |{\mathbf{W}_n^{(k)}}^{H}(\Lambda) \mathbf{H}_n^{(k)}|^2}{\sum_{k=1}^{K} \alpha_{k,n} \sigma_v^2}\right). \quad (14)$$

3.1. A Suboptimal Solution

The exhaustive search for an optimal $\alpha_{k,n}$ solution clearly involves great deal of complexity. A suboptimal solution, whose principle resembles that of the steepest-descent method, is proposed here to reduce complexity. The basic idea is to start with only one $\alpha_{k,n}$ being 1 and find the best user k_1 (the user with the largest $||\mathbf{H}_n^{(k)}||^2$). Then find a user from the rest of users to obtain the largest increase in sum-rate. The algorithm is summarized as follows:

1. Initialize $\kappa_{\text{ones}} = \phi$ and $\kappa_{\text{zeros}} = \{1, 2, \ldots, K\}$, where ϕ denotes the null set.

2. Find k_1 such that

$$k_1 = \arg\max_{k=1,\ldots,K} ||\mathbf{H}_n^{(k)}||^2 \quad (15)$$

and calculate the best throughput for this case:

$$R_{n,o}(1) = \log_2(1 + \frac{P_n||\mathbf{H}_n^{(k_1)}||^2}{\sigma_v^2}). \quad (16)$$

Then update the sets $\kappa_{\text{ones}} \leftarrow \kappa_{\text{ones}} \cup \{k_1\}$ and $\kappa_{\text{zeros}} \leftarrow \kappa_{\text{zeros}} - \{k_1\}$.

3. Set $m = 2$ and begin the following loop

4. Find k_m such that

$$k_m = \arg\max_{l\in\kappa_{\text{ones}}} \sum_{k\in\kappa_{\text{ones}}\cup\{l\}} \log_2\left(1 + \frac{P_n|\mathbf{W}_n^{(k)^H}(\kappa_{\text{ones}}\cup\{l\})\mathbf{H}_n^{(k)}|^2}{(|\kappa_{\text{ones}}|+1)\sigma_v^2}\right) \quad (17)$$

Define $R_n(m)$ as the maximum of the term on the right hand side in the above equation.

5. If $R_n(m) > R_n(m-1)$, then update the sets $\kappa_{\text{ones}} \leftarrow \kappa_{\text{ones}} \cup \{k_m\}$ and $\kappa_{\text{zeros}} \leftarrow \kappa_{\text{zeros}} \cup \{k_m\}$. Then update $m = m+1$ and go to step 4.

6. Otherwise, stop and the solution is $\alpha_{k,n} = 1$ for all $k \in \kappa_{\text{ones}}$ and $\alpha_{k,n} = 0$ for all $k = \kappa_{\text{zeros}}$. The corresponding beamforming vectors are then calculated through (11)∼(13).

3.2. Complexity Issues

We use the number of complex multiplications as a measure of computational complexity. To obtain the optimal solution, there are $\binom{K}{m}$ possible values of $\{\alpha_{1,n},\ldots,\alpha_{K,n}\}$ for the case of assigning m users to a subchannel. For each value, the total complexity in calculating the beamforming vectors and the sum-rate is approximately $m(M_t(3m-2)/2+m^2(1+M_t/2)+\mathcal{O}((m-1)^3))$ complex multiplications. Thus the complexity involved in obtaining the optimal solution is

$$\sum_{m=1}^{K}\binom{K}{m}\left\{m(M\frac{3m-2}{2}+m^2\frac{M_t+2}{2}+\mathcal{O}((m-1)^3))\right\}. \quad (18)$$

For the suboptimal solution, there are only $K-m+1$ possible values of k_m at step 4) with m. To obtain the suboptimal solution, the worst case is that the program stops with $m = K$. Thus the complexity is *upper bounded* by

$$\sum_{m=1}^{M_t}(K-1+m)\left\{m(M\frac{3m-2}{2}+m^2\frac{M_t+2}{2}+\mathcal{O}((m-1)^3))\right\}. \quad (19)$$

In comparison, the complexity for FDMA scheme is KM_t complex multiplications while that of SDMA scheme is $K(M_t(3K/2-1)+K^2(1+M_t/2)+\mathcal{O}((K-1)^3))$. The complexity of the proposed scheme appears to be much larger than that of FDMA scheme. However, the performance improvement over FDMA scheme is substantial, when the SNR is large. The complexity of the proposed scheme is not necessarily larger than that of SDMA scheme, while it always outperforms SDMA scheme.

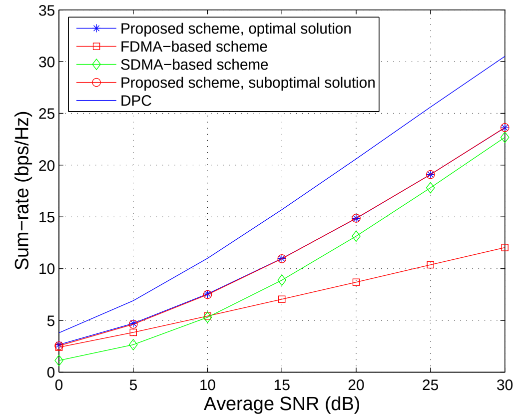

Fig. 1. Sum-rate of a three-user system with three BTS antennas.

4. SIMULATION RESULTS

The proposed scheme is investigated and compared with the FDMA-based and SDMA-based schemes under the assumption of perfectly known channel information. The performance of all schemes is evaluated in terms of sum-rate. The channel model employed here is the SUI-5 model, which is one of six channel models employed in the IEEE 802.16 for the performance evaluation of fixed wireless broadband communications. The size of FFT and IFFT in OFDM is 128. Throughout this section, the SNR is defined as the ratio between the total transmit power and the AWGN variance.

Figure 1 compares the sum-rate of a three-user system with three BTS antennas employing the aforementioned schemes. It is clear that the proposed scheme outperforms the FDMA-based scheme [6]. The improvement increases as the SNR increases. In a three-user system with three BTS antennas, the difference at SNR=30 dB is almost 12 bps/Hz. It is also observed that the difference is larger in a system with more users. At SNR=30 dB, the difference in a four-user system is 18.5 bps/Hz (as shown in Fig.2). This seems to suggest that, when SNR is high, allowing only one user to occupy each subchannel is less effective than sharing each subchannel among different users. Compared to the SDMA-based scheme [7], the proposed scheme consistently offers higher throughput for all SNR values considered here (0-30 dB). This implies that sharing each subcarrier among all users is not always a better solution due to the potential CCI which can sometimes be severe. Furthermore, the improvement is more pronounced when there are more users. As seen in Figs. 1 and 2, at SNR=15 dB, the sum-rate improvement of the proposed scheme over the SDMA-based scheme is 2.1 and 3.8 bps/Hz for three-user and four-user systems, respectively. Dirty paper coding (DPC) is known to achieve the capacity of MIMO broadcast transmission [1]. The sum-rate achieved by DPC with transmit power constraint on each OFDM subchannel is also plotted in Figs.1-2 as an upper bound for comparison. In practice, however, it is difficult to implement DPC in practical systems due to high complexity.

We also investigated the multi-user MIMO-OFDM schemes employing adaptive modulation [12]. In the simulation, adaptive continuous M-QAM modulation was employed. The target BER was at 10^{-4}. Figure 3 shows the sum-rate results for various schemes employing adaptive M-QAM modulation. The system has three antennas and three users. With adaptive modulation, the proposed scheme

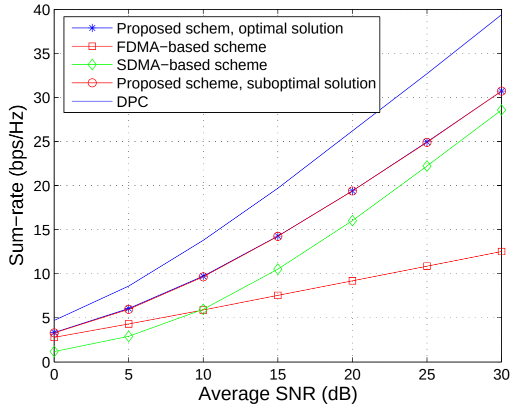

Fig. 2. Sum-rate of a four-user system with four BTS antennas.

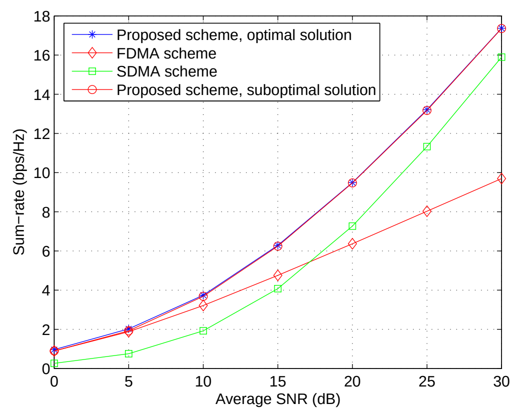

Fig. 3. Sum-rate of a three-user system with three BTS antennas employing adaptive M-QAM modulation.

consistently performs better than the FDMA-based scheme. The improvement becomes more significant as SNR increases. In particular, when SNR=30dB, the difference is almost 8 bps/Hz. The proposed scheme also outperforms an SDMA-based scheme employing adaptive modulation.

The results of the suboptimal solution are also shown in all figures and compared with those of the optimal solution. It is obvious that the suboptimal solution provides almost the same throughput as the optimal solution does. We further compare the performance of SDMA-based and FDMA-based multi-user MIMO-OFDM schemes. It is seen that the FDMA-based scheme always provide higher sum-rate than the SDMA-based scheme when the SNR is low, while the SDMA-based scheme outperforms the FDMA-based scheme when the SNR is high and the difference becomes greater as SNR increases. That implies again that it is more efficient to have each subchannel occupied by fewer users when noise is strong; but when SNR is high, it is better to allow more users using the same subchannel.

5. CONCLUSION

This paper presented a multi-user MIMO-OFDM scheme for downlink transmission in frequency-selective environments. With the assumption of perfectly known channel information at the transmitter, the proposed scheme finds an optimal subset of users for each OFDM subcarrier to maximize the sum-rate with a total transmit power constraint. Both optimal and suboptimal solutions are shown to provide significant sum-rate improvement.

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