LOGIC AND PHILOSOPHY OF SCIENCE IN THE EARLY MARBURG SCHOOL

Abstract: **In this paper I show how the passage from Helmholtz's sign theory to Hertz's picture theory is central to the project of Marburg Neo-Kantianism. I discuss the early Marburgers' distinction between philosophical logic and mathematical logic, which shapes their contribution to the nascent discipline of philosophy of science.**

This paper will analyze the Marburg School's relationship to a set of crucial historical and systematic questions about the relationship between 19th century philosophy and science. The first question is the significance of the shift from sign to picture theory, from Hermann von Helmholtz to Heinrich Hertz. The second question deals with the distinction between mathematical and philosophical logic in the 19th century.[1] I contend that some of the misunderstanding of the neo-Kantian project has stemmed from conflating these two types of logic. I will discuss two figures of the early Marburg School, Friedrich Albert Lange and Hermann Cohen. I will demonstrate how Lange developed his own interpretation of English logic, and how he incorporated it into his idealist philosophy, which was essentially a version of Helmholtz's sign theory. Then I will show how Cohen bridged the gap between Helmholtz's sign theory and Hertz's picture theory, and how he developed an independent interpretation of Hertz's views. Finally, I will examine how Cohen makes a careful distinction between "transcendental," or philosophical, logic and mathematical logic, and how this distinction has led to rampant misunderstanding of his project. Despite its unfortunate consequences, I would submit that it is a useful distinction in the history of the philosophy of science.

[1] For this distinction, see Peckhaus, "19th century logic between philosophy and mathematics," *The Bulletin of Symbolic Logic*, Volume 5, Number 4, Dec. 1999.

Sign to picture theory: the shift from Helmholtz to Hertz, Lange to Cohen.[2]

The story of sign theory begins around the 1830's: as many German historians have picturesquely put it, after Hegel's death in 1831. Philosophers and scientists withdrew from the excesses of speculative philosophy, spurring on a return to materialism in philosophy, even as scientists like Gustav Fechner and Hermann von Helmholtz used Fichte and Schelling to interpret their empirical results. Long before Otto Liebmann sounded the cry "Back to Kant!" in 1865, the founders of empirical psychology, Friedrich Herbart and Fechner, and pioneers of the physiology of perception, Helmholtz and Hermann Lotze, among others, tried to reconcile idealist philosophy with the undeniable challenges it faced from their own research. However, as Liebmann pointed out, they did so by following the so-called German Idealists, Fichte and Schelling most prominently, rather than Kant himself.

When attention shifted to Kantian doctrines in the 1850's and 1860's, two elements of Kant's theory became the center of contention.[3] First, Kant's limitation of logic to the Aristotelian syllogistic, and his related doctrine of the stability of the categories of the understanding, as well as his description of the space described by Euclidean geometry and time as the forms of intuition, came under fire from diverse quarters. Neo-Aristotelians, like Adolf Trendelenburg and Friedrich Überweg, attacked the doctrine that space and time did not apply to things in themselves. Helmholtz, Herbart et al objected to the view that space and time were "pure" intuitions, instead of the formal principles that order our physiological or psychological

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[2] For accounts of the shift from sign to picture theory, see Michael Heidelberger, 1998. "From Helmholtz's Philosophy of Science to Hertz's Picture-Theory," in *Heinrich Hertz: Classical Physicist, Modern Philosopher*, edited by Davis Baird, R.I.G. Hughes, and Alfred Nordmann. Boston Studies in the Philosophy of Science Vol. 198. Dordrecht: Kluwer Academic Publishers, and Jean Leroux, 2001. "'Picture theories' as forerunners of the semantic approach to scientific theories," *International Studies in the Philosophy of Science*, 15.2: 189-197.

[3] See, e.g., Eduard Zeller's 1862 inaugural address as chair at Heidelberg, "On the Meaning and Task of Epistemology," Hermann von Helmholtz's 1855 essay "On Human Sight," and Otto Liebmann's 1865 book, *Kant and the Epigones*.

reactions to stimuli.[4] New developments in English logic challenged the idea that logic had reached its apotheosis with Aristotle. (A long-standing tradition has it that the neo-Kantians shared this view with Kant, but it's not true—see below.) Second, Maxwell's and Hertz's research into electromagnetism in the mid-1800's, which reached Germany quickly, was a potent challenge to Kant's view of philosophy as handmaiden to the Newtonian sciences.

Scientific developments called more and more for a logical approach, for reasons I'll explain in a moment. But logic in Germany wasn't up to it. Among the first people to recognize this were the founders of *Erkenntnistheorie*, a 19th century program of epistemology *avant la lettre*: Friedrich Beneke, a neo-Kantian, Adolf Trendelenburg—but also Helmholtz and Herbart. Early on, they recognized that Kantian and Hegelian philosophical logic were not sufficient as they stood (as Volker Peckhaus points out, Beneke suggested quantifying predicates before Hamilton and/or DeMorgan did it).[5] In what follows, I will present the backdrop to Friedrich Albert Lange's work in logic and the physiology of perception, and its sources in the work of Helmholtz and Hermann Lotze. I will show how Lotze's criticism of Helmholtz's sign theory provoked Lange to develop a logical revision of the sign theory.

Helmholtz claims that the fundamental questions of his epistemology are "common problems" of philosophy and of science, though the two disciplines "attack these questions from opposite directions." While philosophy describes mental relations that are independent from

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[4] For a detailed discussion of these debates, see Lydia Patton, "The Critical Philosophy Renewed: the Bridge between Hermann Cohen's Early Work on Kant and Later Philosophy of Science," *Angelaki* (Routledge) **10**.1 (April 2005): 109-118.

[5] "Application of the diagrammatic methods of the syllogism proposed e. g., by the 18th century mathematicians and philosophers Leonard Euler, Gottfried Ploucquet, and Johann Heinrich Lambert, presupposed quantification of the predicate. The German psychologistic logician Friedrich Eduard Beneke (1798–1854) suggested quantifying the predicate in his books on logic [*Syllogismorum analyticorum origines et ordinem naturalem*, Berlin: Mittler, 1839 and *System der Logik als Kunstlehre des Denkens*, 2 volumes, Berlin: F. Dümmler, 1842], the latter of which he sent to Hamilton" (Volker Peckhaus, "19th century logic between philosophy and mathematics," *The Bulletin of Symbolic Logic*, Volume 5, Number 4, Dec. 1999: 437).

reality, natural science tries to isolate the "laws of reality" from our ideas, that is, from "definitions, systems of symbols, patterns of representation, and hypotheses."[6] Helmholtz's "sign theory" answers the first question, "*What is true in our sense perceptions and thought?"* According to the sign theory, sense perceptions and concepts are signs, not images, of their objects.[7] Helmholtz answers the second question—"*In what way do our ideas correspond to reality*?"—by arguing that "conformity to law is the only condition which something must satisfy in order to be real."[8] The general law of causality is for Helmholtz a transcendental law, an *a priori* condition for constructing any theory that corresponds to reality.[9]

Helmholtz argues that what is *real* in our perceptions and thoughts is the "lawful regularity of phenomena."[10] One can represent this lawful regularity by means of signs, and then interpret the signs as part of a scientific theory. The account of knowledge in Helmholtz's epistemology depends on the claim that the causal relations of the external world can be represented by means of signs. Helmholtz claims that we have access to reality only through representations. He argues, nonetheless, that intuitions and concepts can represent causal relations directly, and that therefore we have access to real lawful relations between objects:

> I need not go into the fact that it is a *contradictio in abjecto* to try to present the actual [*das Reele*] or Kant's thing in itself [*Ding an sich*] in positive statements without comprehending it in our forms of representation. This fact has been pointed out enough already. What we can attain, however, is knowledge of the

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[6] Helmholtz, Hermann von, 1971 [1878], "The Facts of Perception," in *Selected writings of Hermann von Helmholtz*, edited, with an introduction, by Russell Kahl. Middletown, Conneticut: Wesleyan University Press, 368.
[7] Helmholtz glosses the sign theory as the claim that "our sensations are qualitatively only *signs* [*Zeichen*] of the external objects, and certainly not *copies* [*Abbilder*] with any degree of similarity" (Helmholtz 1968 [1869], „Über das Ziel und die Fortschritte der Naturwissenschaft," in *Das Denken in der Naturwissenschaft*. Darmstadt: Wissenschaftliche Buchgesellschaft, 56). He argues that the signs that are our sensations need not resemble the objects they symbolize, any more than the words of a natural language must resemble their objects. Rather, we learn through effort to interpret the signs as we would a language.
[8] Helmholtz 1971 [1878], 388.
[9] *Ibid.*, 389.
[10] *Ibid.*, 386.

> lawful order in the realm of reality, since this can actually be presented in the sign system of our sense impressions.[11]

According to Helmholtz's account, spatial and temporal relations are not properties of the objects themselves, but are ways to organize signs. Helmholtz explains spatial properties by means of a genetic account, according to which the spatial properties of our representations are produced by the physiological process of perceiving. These properties are certainly signs of their objects, but they do not refer directly to properties of external objects.[12]

Helmholtz believes that we learn how to interpret spatial concepts through experience, which means that he has what he calls an empirical theory of spatial perception. This theory coexists, in Helmholtz's epistemology, with his above commitment to the sign theory, according to which spatial properties are only properties of representations. Helmholtz's epistemology commits him to the view that representations arise in a physical process, but are merely signs and not copies of their objects and the relations between objects. According to Helmholtz's explanation of the physiology of perception, even such relations as separation in space and relative spatial position are not real properties of objects.

There are at least two fronts on which to attack Helmholtz's sign theory: his empirical account of how the sign system is produced, and the argument that signs, and the relations between them, are not properties of things in themselves but are nonetheless applicable to reality. In the *Outlines of Psychology*, a transcription of Hermann Lotze's lectures first published in 1881, we find an attack on Helmholtz's program on the first front: Lotze argues that Helmholtz's empirical explanation of the synthesis of representations is inadequate. Lotze begins the second

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[11] Helmholtz 1971 [1878], 388.

[12] For detailed and persuasive accounts of Helmholtz's views on perception and epistemology, see Gary Hatfield, *The natural and the normative: theories of spatial perception from Kant to Helmholtz*. Cambridge, MA : MIT Press, 1990, and Robert DiSalle, "Helmholtz's Empiricist Philosophy of Mathematics," in David Cahan, ed., *Hermann von Helmholtz and the Foundations of Nineteenth-Century Science.* Berkeley: University of California Press, 1994: 498-521.

part of his lectures, on Theoretical Psychology, with a disquisition on the mind [*Geist*]. The first part had been dedicated to "The single elements of the inner life," or sense-impressions; we will return to them later. In §61 of the second part, Lotze addresses "the constant connection of the spiritual [*mental*] life with that of the body."[13] The connection of the mental [*Geistige*] self to the body makes us try to see the "inner" life, the life of sense-impressions and thought, as "simply the product of the bodily functions."[14] Here Lotze makes an observation that will be fundamental:

> Nevertheless it is an ancient truth…that the origin of a mental condition is never analytically comprehensible from all possible combinations of material conditions. Or, more simply said: If we conceive of material elements in such a way that we presuppose nothing in them which does not strictly belong to the conception of *matter*; if we therefore apprehend them merely as space-filling realities, which are movable and can produce motions in each other by means of the forces belonging to them; if, finally, we conceive of these motions, whether of one or of many elements, as no matter how much varied or combined; still the moment never comes when we should be able to say: Now it is self-evident that this motion last produced can no longer remain motion, but must pass over into sensation.[15]

A basic assumption of the sign theory is that signs are produced by material objects or events. Otherwise, there is no way to show that the signs are even indirectly related to real objects. But Lotze argues that we cannot *justify* the transition from material condition to mental condition in the sign theory, because we can't reconstruct the "origin of the mental condition" analytically from the material conditions. (This is an argument found earlier in Hume, by the way, as well as in the "ancient tradition" Lotze cites.) The first problem Lotze finds with Helmholtz, then, is that

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[13] Lotze 1886 [1881]. Originally published as *Grundzüge der Psychologie. Diktate aus den Vorlesungen*, Leipzig. Citation from *Outlines of Psychology: Dictated portions of the lectures of Hermann Lotze*, trans. and ed. George Ladd. Boston: Ginn & Co., §61, 91.
[14] *Ibid*. 91-2.
[15] Lotze 1886 [1881] §61, 92, translation corrected.

explanations depending on the physiology of perception don't explain how we acquire knowledge through perception.

Lotze's account of the progress of material explanation, then, is that it yielded inexorably to a "new form of the attempt": namely, "exactly as physiology deduces the corporeal life from the reciprocal actions of the *physical* forces of all the corporeal elements, exactly so has psychology to explain the mental life from the joint action of the *psychical* forces of these elements."[16] So instead of explaining the origin of the sign system in the reaction of our senses to stimuli, we can try to show how our mind's activity organizes our reactions to physical stimuli into a coherent system. Then we can find the "principles" of the mental sign system. This is the "psychologistic" approach of Johann Friedrich Herbart, among others. Lotze continues, though, to say that this approach, of trying to find a *mental* unity, founders exactly where it should not: on the case of the unity of consciousness. Lotze argues that the disparate sensations and experiences we undergo in what he calls "inner" experience have no principled unity, either. For instance, he points out that:

> The various classes of sensations (colors, tones, smells) exist without intervening terms and as a bare matter of fact, side by side with each other; and they do not constitute a closed system. For example, from the fact that we experience waves of ether as light, it does not in the least degree follow that we must consequently experience waves of air as sound.[17]

Lotze's argument is that if we try to find some *principled* unity of consciousness, or some conceptual point of origin, we will fail. There is no single system of interpretation that will yield a *logic* of sensation. Therefore, he concludes, we must appeal to "the consciousness of the most preëminent element of all, -- the central monad of our body according to Leibnitz."[18] Only a

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[16] Lotze 1886 [1881] §61, 93, corrected translation.
[17] Lotze 1886 [1881] §5, 10.
[18] Lotze 1886 [1881] §63, 96.

monadic, simple mind or soul, he argues, can explain the basic fact of the *connection* of our sensations to one another to form a single system.

Friedrich Albert Lange[19] picks up on Lotze's argument here, and responds that Lotze has ultimately constructed a reductio of the sign theory. Helmholtz and Lotze were missing a key component: neither could demonstrate the possibility of a *logical* foundation for the unity of our representations, or at least, for the connections between them. Instead, Helmholtz relied on the physiological or psychological organization of the perceiving and knowing *subject*. While Lotze realizes that this is hopeless, he becomes bogged down in the argument for a Leibnizian monad as the source of conceptual unification. Lange argues that the idea that the subject is a monad that is unified in essence is not well founded.[20] But perhaps we can give a set of logical principles that organize our observations into a single system?

Lange comes very close to achieving an answer to this problem, or at least as close to an answer as anyone came up with at the time. In the *Logische Studien* of 1877, he responds to the new English work in logic. (The *LS* is an interesting counterexample to the claim that the neo-Kantians were behind the times—especially since John Venn calls the work "admirable" and

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[19] Friedrich Albert Lange was born in 1828 in Wald, near Solingen. He studied a wide range of subjects over his lifetime: pedagogy and its history, the history of the teaching of gymnastics, psychology, moral statistics. In 1841 his family moved to Zürich so that his father, a theologian, could take up a chair. Lange went to the university in Bonn from 1848 to 1851 to study theology, philosophy and philology. Afterward he taught in a gymnasium in Köln (until 1855), and then taught as Privatdozent for two years at the Bonner Universität. There in the summer of 1857 he held a seminar on the history of materialism that attracted nineteen participants. That was the beginning of a book project, the *Geschichte des Materialismus* or *History of Materialism*, that would finally come out in 1865 and go through multiple editions, though only two in his lifetime. In the 1860's he moved to the university at Marburg, where he stayed until his death in 1876.

[20] In the *History of Materialism*, his *magnum opus*, Lange attacks Lotze's concept of the monadic self as "a unitary connecting point" that makes possible "the fusion of the functions of all sensoria" (1881 [1866]. Originally published as *Geschichte des Materialismus und Kritik seiner Bedeutung in der Gegenwart*, citation from *History of Materialism and Criticism of its Present Importance*, trans. Ernest Thomas. London: Trübner & Co., 213). Lange's argument is that the synthetic unity of apperception, and indeed the synthetic unity of our representations, presents us with a "metaphysical riddle." Even *if* we allow for a central connecting point of all the sense mechanisms in the brain, Lange argues, we would not be any further along to explaining in material terms how "out of the multiplicity of the atomic movements there arises the unity of the psychical image" (*Ibid*.). That is precisely the riddle that Lotze was trying to solve with the monadic self.

concedes that Lange had achieved one of Venn's own, admittedly minor, results concurrently, but Venn hadn't known about it to give Lange credit, because the work was published in Germany.[21]) The *Logische Studien* is an example of a diagrammatic proof-system. In it, Lange gives a deduction of all the major forms of the syllogistic using Venn diagrams, which is related to the earlier work of William Hamilton.[22] Lange also jettisons several key elements of the Kantian system, which had hampered earlier attempts to give a logical foundation for science based on the sign system. The result is a real step forward, though the final nail in the sign theory coffin would have to wait for Heinrich Hertz. In what follows, I will present Lange's theory briefly, and show how it contributes to the debate.

For Lange, all formal descriptions are given in terms of extension. Extension can be described either as the literal extension of a figure in space or as a class of objects contained in a general concept.[23] Lange argues that the "new logic" is a logic of extension rather than of content, and that this change is what allows him to build his system. In particular the new practice of determining the extension of a concept rather than its essential characteristics allows Lange to argue that a single activity, that of determining and evaluating the extension of a concept, is fundamental to apodictic reasoning. Lange does not mean to refer only to the literal *drawing* of a diagram (a demonstration in psychological space) when he speaks of determining the extension of a concept by means of demonstrations in intuition. But what evaluations of extension are available to us? How should we regulate their use?

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[21] John Venn, *Symbolic Logic*. London: Macmillan and Co., 1881, 6n and 17n: "When the substance of this chapter was first written out for *Mind* I was unable to ascertain that any attempt had been made to reconstruct the syllogistic figures upon this propositional scheme. I have since found that almost exactly the same results as are given here had been already obtained by F.A. Lange, in his admirable *Logische Studien*, though from a somewhat different point of view."

[22] *Logic. In reference of the recent English treatises on that science*, *Edinburgh Review*, vol. 66 (April 1833), pp. 194–238. I am grateful to Iulian Toader for mentioning the Hamilton book in this connection.

[23] See, e.g., Lange, *Logische Studien: Ein Beitrag zur Neubegründung der formalen Logik und der Erkenntnistheorie*, ed. Hermann Cohen. Iserlohn: Verlag von J. Baedeker, 1877, 145.

Through mathematics (for Lange, both geometry and algebra) and logic, we can manipulate our descriptions of our observations in formally regular ways. These manipulations are what Lange refers to as "apodictic" reasoning. The apodictic, Lange says, is not just any conclusion "supported by whichever 'proofs' there may be, but only those [conclusions], that any person, who has understood the sense of the relevant assertions, must also truly accept and will accept; that, therefore, *over which there can be no further argument.*"[24] In a review of *LS* for *Mind*, J.A. Stewart points out the ramifications of this text for Lange:

> The fact that metaphysicians are not agreed, proves that we must not look for the apodeictic in their various systems, for the apodeictic is self-evident and beyond dispute. The metaphysicians have had it so much their own way since Aristotle's time that the mere *form* of deduction has come to be identified with the apodeictic, however disputed in each system the principles may be and the conclusions derived from them. The professor of a systematic metaphysic thus elevates himself above the man of science to whom he denies the apodeictic. It is the object of Lange in the present work to vindicate against this professorial apodeictic that of μαθηματικη ακριβολογια.[25]

Here Stewart makes a clear distinction between the *metaphysical apodictic* and the *philosophical apodictic*. The "metaphysical apodictic" relies only on the "form of deduction," and not on a logical demonstration of that deduction. Lange uses Trendelenburg and Herbart as examples of proponents of *Erkenntnistheorie* who rely on the metaphysical apodictic: he goes so far as to call them the new "scholastics." Lange argues that no *metaphysical* system can lay claim to an apodictic system of reasoning, whereas most mathematical theories can. For Lange, it is not a system of *principles* that are apodictic, but the method of reasoning. The goal of Lange's formal logic is to make logic more like mathematics in that sense: rather than appealing to a deductive

[24] Lange 1877, 3.
[25] J.A. Stewart, 1878, "Review of Lange's *Logische Studien*," *Mind* **3** (9): 112.

and closed system of axioms, the point is to provide a grounding for a particular *method* of thought.

Lange sets out that method, in four steps. The first is his argument that all truly apodictic reasoning is the same in kind. The second step is his argument that since the truths of mathematics (always meaning both arithmetic and geometry, for Lange) and logic are the same in kind, Kant's distinction between analytic and synthetic reasoning fails. The third basis of Lange's theory is his claim that even logical results alleged to be analytic such as the law of non-contradiction in fact are synthetic, that is, they must be demonstrated or constructed a priori.[26] This last argument rests on an important distinction between the *descriptive* psychological law of non-contradiction, which does not require demonstration, and the *normative* logical law, which does.[27] On the basis of Lange's results he argues that all formal relations, including the syllogistic, must be constructed or demonstrated a priori. Lange develops a system of formal relations that, he says, allow us to give such a demonstration.

Lange argues that Kant's demonstrations that mathematics can be apodictic and synthetic a priori did not go far enough. Lange contends that Kant should have argued further that *all* apodictic reasoning a priori is synthetic, including the fundamental principles of formal logic. For if we remember Lange's definition of apodicticity, it depends on a certain idea of *demonstration*: that, like in a mathematical proof, the steps of reasoning must be such that the

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[26] In my presentation I follow the account given by Christian Thiel in his "Friedrich Albert Langes bewundernswerte *Logische Studien*" (1994, *History and Philosophy of Logic* **15:** 105-126).

[27] In his *Vorlesungen über die Algebra der Logik*, Ernst Schröder identifies one of the main goals of the logic of the time: "We must distinguish physical-physio-psychic, '*psychological*' or *subjective* formal necessity [*Denknotwendigkeit*] from 'logical' or *objective* formal necessity" and remarks, "In striving for our goals, we can thus take comfort in the belief that under certain knowable conditions the objective necessity that we are searching for will always also become subjective. Namely, if we are talking about the unification of unmediated contradictions, then both formal necessities will always occur together. In this context, F.A. Lange remarks very aptly, 'The law of non-contradiction is the point at which the *natural law* of thought comes into contact with the *normative law*'" (Schröder*, Vorlesungen*, Vol. I, Leipzig: Druck und Verlag von B.G. Teubner, 1890, 11-13, citation from Lange 1877, 27-28).

inference is obvious to anyone who has understood the presuppositions. For Lange, formal logic must be demonstrated to be true as well, but he argues that the demonstration can be given a priori.

Lange says in several places that logical and mathematical reasoning a priori must be demonstrated in *spatial intuition*.[28] Does that mean, then, that Lange is restricting logical and mathematical proofs to demonstrations within the "fixed subjective sphere" of *psychological* perception, as he puts it in the *History of Materialism*? No. Lange's account of spatial intuition here is not restricted to particular judgments, but can incorporate what he refers to as a typical intuition [*typische Anschauung*] – an intuition, broadly speaking, of the general in the particular to which we have access legitimately in apodictic reasoning. However, here is where Lange has not quite rid himself of Helmholtzian sign theory, as Ernst Cassirer will later lament![29] He appeals to our "innate" psychological and physiological organization, that is, to our innate concept of spatial organization, to ground his proofs. Lange draws an analogy between the extension of a geometrical figure and the extension of a concept. He argues that logical concepts, like geometrical figures, can be constructed in accord with a given intellectual problem, and that like a geometrical demonstration, the fact that a logical proposition or proof can be constructed is *ipso facto* a proof that the construction is valid (within its own extension).

Here is where Lange's lingering affiliation with Helmholtz raises problems. Lange thinks that the justification for the demonstration is simply the fact that "any person" can follow the proof, because the demonstration rests on the formal aspects of our innate spatial organization.

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[28] See, e.g., Lange 1877: 27-8, 78.

[29] "As much as *Friedrich Albert Lange* attempts to overcome the dogmatism of naturalism, 'psychophysical organization,' which surely describes the puzzle of cognition rather than solves it, remains the last word to him nonetheless" (Cassirer, orig. pub. "Hermann Cohen und die Erneuerung der Kantischen Philosophie," *Kantstudien* **17** (1912), citation from "Hermann Cohen and the Renewal of Kantian Philosophy," trans. Lydia Patton, *Angelaki* (Routledge) **10**.1 (April 2005): 96.

But the question remains, how do we show that these formal connections are real properties of objects? So, for instance, say we observe a triangular object. We construct a mathematical sketch of that object, and prove all sorts of things about it geometrically by means of formal reasoning (following, for instance, the "Platonic" method from the Academy). But how can we prove that the geometrical properties of the object are properties of the real object? For instance, to use a later example, if two lines intersect in our drawing, how do we know they really intersect? What if, in the real object, they somehow slip into another dimension and don't cross at all? (Or, for a less science-fiction example, what if the lines are not perfectly continuous, and so there is no point where there needs to be one.) The problem with Lange's approach, according to this objection, is that he *assumes* that if we must perceive the lines as intersecting according to the laws of our "psychophysical organization," then they must intersect. But he doesn't *prove* it: that is, he doesn't show that the formal principles of "spatial intuition" capture all the relevant formal aspects of external reality, for any given proof.

Now, at this point in German philosophy, there was a clear problem. Helmholtz's sign theory had been well and truly challenged on all sides, and many philosophers saw it as their job to address those challenges. To use a common way of speaking, trying to find a solution to *this* complex of problems was "in the air." One way to address these problems is to give a logical foundation for geometry and mechanics, according to which, if you can show that it's not logically possible for one line to slip behind another, then the problem disappears. So if (after having given a logical foundation for geometry) you can show that the mathematical description of a mechanical process captures every relevant aspect of that process, then mere manipulations of the equations will tell us about real events, without any appeal to the formal principles of intuition. Another way to address the problem is to give the missing proof from Lange's method:

the missing proof that the formal principles of "spatial intuition" capture all the relevant formal aspects of external reality.

Within this very broad historical context, the early analytic tradition and neo-Kantianism could come together, in the sense that they were answering the same question. However, they took the very different paths that I described in the paragraph above. Frege addressed the first, narrowly mathematical question: how can we found geometry and arithmetic logically? On the other hand, Helmholtz's graduate student, Heinrich Hertz, will argue that any system of mechanics that can be deduced from a single mathematical principle a priori forms a *Bild* or picture of reality, and that the formal connections constructed a priori are real connections between real objects, but the content of our observations probably does not apply to reality. Thus, Hertz provides the proof missing from Lange's system, and constructs the first "picture" theory to replace Helmholtz's sign theory. Hermann Cohen, as Lange's intellectual heir, will follow Hertz's trajectory, rather than Frege's logicist project.[30]

In an early work, *The Principle of the Infinitesimal Method (*hereafter *PIM*), Cohen cites Maxwell's equations, Faraday's research, and their nascent "field" idea as a potent challenge to classical mechanics, and even to traditional ontology. Cohen does not consider constructing a

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[30] Cohen attended university at the Friedrich-Wilhelms-Universität, now the Universität Humboldt, in Berlin. Trendelenburg was his dissertation supervisor. After obtaining his doctorate, Cohen used his first Kant-book, *Kant's Theory of Experience*, as his Habilitationsschrift, which would have qualified him for a position as professor. In the text Cohen criticized Trendelenburg's Kant interpretation sharply. It was hardly surprising, then, that Trendelenburg and the others on his committee rejected his petition. After Trendelenburg died, Cohen tried again in 1873. Again Cohen's submission was rejected, this time by Trendelenburg's successor, the prominent historian of philosophy Eduard Zeller, among others. Fatefully, and fortunately for Cohen, he decided to send a copy of *Kant's Theory of Experience* to Lange in Marburg, where Lange was now teaching. Lange was persuaded of the merit of Cohen's work on first reading. Also, Lange was a political dissident and was sympathetic to Cohen's professional near-ostracizing. With Lange's support, Cohen successfully submitted a Habilitationsschrift in Marburg, after some resistance from Kuno Fischer, whose oxen Cohen had gored in an essay on a debate between Trendelenburg and Fischer. Lange made many efforts to get Cohen an academic place at Marburg, but to no avail. However, when Lange died in 1875, the faculty needed someone to carry on his work, and Cohen had been associated closely with Lange. There were objections on religious grounds from the botanist Albert Wigand and the theologian Ernst Ranke. Nonetheless, in 1876 Cohen was appointed to Lange's former professorship. In 1877, Cohen posthumously edited and produced the *Logische Studien*, which Lange had delivered to Cohen three weeks before Lange's death.

logicist foundation for the new mechanics because, as he notes, if mathematics is to be the foundation of mechanics, it must be the tool we use to measure forces. And if mathematics is to measure forces, in turn, it must be linked inextricably to the fundamental mechanical notions of space, time, and matter, which are based on observation. In a revised, novella-length introduction to a posthumous edition of Lange's *History of Materialism*, Cohen attempts to put Lange's work into the new scientific context. In assessing Hertz's *Principles of Mechanics*, Cohen remarks approvingly that

> In the introductory "Overview" that he appended to the complete edition of "Inquiries into the Propagation of Electrical Force," he [*Hertz*] himself deemed the character of his investigations philosophical. "All of these well-supported experiments deliver a proof for the temporal propagation of a so-called *Fernkraft* [*action at a distance*]. This fact is the philosophical, and at the same time the most important result of the experiments, in a certain sense." This judgment is simply an expression of the precise and clear insight that the fundamental hypothesis of the "Faraday-Maxwell theory" rests on those philosophical grounds. According to this fundamental philosophical insight the measure of electrical forces is associated with that of weighable matter, and further, the measure of forces is connected to space, to the nature of space itself, and concurrently to time – thus, to the most significant basic concepts of mechanics.[31]

In 1894, Hertz published *The Principles of Mechanics*.[32] In it Hertz attempts to clear up the confusion over the basic principles of mechanics, not only on the basis of his groundbreaking experimental results, but also by giving a philosophical analysis of the fundamental concepts and principles of mechanics. Hertz describes the precise character of his conceptual analysis in the Introduction to the *Principles*, in which he observes,

> Strictly speaking, what was originally termed in mechanics a principle was such a statement as could not be traced back to other propositions in mechanics, but was

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[31] Cohen, 1928 [1914]. *Einleitung, mit kritischem Nachtrag, zur neunten Auflage von Langes Geschichte des Materialismus*, in *Hermann Cohens Schriften zur Philosophie und Zeitgeschichte*, Albert Görland and Ernst Cassirer, eds. Berlin: Akademie Verlag: 246, cited hereafter as *Einleitung*. The citation from Hertz is from "Inquiries into the Propagation of Electrical Force," in Hertz, *Gesammelte Werke* Bd. II, Leipzig: J. A. Barth, 20.
[32] Hertz 1956 [1894], *The Principles of Mechanics*, translated D.T. Jones and J.T. Walley. New York: Dover Publications, Inc. Cited hereafter as *Principles*.

> regarded as a direct result obtained from other sources of knowledge…Since Lagrange's time it has frequently been remarked that the principles of the center of gravity and of areas are in reality only propositions of a general nature…. Thus the idea of a mechanical principle has not been kept sharply defined. We shall therefore retain for such propositions, when mentioning them separately, their separate names. But these separate concrete propositions are not what we shall have in mind when we speak simply and generally of the principles of mechanics: by this will be meant any selection from amongst such and similar propositions, which satisfies the requirement that the whole of mechanics can be developed from it by purely deductive reasoning without any further appeal to experience.[33]

At the most general level, Hertz's attention to the relationship of principle to theory is what Cohen takes notice of in his discussion of the "Inquiries," that "According to this fundamental philosophical insight…the measure of forces is connected to space, to the nature of space itself, and concurrently to time—thus, to the most significant basic concepts of mechanics." It is clear from the structure of the *Principles* that when Hertz says that every *element* of the theory should be deducible from the mathematical principle, that does not mean every *theorem* but rather the relations described in Book One:

> The subject matter of the first book is completely independent of experience. All the assertions made are a priori judgments in Kant's sense. They are based upon the laws of the internal intuition of, and upon the logical forms followed by, the person who makes the assertions; with his external experience they have no other connection than these intuitions and forms may have.[34]

Hertz's initial requirement for the construction of a mechanics is that each basic concept, such as space, time, and mass, should be defined by reference to the fundamental principle of the system, without appeal to experience. His criteria for evaluating distinct theories based on distinct principles are based on the logical permissibility, correctness and appropriateness of these systems of principles and fundamental notions.

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[33] *Principles* 4-5.
[34] *Principles* 45.

Hertz discusses three *Bilder* of mechanics. In his presentation of the basic principles of mechanics, Hertz relies on what I will call the "principle condition" mentioned above: a proposition can be counted as a principle of mechanics if and only if it "satisfies the requirement that the whole of mechanics can be developed from it by purely deductive reasoning without any further appeal to experience."[35] As Cohen remarks,

> A "selection" of the propositions set at the foundation [of mechanics] could satisfy this condition, and thus different presentations can be given of the principles of mechanics, or different *Bilder* "of the things of the sensible world and the processes that occur in it."[36]

Hertz identifies three possible *Bilder* for mechanics, and finds the mathematical principles at the basis of each of the *Bilder*. The classical picture (what Cohen refers to as "atomism") is based on d'Alembert's generalization of Newton's third law of motion. The energy-based picture (Cohen's "dynamism" or energetics) is based on Hamilton's principle, which can be seen as a combination of d'Alembert's principle and the principle of the conservation of energy. Finally, Hertz presents his own *Bild*, based on a mathematical principle combining Galileo's law of inertia [*Trägheit*] and Gauss's principle of least constraint. Hertz believes that this choice at the level of principle gives him the freedom to reduce the number of fundamental concepts included in his *Bild*, and thus to address the logical shortcomings of the *Bilder* of classical mechanics and of energetics.

Cohen cites Hertz's three requirements for the so-called "Eindeutigkeit der Bilder." Hertz's use of the term "Bild" differs somewhat from Boltzmann's use of the term above. Hertz's "Eindeutigkeit der Bilder" requires lack of ambiguity and uniqueness – that the *Bild*

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[35] *Principles* 4.
[36] *Einleitung* 249. Citation is from *Principles* 4. In Cohen's original text, the citations are from Hertz's *Gesammelte Werke*.

should paint a direct and unique picture of the phenomena it describes.[37] Hertz's usage also leaves open the possibility of alternative *Bilder*, something that will become crucial to his analysis. Cohen reports Hertz's own criteria for the conceptual analysis of a *Bild* as follows:

> The "Eindeutigkeit der Bilder" is determined through *three* criteria: first through that of *permissibility*, "that all our *Bilder* be logically permissible or, briefly, that they shall be permissible." This permissibility rests on the validity of the "laws of our thought" or: "What enters into the *Bilder*, in order that they may be permissible, is given by the nature of our mind." To mention the works in which we ourselves have clarified Kant's terminology [*that is, Cohen's own Kant-scholarship*], one can see that Hertz does not distinguish consciously between the metaphysical and the transcendental *a priori*; but, at any rate, his further criteria are sufficient to construct boundaries against the ill effects of this confusion. The second criterion is that of *correctness*. The requirements for this are "contained in the *facts of experience*, from which the *Bilder* have been built up." Correctness is, then, not within the power of the *a priori*. One the other hand, the *third* criterion leads back to it again. He describes it as that of *appropriateness*. It expresses the number of "*essential* relations" that are "reflected" in the *Bild*. Appropriateness is manifest, then, as "clearness" and as "simplicity," as the latter, insofar as a "smaller number of superfluous or empty relations" are contained in the *Bild*.[38]

These criteria for *Eindeutigkeit* are Hertz's tools for evaluating the distinct *Bilder* of mechanics given by classical mechanics and by some form of energetics. He presents these *Bilder* with the aim of evaluating them according to the criteria of *Eindeutigkeit*.

It is not possible here for me to give an exhaustive account of Hertz's view in the *Principles.* However, it should be clear from what has gone before that Hertz is able to do what Lange couldn't: to provide a logical criterion to judge between different pictures, or *Bilder*, of reality as described by our mechanical theories. While Hertz retains the idea that all that is *real*

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[37] For a detailed discussion of this notion of "Eindeutigkeit" see Howard 1996.

[38] My translation of the passage from Cohen's *Einleitung* 248. The English translation of Hertz's *Prinzipien* is amended from the 1956 translation as follows: In the original "*Bild*" is translated as "image;" I have left it in the original German, here and in the citations from the Jones and Walley translation that follow. The original for the description of "correctness" reads "enthalten in den *Erfahrungstatsachen*, welche beim Aufbau der Bilder gedient haben." Jones and Walley translate *Erfahrungstatsachen* as "results of experience." I have replaced it with "facts of experience," where "facts" is an equally valid translation of "Tatsachen." Further, in the description of "appropriateness" Jones and Walley do not translate "wiedergespiegelt" directly, whereas I have here, since Cohen cites it directly.

about our *Bilder* are the connections between objects that the *Bild* describes, he is nonetheless able to give rigorous standards with which to judge any given *Bild*.

Mathematical and philosophical logic: the independence of philosophy of science

My final goal is to show how these debates contribute to our understanding of 19th century philosophy and its interpretation of science. In this context, I want to show how Cohen was able to forge a neo-Kantian view, distinct from Frege's, yet responsive to progress in the sciences. My final point is that Cohen distinguishes carefully between mathematical logic, as a *tool* for the sciences, and philosophical logic, as an *interpretation* of the facts of science. (Another way of putting this difference is as the distinction between epistemology, as philosophical logic, and logic proper, as mathematical logic.) As a result of this distinction, Cohen is able to revise Kant's theory, for instance, without violating his Kantian conscience. Since Kant was interpreting different scientific facts, his picture of *mathematical* logic was different from Cohen's. However, Cohen thinks that since their *philosophical*, or transcendental, logic is the same, he and Kant still have a common project.

Early in his career, in 1871, Cohen had intervened in a debate between Trendelenburg and Fischer about the possibility of reconciling Kant with new empirical research. Cohen had argued, echoing Liebmann's "Back to Kant!", that returning to Kant's original doctrines would solve the problem. However, as his career progressed, Cohen embarked on a project that could in all fairness be called scholastic: he began to re-interpret Kant's doctrines, one by one, to bring them in line with his own reasoning. It goes beyond the scope of this paper to evaluate Cohen's revisions to Kant. The significant point is that Cohen, like Kant, makes a clear distinction between epistemology and logic, that is, between logic, which evaluates the relations between

concepts only, and the epistemological methods that investigate the relations between concepts and empirical observations. In *PIM*, Cohen argues,

> The revision of the boundaries of such general methods, or rather, the conditions of any sort of scientific methods, must be expressly removed from logic; for logic should investigate only those *relations of thought* that are separate from observation.[39] All help that logic is able to give to knowledge is thus restricted strictly and exclusively to *securing knowledge from the side of those groundings that lie in thought* .[40]
>
> And since mathematics either can or should carry out a grounding of its concepts that goes beyond definitions and axioms, the grounding of the basic concept [*of the infinitesimal*], also presupposed in the limit method, demarcates an area of enquiry that must be distinguished from logic.[41]

Cohen distinguishes deliberately between the foundations of mathematics and logical methods. And yes, he does so because he thinks that logic should investigate only "relations of thought" that are not related to observation, and mathematics must be related to observation. However, note the qualification in the second quotation above: "since mathematics either can or should carry out a grounding of its concepts that goes beyond definitions and axioms." We now have a better idea of what that grounding needs to be: we need to show how mathematical concepts can apply to real objects, without resorting to a sign theory.

Notice that Cohen's distinction between logic and epistemology, above, requires him to distinguish between the grounding for the truths of mathematics and for the truths of logic, which Lange saw as being identical, since he saw all apodictic truth as the same in kind. This difference results from Cohen's engagement in the task of figuring out the philosophical consequences of Maxwell's and Hertz's new research into electromagnetism, and Hertz's new groundwork for

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[39] In Cohen's work, the word "Anschauung" means *only* observation, i.e., the scientific method of observation that can in theory be purified of all psychological or physiological content.
[40] Cohen 1928 [1883], §4: *Distinction between Logic and Theory of Cognition [Erkenntnistheorie]*.
[41] Cohen, *PIM*, reprinted in *Hermann Cohens Schriften zur Philosophie und Zeitgeschichte*, Vol. 2, Albert Görland and Ernst Cassirer, eds. Berlin: Akademie Verlag, 1928 [1883], §2.

that research in the *Principles of Mechanics*. Thus, there is a secondary historical question, namely, what can we say about the progress in neo-Kantian understanding of the difference between mathematical and philosophical logic in the 19th century? An investigation of *this*

question will be directly relevant to the core of the neo-Kantian philosophical interpretation of