YALE INTERNATIONAL CENTER FOR FINANCE

Yale ICF Working Paper No. 03-03
March 10, 2003

ESTIMATING THE DYNAMICS OF MUTUAL FUND ALPHAS AND BETAS

Matthew Spiegel
Yale School of Management
Harry Mamaysky
Morgan Stanley
Hong Zhang
Yale School of Management

Estimating the Dynamics of Mutual Fund Alphas and Betas *

Harry Mamaysky †
Morgan Stanley
1585 Broadway
New York, NY 10036

Matthew Spiegel ‡
Yale School of Management
P.O. Box 208200
New Haven, CT 06520-8200

Hong Zhang §
Yale School of Management
P.O. Box 208200
New Haven, CT 06520-8200

March 10, 2003

*We thank Robert Engle, Wayne Ferson, Will Goetzmann, and Geert Rouwenhorst for their comments. Much of this work was done when Harry Mamaysky was at the School of Management at Yale University. We would like to thank participants at the Rutgers' Conference honoring David Whitcomb, and seminar participants at Boston College, and the University of Wisconsin-Madison.

†Email: harry.mamaysky@morganstanley.com, phone (212) 761-1022, fax (212) 761-9589.

‡Email: matthew.spiegel@yale.edu, phone (203) 432-6017, fax (203) 436-0630, som.yale.edu/∼spiegel.

§Email: bill.zhang@yale.edu, phone (203) 436-0648, fax (203) 436-0630.

Estimating the Dynamics of Mutual Fund Alphas and Betas

Abstract

Consider an economy in which the underlying security returns follow a linear factor model with constant coefficients. While portfolios that invest in these securities will, in general, have a linear factor structure, it will be one with time-varying coefficients. However, under certain assumptions regarding the portfolio's investment strategy, it is possible to estimate these time varying alphas and betas. Importantly, this can be done without direct knowledge of either the portfolio manager's exact investment strategy or of the alphas and betas of the individual securities in which the portfolio invests. As other papers in the area of mutual fund performance measurement have found, overall there appears to be little evidence that, in aggregate, fund investors earn superior returns. Of course, even though the average fund may not produce a superior expected return, this need not be true of sub-populations. Using a dynamic coefficient model to find funds with superior expected returns produces fund of fund portfolios that substantially outperform the market benchmark. Furthermore, these portfolios outperform portfolios selected using the traditional OLS approach. Bootstrapped estimates indicate that the median return produced by the Kalman filter selected funds exceeds those selected via OLS by over 1.6% under the single factor market benchmark, and 1.2% under the four factor Carhart benchmark.

JEL Classification: G12, G13.

Over the last twenty years the mutual fund industry has grown at an incredible rate, and this has naturally attracted a lot of attention from the academic and financial community. Because most of these funds are actively managed two questions have arisen: First, does the average mutual fund produce superior returns? Second, can funds with superior future returns be identified ex-ante? For the most part the answer to the first question seems to be no, at least after expenses are taken into account (see, for example Lehmann and Modest (1987), Carhart (1997), Daniel, Grinblatt, Titman, and Wermers (1997), Wermers (2000), and Pástor and Stambaugh (2002a)). The answer to the second question is not as clear, with different studies coming to different conclusions. Hendricks, Patel, and Zechhauser (1993), and Brown and Goetzmann (1995) conclude that finding funds with future expected excess returns is a difficult but perhaps not impossible task. More recently Teo and Woo (2001) find that allowing for the trading restrictions imposed by a fund's advertised investment style helps predict out of sample returns.[1] In contrast, Carhart (1997) argues that whatever selection ability can be found is due to portfolio momentum rather than managerial ability.[2]

What most studies have in common is the maintained hypothesis that past factor loadings reasonably forecast future factor loadings.[3] While this assumption may or may not be true at an individual security level, it seems rather unlikely to hold for managed portfolios. Investors presumably employ portfolio managers to move assets into and out of various sectors and securities as part of a dynamic strategy.[4] Absent some mathematical coincidence, the simple act of shifting funds across securities will lead to time varying portfolio loadings on any benchmark. As noted by Admati and Ross (1985), and Dybvig and Ross (1985) a model with static coefficients may then lead to the erroneous conclusion that a manager with market timing abilities produces negative abnormal returns. In response, Grinblatt and Titman (1989a) (hereafter GT) propose a technique that can detect market timing abilities under such circumstances, and implement it in their 1994 paper. However, as Ferson and Schadt (1996) point out correlations between factor loadings and market returns may also be due to predictable changes in time varying expected returns, and thus implement a technique for handling this case.

[1]One might think that professional rating agencies might be able to select funds with superior performance. But Blake and Morey (2000) do not find any evidence MorningStar ratings help in this regard. Another approach has been to look at overseas data. Dahlquist, Engström, and Söderlind (2000) find they can, to a limited degree, identify Swedish mutual funds with future superior performance. Chevalier and Ellison (1999) examine whether or not measures related to the fund manager such as SAT scores can help predict superior stock picking ability. While they find the answer to that question is yes, the evidence that fund investors capture any of it is considerably weaker.

[2]Historically, stock returns with super normal returns in the previous six months, tend to outperform in the following six months. Thus, to the degree that managers simply hold onto a winning portfolio from one year to the next they will appear to outperform their benchmark.

[3]One exception is Grinblatt and Titman (1993). The methodology they use avoids a direct comparison against a specific portfolio, and instead uses an "endogenous" benchmark. However, their technique requires knowledge of the fund's actual composition, which may not always be available. Ferson and Khang (2002) extend the technique to condition the portfolio betas on exogenous variables such as macro economic data.

[4]See Breen, Glosten, and Jagannathan (1989) for an empirical estimate of the potential value of such actions, and Mamaysky and Spiegel (2002) for a theoretical treatment.

This paper extends the mutual fund performance literature along the lines of Ferson and Schadt (1996), hereafter FS. In order to estimate time variation in a portfolio's risk loadings, FS project the latter onto a set of observable macro variables. For example, suppose credit spreads can be used to forecast future expected stock returns and a portfolio manager uses this information to allocate assets. The FS technique is designed to estimate the manager's implicit strategy with respect to credit spreads and then allow for the resulting correlations when judging performance.[5]

However, in contrast to FS, the goal here is to allow for portfolio shifts due to factors unobservable by the econometrician. This is accomplished by assuming that assets are reallocated on the basis of some unobserved factor, and then estimating the system of equations via a Kalman filter. Of course, one can also include the macro economic factors FS use, thereby allowing for both observable and unobservable factors in the specification. Relative to the typical OLS model, this may allow researchers to estimate a portfolio's alpha and betas with less misspecification bias, and thus produce models with better in and out of sample properties.

Using the CRSP mutual fund database, cross referenced to Morningstar's mutual fund classifications, this paper estimates a dynamic model with time varying parameters for a large subset of all U.S. mutual funds. Using the resulting alpha and beta time series, the paper shows that the Kalman filtering approach produces considerably better estimates of their instantaneous values than do standard OLS models. It appears that depending upon the mutual fund category (and thus implicitly the strategy followed) static OLS alphas can be off anywhere from 5 to 87 percent from a fund's time *averaged* alpha.[6] These results imply that previous performance estimates may be very sensitive to the security classes a fund trades in. In addition, they show the potential value of explicitly allowing for managerial portfolio reallocation not only on publicly observed variables as in FS, but also on unobserved factors.

As with the FS model the current model does a better job of fitting the data in sample, and appears to pick up a number of statistical patterns relative to an OLS model with constant coefficients.[7] In fact, the decompositions of the alpha and betas described above follow along the same lines in both papers. However, note that the in sample tests presented here offer the OLS model a better chance than would a direct comparison with FS. By its very nature the FS model employs data that the OLS model does not. Here comparisons between the Kalman filter estimates and those of the OLS model use exactly the same predictive data. Even so, the Kalman filter does a better job of picking up the statistical patterns in the data. More importantly, *out of sample tests* show the empirical model presented here

[5]Several recent papers have adopted this technique for performance evaluation. For example, Christopherson, Ferson, and Glassman (1998), and Blake, Lehmann, and Timmermann (2002).

[6]In contrast, static OLS beta estimates are much more reliable, in that they are never estimated to be off by more than 8% from their time averaged values. However, the dynamic estimates indicate that at any one point in time the OLS betas can lie far from their current values. As with the alphas there is considerable variation across fund types.

[7]This empirical result holds whether one estimates the OLS model on the entire data set or that contained within a rolling window.

does a much better job of predicting future alphas and betas than the standard OLS model with constant factor loadings. Again, this is true even though both use the same data for making their predictions.

The empirical analysis presented here also has a number of normative implications. Past research has traditionally found little evidence of persistence in mutual fund performance. That is, funds with high alphas today have only a weak tendency to have high alphas in future time periods (see Carhart (1997) for example).[8] From a practical point of view, this is discouraging because it offers little hope of finding those mutual funds that tend to be the consistent winners, on a risk-adjusted basis.[9] One potential explanation for this state of affairs is that the traditional approaches for alpha estimation (i.e. OLS regressions) are fraught with statistical problems when applied to portfolio returns. Since the methodology presented here serves to alleviate some of these estimation problems, it stands to reason that it should be able to better identify those funds whose *true* instantaneous alphas are positive.

To test the above proposition, the paper conducts the following experiment. First, select a random subset of the available mutual funds. Second, estimate alphas for these funds via both the standard OLS approach, and the methodology developed here. Third, form out of sample portfolios that go long the five funds that each methodology identifies as having the largest alphas. Fourth, and finally, hedge out the market exposure of each portfolio by using the historical beta estimates from both estimation strategies. By repeating the random selection process many times, bootstrapped return distributions of positive alpha, and zero beta portfolios are constructed on the basis of each empirical methodology. The resulting data shows that the return distribution from the dynamic model both first order stochastically dominates the OLS return distribution, and lies above the risk free rate over 52% of the time. (The OLS selections only outperform the risk free asset 42% of the time.) These relative performance results also hold true when the sample years with negative excess market returns (1994, and 2000) are examined. Altogether, the evidence strongly points to the conclusion that relative to the traditional OLS approach the dynamic model developed here does a better job of estimating portfolio return parameters.

Given the current popularity of the Carhart (1997) four factor model, the out of sample tests also include it as a benchmark. In this case the bootstrap begins by estimating both a four factor Kalman filter and OLS model. Next, using the estimated in sample parameters all four factors are hedged out of any selected mutual fund to produce multi-factor zero beta portfolios. As with the one factor model, the Kalman filter better predicts out of sample returns. However, in the four factor case the results are considerably closer. In general, the Kalman filter selects funds with returns of about 114 basis points per year in excess of those

[8]Interestingly, this result does not hold in the other direction: Poorly performing mutual funds in this time period tend to be the poorly performing mutual funds in subsequent time periods.

[9]One solution taken by the literature has been to seek funds whose stock picks perform well prior to expenses. A positive finding would, at least, show that managers can earn back their fees. See Wermers (2000), and Grinblatt and Titman (1989). A similar approach is taken by Chen, Jegadeesh, and Wermers (2000) who examine the performance of securities recently transacted by a fund.

selected by the four factor OLS model. Importantly, the out of sample return differences remain nearly unchanged when regressed against the four factor returns. Once again, this casts doubt on the possibility that the Kalman filter returns (relative to the OLS returns) arise from the type of return persistence associated with factor loading estimation errors. The Kalman fund of fund portfolios also appear to be somewhat less volatile than those selected by the OLS model, as can be seen from the resulting Sharpe ratios. One possible explanation is that the Kalman filter may be less reliant on finding funds that focus on one particular strategy.

The final test in the paper looks at the degree to which conditioning information, as in FS, adds to the model's ability to fit the data within sample. Overall, the conditioning information does not improve the model's fit (as measured by the R^2 statistic). But this is not true of every fund. The number of funds with significant parameter values somewhat exceeds that which would be produced by chance. From an economic point of view, these findings indicate that while some funds condition on the type of macro information tested here, many do not. For those that do not, the Kalman filter picks up the time variation in their betas and alphas via estimates of the unobserved factor's value. The tests in this paper suggest that perhaps 12% of all mutual funds exhibit investment strategies with some dependence on the lagged treasury bill rate, and on the market dividend yield. Of course, the other funds may be conditioning on macro information not included in this paper's tests, a possibility which offers intriguing avenues for future research.

The remainder of the paper proceeds as follows. Section 1 derives our empirical specification for the dynamic alpha–beta model for portfolio returns. Section 2 derives the alphas and betas of an OLS regression for a dynamic coefficient, linear model. Section 3 describes the data used to estimate the model. Section 4 examines the model's performance across a number of simulated portfolio strategies. Section 5 discusses the model's ability to remove intertemporal patterns from the estimated residuals across fund categories. Section 6 presents our decomposition of OLS alphas and betas for a large cross-section of mutual funds. Section 7 reports the results of our bootstraps for out of sample performance. Section 8 explores the impact of adding macro economic factors like those used in FS to the model. Section 9 concludes. All proofs are in the Appendix.

1 Statistical Model

Portfolio returns and the returns of those securities which constitute them may behave in quite different ways. Therefore a model which appropriately describes the returns of individual securities may poorly describe a portfolio holding those same securities.

Consider, an economy in which the return on asset i is generated by a linear factor model with constant factor loadings, or

$$r_i(t) - r = \alpha(t) + \beta_i{}'(r_m(t) - r) + \epsilon(t). \tag{1}$$

Here β_i is an n by 1 vector of factor loadings, r_m the corresponding per period factor returns, r the risk free rate, and ϵ a random shock. Throughout the paper it is assumed that the errors are normally distributed and independent over time. Note that while returns change over time, their loadings on the economy wide risk factor returns (here, the r's) remain constant.[10] If the r_m's are known, estimates of a security's loadings on the economy's risk factors can be obtained by regressing security returns on factor returns. If one additionally imposes some type of equilibrium or no-arbitrage condition on the economy in question, then knowledge of a security's β's, and of the values of the risk premium in the economy, completely determines that security's expected excess returns.

However, consider a portfolio which holds securities A and B, each of whose returns are given by (1). At any time t the portfolio's return (r_P) equals

$$r_P(t) \quad = \quad f_A(t)r_A + f_B(t)r_B$$

where the f terms equal the fraction of the portfolio invested in each asset. Using this, and equation (1), it is straightforward to see that portfolio returns are also linear in the factor returns $r_i(t)$'s. However, unless the returns on A and B at time t happen to be the same, then the portfolio weights for securities A and B will be different at time $t+1$ than they were at time t. Thus, while time $t+1$ portfolio returns remain linear in the $r_i(t+1)$'s, the weights attached to each factor's return will have changed from the time t weights. Clearly, even in this simple example, security returns and portfolios returns may not be well described by the same model (in particular, a linear factor model with constant coefficients).

Now suppose one wishes to estimate the alphas and betas of the above portfolio, rather than the alphas and betas of its constituent securities. In this case, an OLS estimate of the portfolio's loadings on the r_i's can produce answers that are quite far from the portfolio's true loadings on the factor returns in question.

To address the above problem a statistical model needs to explicitly allow for variation in the fund's portfolio weights over time. In order to remain as close as possible to the traditional OLS approach, start by considering an economy in which security returns are given by a linear factor model. Further assume these coefficients remain constant over time, and that the portfolio satisfies an intertemporal budget constraint. Then the portfolio's time t return equals the weighted average of the returns from the underlying I assets:

$$\begin{aligned} r_P(t) - r(t) &= f(t-1)'\Big(\alpha(t) + \beta'(r_m(t) - r(t)) + \epsilon(t)\Big) - k(t) \\ &= \alpha_P(t) + \beta_P(t)'\Big(r_m(t) - r(t)\Big) + \epsilon_P(t), \end{aligned} \tag{2}$$

[10]Many studies like those of Ferson and Harvey (1991, and 1993), and Ferson and Korajczyk (1995) question whether or not individual security loadings are constant. However, this will not qualitatively alter this paper's conclusion that fund loadings change over time. If anything such underlying intertemporal variation in the underlying securities will only add to the importance of allowing for time variation in the mutual funds themselves.

where the variables α_P, β_P, and ϵ_P are defined by

$$\alpha_P(t) \equiv f(t-1)'\alpha(t) - k(t), \quad (3)$$

$$\beta_P(t) \equiv \beta f(t-1), \quad (4)$$

$$\epsilon_P(t) \equiv f(t-1)'\epsilon(t), \quad (5)$$

with f, α, and ϵ, the I by 1 vectors containing their corresponding firm specific elements f_i, α_i, and ϵ_i. The β term represents a matrix with I columns containing the vectors β_i. Finally, k equals the transactions costs incurred by the portfolio, which for mathematical tractability are assumed to be proportional to the funds under management. In (2), if the CAPM or APT holds period by period, then $\alpha(t)$ equals a vector of zeros for all t. Equation (2) is the main focus of the econometric analysis in this paper, and as such, deserves some discussion.

Thus far the model has employed two important assumptions:

1. The evolution of portfolio wealth must satisfy an intertemporal budget constraint.

2. All stocks have constant betas.

These two assumption together imply that portfolio returns will satisfy a linear factor model, but with time varying coefficients, and with a heteroscedastic innovation term. This suggests that linear-factor, constant-coefficient models for portfolio returns, a common paradigm for empirical work in asset pricing, are misspecified.

Absent information about a fund's holdings and the alphas and betas of the underlying assets, the empirical system in (2) through (5) cannot be estimated. However, these problems can be overcome by adding some additional assumptions. As will be shown, with the proper specification of the dynamics governing a fund's portfolio weights, knowledge of the individual weights, alphas and betas is not necessary.

Let $F(t)$ represent some signal (normalized to have an unconditional mean of zero) that the fund uses to trade. Assume that it follows the AR(1) process (though more general specifications are possible)

$$F(t) = \gamma_F F(t-1) + \eta_F(t) \quad (6)$$

through time. The $\gamma_F \in [0, 1)$ coefficient measures the degree to which the signal's value persists over time, and $\eta_F(t)$ represents an i.i.d. innovation.

If the signal F has value then one expects it to influence both the fund's holdings, and future expected stock returns. Statistically, these dual impacts can be represented by assuming that the portfolio weights follow:

$$f_i(t) = \bar{f}_i + l_i F(t), \quad (7)$$

and that stock alphas equal

$$\alpha_i(t) = \bar{\alpha}_i F(t). \quad (8)$$

Here $\bar{f}_i$ represents the steady-state fraction of the strategy invested in a given security. Alternatively, $\bar{f}_i$ can depend upon any set of observable variables, in which case it may be time dependent. The variable l_i is stock i's loading on a common unobservable factor $\tilde{F}(t)$ which shifts the portfolio weights from their steady-state values. This formulation is generally consistent with Blake, Lehmann, and Timmermann's (1999) finding of mean reversion in fund weightings across securities among UK pension funds. Finally, $\bar{\alpha}_i$ represents the degree to which a stock's expected return is predictable by the signal F. If the signal has no value then all of the $\bar{a}_i$ terms equal zero. Also, the present specification insures that the steady state alpha values equal zero.[11]

Now use (3), (4) and (8) in the above formulation. Also, define $\bar{f}$, l, and $\bar{\alpha}$ as the I by 1 vectors with elements $\bar{f}_i$, l_i, and $\bar{\alpha}_i$ respectively, one finds that

$$\begin{aligned} \alpha_P(t) &= \bar{f}'\bar{\alpha}F(t-1) + l'\bar{\alpha}F(t-1)^2 - k(t) \\ &= \bar{\alpha}_P F(t-1) + b_P F(t-1)^2 - k(t), \end{aligned} \tag{9}$$

for the appropriately defined $\bar{\alpha}_P$ and b_P. Similarly, one has

$$\begin{aligned} \beta_P(t) &= \beta\bar{f} + \beta l F(t-1) \\ &= \bar{\beta}_P + c_P F(t-1), \end{aligned}$$

for the appropriately defined $\bar{\beta}_P$ and c_P.

The $\bar{\alpha}_i$, $\bar{\alpha}_P$, and b_P each play a unique economic role in the analysis. In equation (8), $\bar{\alpha}_i \neq 0$ implies that a given fund's signal has a systematic relationship with the instantaneous excess returns of individual stocks in an economy. Therefore, one can alternatively write $\bar{\alpha}_i^P$ to indicate that this coefficient is both stock *and* fund dependent. The point, though, of having non-zero $\bar{\alpha}_i$'s is to allow the fund's α_P to systematically depend on the fund's trading strategy F. This dependence comes about through a linear term, the $\bar{\alpha}_P$ and a quadratic term b_P. There is no constant alpha term in α_P because in the long-run all alphas are assumed to be zero (their unconditional value). The linear term $\bar{\alpha}_P$ simply measures the degree to which a given fund's strategy is actually related to the instantaneous alphas of individual stocks. Since F can be positive or negative, a non-zero α_P does not indicate either under- or overperformance. The quadratic term b_P, on the other hand, does indicate exactly this – it measures the degree to which a fund is able to systematically go long (short) positive (negative) alpha stocks.[12] Note that this is a sufficient, though not necessary, condition for

[11] Beyond the asset allocation case outlined above, the modeled interaction between the signal $F(t)$ and security alphas can also accomdate market timing strategies. Imagine a fund manager that uses macroeconomic information to move in and out of the market index. In this case $F(t)$ equals the current value of the macroeconomic variable, $\bar{\alpha}_1$ its impact on next period's market return, and l_1 the fraction of the fund the manager invests in the market (with 1-l_1 invested in the risk free asset). Within this setting a high value of $F(t)$ implies an expected period $t+1$ market return that the manager's information indicates will be higher than the overall market expects.

[12] Intuitively, b_P can be thought of as the covariance between a fund's security weights ($f(t)$) and the underlying security alphas.

a given fund to exhibit occasional (as opposed to systematic) risk-adjusted outperformance. A weaker and necessary condition is that a fund's α_P is persistent and occasionally positive (which obtains when $\bar{\alpha}_P \neq 0$ and when $\gamma_F > 0$).

The empirical model derived above is very flexible. For example, if one assumes that $\eta_F(t)$ has a variance of zero, or that γ_F equals zero the FS specification can be reproduced. Importantly, however, even absent these assumptions the model can still be estimated. Also note that nowhere does the econometrician need data on the actual portfolio weights used to produce the observed returns.[13]

The above set of equations provides an empirically implementable structure with which to estimate a fund or strategy's performance. For convenience, the relevant equations are presented below:

$$r_P(t) - r(t) = \alpha_P(t) + \beta_P(t)'\Big(r_m(t) - r(t)\Big) + \epsilon_P(t), \tag{10}$$

$$\alpha_P(t) = \bar{\alpha}_P F(t-1) - k(t) + b_P F(t-1)^2, \tag{11}$$

$$\beta_P(t) = \bar{\beta}_P + c_P F(t-1), \tag{12}$$

$$F(t) = \gamma_F F(t-1) + \eta_F(t). \tag{13}$$

Equations (10–13), can be estimated via extended Kalman filtering. To obtain the observation equation, use (11) and (12) in (10) to eliminate $\alpha_p(t)$ and $\beta_p(t)$ and produce:

$$r_P(t) - r(t) = b_P F(t-1)^2 - k(t) + \bar{\beta}_P\Big(r_m(t) - r(t)\Big) + \Big(\bar{a}_P + c_P\Big(r_m(t) - r(t)\Big)\Big)F(t-1) + \epsilon_P(t) \tag{14}$$

after some minor algebra. Due to the $F(t-1)^2$ term standard Kalman filtering techniques will fail as the conditional variance of $r_P(t) - r(t)$ will no longer be independent of the estimated values of $F(t-1)$. The standard solution is to use a first-order Taylor expansion around the conditional expectation of $F(t-1)$, or

$$\begin{aligned} F(t-1)^2 \approx\ & 2\,\mathbb{E}\Big[F(t-1)\Big| r_P(t-1) - r(t-1), F(t-2)\Big]F(t-1) \\ & - \mathbb{E}\Big[F(t-1)\Big| r_P(t-1) - r(t-1), F(t-2)\Big]^2 \end{aligned} \tag{15}$$

to replace the $F(t-1)^2$ term in equation (14) where $\mathbb{E}$ is the expectations operator.[14] Equation (13) then forms the state equation.[15] Note, the vector c_P has n elements (one for

[13] Of course, other modeling choices are possible, and this is an interesting area for future research. For example, some portfolio strategies lead to known security weightings. In such cases the portfolio alpha and beta in (3) and (4) may be calculated directly, as long as alphas and betas of individual stocks are known.

[14] For details about extended Kalman filtering see Harvey (1989).

[15] The estimated dynamic Kalman filter model bears some philosophical resemblance to the Bayesian approaches found in Baks, Metrick, and Wachter (2001), and Pástor and Stambaugh (2002b). In those papers, the authors wish to investigate optimal fund holdings across investors with different priors regarding managerial ability. As with this model, past data is used to form forecasts of future performance. However, the focus of the present model is on inferring the dynamics of mutual fund holdings, rather than on identifying skilled or unskilled managers.

each risk factor) but only n-1 degrees of freedom. Thus, in the scalar case (as in the CAPM) it can be normalized to one when estimating the model. In the case where n is greater than one, at least one element's value must be fixed or some other normalization must be applied. The other fact needed for estimation is that the variance of $\epsilon_p(t)$, conditional on time $t-1$ information, is given by

$$\mathrm{Var}_{t-1}\Big(\epsilon_P(t)\Big) = \sum_{i=1}^{I} f_i(t-1)^2 \mathrm{Var}_{t-1}\Big(\epsilon_i(t)\Big).$$

This follows from (5), and from the fact that all $\epsilon_i(t)$'s are independent.

The system specified in equations (10–13) imbeds an important timing convention. The alphas and betas which determine time t returns are known at time $t-1$ (assuming that $k(t)$ is deterministic). Therefore any covariance which exists between a portfolio's time t alphas and time t market returns indicates an ability of the portfolio manager to make investment decisions at time $t-1$ which successfully anticipate market returns at time t. Similarly for time t betas and time t market returns. Whether a portfolio manager has such ability or not will effect the interpretation of our results in Section 2.

2 Problems with Constant Coefficient Models

If funds dynamically adjust their portfolio holdings in response to changes in the economy then estimates from a constant coefficient model will generally be systematically biased. As it turns out these biases are readily quantifiable. Roughly, the estimated OLS coefficients can be decomposed into a number of elements, which themselves can be estimated. Thus, it is possible to determine just how biased a particular OLS coefficient may be, and what part of the dynamic structure is responsible. The analysis that follows is similar to that in both FS and GT, but is reproduced here to accommodate this paper's particular setting and notation.

Assume that the return generating model for a given strategy is the following

$$r_P(t) - r(t) = \alpha(t) + \beta(t)\left(r_m(t) - r(t)\right) + \epsilon(t). \tag{16}$$

One example of a structural derivation of such a specification is in the previous section of the paper. However, for the analysis which follows, no assumptions about the dynamics of the above coefficients and error term are necessary, other than the usual regularity conditions needed for the law of large numbers.

Now, assume that for data generated using equation (16), one estimates a single factor, constant coefficient, linear model as follows

$$r_P(t) - r(t) = \hat{\alpha} + \hat{\beta}\, x(t) + \eta(t), \tag{17}$$

where $x(t) \equiv r_m(t) - r(t)$. The following proposition shows that asymptotically the above coefficient estimates converge to expressions which depend on the co-dynamics of $\alpha(t)$, $\beta(t)$, and $(r_m(t) - r(t))$ in (16).

Proposition 1 *Using data originating from equation (16), ordinary least squares estimates of the regression in (17) converge in probability to the following limits:*

$$\begin{aligned}\text{plim}(\hat{\alpha}) &= \mathbb{E}[\alpha(t)] - \frac{\mathbb{E}[x(t)]}{\text{Var}(x(t))}\Big(\text{Cov}(\alpha(t), x(t)) + \text{Cov}(\beta(t), x(t)^2)\Big) \\ &\quad + \left(1 + \frac{(\mathbb{E}[x(t)])^2}{\text{Var}(x(t))}\right)\text{Cov}(\beta(t), x(t)), \end{aligned} \tag{18}$$

and

$$\begin{aligned}\text{plim}(\hat{\beta}) &= \mathbb{E}[\beta(t)] + \frac{1}{\text{Var}(x(t))}\Big(\text{Cov}(\alpha(t), x(t)) + \text{Cov}(\beta(t), x(t)^2)\Big) \\ &\quad - \frac{\mathbb{E}[x(t)]}{\text{Var}(x(t))}\text{Cov}(\beta(t), x(t)). \end{aligned} \tag{19}$$

The proof is in the Appendix. Note as well that the proof easily generalizes to the multi-factor case.

The following sections decompose the estimated OLS alphas and betas into their constituent parts as given by (18), and (19).

3 Data Description and Model Estimation

Monthly mutual fund data from 1970 to 2000, as supplied by CRSP, is used to estimate the model. A fund is only included if it has more than 48 months of return data. Some of the tests in the paper use data from MorningStar. For those tests, a fund must also have a MorningStar assignment into one of nine categories as of the end of 1999. The particular categories used in this study (the set of domestic equity funds) can be found in Table 1. These criteria leave a total of 572 funds with which to conduct the estimation. Other data includes the market factor returns, and T-bill returns from Ken French's web site (http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html), and the CRSP stock decile returns.

The empirical model also uses the dividend yield on the market which is constructed via a three step process. First, the dividends from the previous twelve months of the CRSP value weighted index is divided by the "with dividends" index level. Second, the same is done using the without dividends index level as the divisor. Third, the result from the second step is subtracted from the first to get the dividend yield.

Most of the tables and graphs presented here derive from estimating the dynamic model discussed in Section 1 within a single factor structure. Unless otherwise stated, estimates are conducted under the assumption that the $\bar{f}_i$ are constants. Also, unless otherwise stated, the estimates assume that stock returns are determined via a single factor model with the CRSP value weighted market portfolio as that one factor.

A note is in order at this point about the use of MorningStar data. Since requiring that a MorningStar assignment for a given fund should exist as of 1999 introduces survivorship bias into the sample, care must be taken as to the tests that use this classification and those that do not. For analyses that look only at characterization of the mutual fund alphas and betas, or at model comparisons (but not from a performance point of view), and hence are not sensitive to survivorship issues, the MorningStar classification is used in order to provide further insights into the results. For those tests where statements about performance of a given strategy are made, no classification into MorningStar categories is done. Hence these tests use the entire CRSP mutual fund sample, thereby maintaining to the greatest possible degree unbiasedness of the data, and rendering the results comparable with those of other studies.[16]

4 Simulation Study

The dynamic model presented trades off the simplicity of an OLS estimator for an ability to capture the dynamics associated with managed portfolios. This naturally leads to the question of whether the Kalman filtering technique used here can in fact capture such dynamics. To test this a number of simulations were conducted. Each simulation begins with a specific trading strategy across twenty individual stocks. The individual stocks have constant, but randomly assigned $\bar{\alpha}$'s and β's, and produce returns based upon the economy described in Equations (1), (6), and (8) where the conditional alphas are the product of $\bar{\alpha}$ and a latent factor. This latent factor provides the signal used by the portfolio manager. The $\bar{\alpha}$'s and betas are drawn from normal distributions with means of 0 and 1, and standard deviations 0.005 and 0.8 respectively. The latent factor follows an AR(1) process, with an initial value of zero and an AR(1) coefficient of 0.8 with an error drawn from a independent normal distribution with mean zero and variance .001. Market monthly excess returns from January 1994 to December 1998 are used as the CAPM market factor. Different portfolios strategies are then constructed based upon how the signal is used.

Two types of simulations are conducted labeled "cross sectional" and "time series." In the cross sectional case each run draws a new time series for $F(t)$, and the $\epsilon_i(t)$'s. In addition, the realized market returns from January 1994 to December 1998 are drawn randomly without replacement to produce a run specific sequence of $r_m(t)$ terms. After this portfolios are constructed, and the Kalman filter model estimated. In the time series case the same sequence of values for $F(t)$ is used across all runs, and the market return is the realized sequence (in order) of the actual market returns from January 1994 to December 1998. The only random variables that differ across runs are the $\epsilon_i(t)$'s. This proceedure has the advantage of producing a single time series for the true portfolio alphas and betas, thus allowing one to determine how well the model is likely to fit any one realization of the economy.

Figure 1 displays the results from a simulation in which portfolio shifts follow the postulated empirical model exactly. This is the case where the portfolio manager identifies the signal and incorporates it into the portfolio strategy. More specifically, the portfolio

[16] See Elton, Gruber and Blake (2001) for a discussion of biases in the CRSP mutual fund database.

weight for any security is determined via a steady state and a dynamic part as described by Equation 7. The steady state part is an equally weighted portfolio. The dynamic part is driven by constant exposures to the prespecified factor, and a noise term. The exposures are randomly generated from a standard normal distribution and the noise term from a normal distribution with mean zero and standard deviation 0.0001. Stocks are then bought and sold on the basis of the latent factor's realization.

Panels A and B plot the standard devations from the cross sectional simulations for the true portfolio alphas and betas (Solid Line) and the estimation errors (Dotted Line). The estimation errors are defined as the Kalman estimated parameters minus the true values. Panels C and D plot the cross sectional mean of the estimation errors (Solid line) and their 10% and 90% intervals (Dotted Lines). The beta estimations appear to be unbiased, with the estimation errors varying within a much narrower range than the true betas. This implies that the model is able to capture very volatile beta processes. The alpha estimations contain more noise, but are generally unbiased too.

To further demonstrate how well the model can capture a specific alpha and beta time series, a second set of time series simulations were run. Panels E and F plot the true portfolio alphas and betas as well as their 10% and 90% estimation intervals (Dotted Lines). As the pictures show the model does a very good job of tracking the dynamic alphas and betas. The true time series values not only fall within the 90% boundaries, but generally lie well within. Consistent with the cross sectional simulation, the beta estimates seem to be more accurate. Overall, when the underlying model's assumptions hold exactly, the model does a very good job of estimating the dynamic alphas (α_P) and betas (β_P). This is useful to know since the estimated model uses a linear approximation to the true model and these results indicate that the approximation works extremely well.

For Figure 2 the portfolio managers are assumed to switch between two groups of ten stocks with individual factor loadings of plus or minus .1. When the factor has a realized value of 0.5 or -0.5, the portfolio holds only the first or the second ten stocks, respectively. One can think of this strategy as either representing a fund that switches between sectors, or one that times the overall market by switching between bonds and stocks. As in the previous two simulations the steady state holdings are equally weighted among the twenty stocks. Once again, the cross sectional simulation shows that the model is able to identify very dynamic alphas and betas produced by this strategy. Furthermore the estimations are on average unbiased. In the time series simulation, the true parameters generally lie within the 90% confidence interval.

Figure 3 looks at the results from estimating the dynamic model when a fund selects stocks at random. In this case the portfolio manager omits any useful information. The portfolio weight of any security is one twentieth plus a noise term which is randomly generated from normal distribution with mean 0 and variance 0.04. In this case there is no significant correlation between the portfolio alphas and betas. Both the cross sectional and time series simulations show that the model does not capture the alpha values very well. The problem lies in the volatility of the alphas, which swing wildly up and down without any predictable pattern. To compensate for these swings the Kalman filter tries to fit the fund alpha within a fairly wide band. In terms of statistical inference, the confidence interval appears to be about right. The cross sectional simulation shows that the estimation errors for alphas follow as wide a distribution as the true alphas do. This is because the portfolio alphas contain very little information and can be practically viewed as noise. Nevertheless, the model does a reasonable job of picking up the fund betas.

So far all the simulations have been conducted under the model's null hypothesis that fund managers actively trade their portfolios. This leaves open how well the model preforms with static portfolios like index funds (which are included in the database). The simulation results for such a strategy are displayed in figure 4. The static alphas and betas and the return residuals are generated from distributions as described previously. Funds start with an equally weighted portfolio of the twenty stocks and then hold it for seven years. Since all stocks are assigned a market value of one in the first month, the portfolio is value weighted. Panels A through D report the results from bootstrapping the CAPM factor five hundred times to produce cross sectional results. Panels E and F report the results from two hundred time series simulations. While the cross sectional results indicate that the model underestimates the fund alpha by about 0.028, the time series simulations produce unbiased estimates. It is thus possible that for index funds, or funds that engage in very limited trading the model may produce performance predictions that are too low by a small amount. The results regarding the fund betas are overall unbiased, however for any one time period they may not be. The cross sectional results indicate that early on the model overestimates beta and later on underestimates it. The time series simulations show why. Over time a buy and hold strategy (that does not include the entire market) tends to increasingly weight higher beta securities as, on average, they tend to produce higher returns. Thus, such funds will have a time series beta that drifts upward. The model, however, assumes that the expected beta for a fund has the same unconditional mean each period. Apparently, the model accomodates the slowly increasing betas from the buy and hold strategy by overestimating the fund beta early on, and underestimating it later on. Fortunately, to the degree that index funds hold the "market portfolio" this will not be a problem as such funds have constant portfolio betas equal to one.

Overall it appears that the Kalman filtering technique does a reasonable job of picking up the time variation in fund alphas and betas. What problems exist seem concentrated in the simulations using random stock selection strategies. In terms of drawing economic inferences from the results, there does not appear to be a systematic bias in the estimated values. This is itself of value since the OLS estimates are known to lack this property.

5 Fund Dynamics

Table 1 breaks down the funds by MorningStar category. For each category the last column displays the number of funds for which the Kalman filter estimates diverge from the static OLS estimates. Throughout the tables these are referred to as "dynamic funds" in that they appear to employ strategies that produce time varying alphas and betas. Note that within each category the vast majority of funds fall within the set of dynamic funds. This should not be too surprising. Fund managers are generally active traders, and as the discussion in Section 1 shows such activity will produce time varying return parameters.

Table 2 reports the results of a CUSUMSQ (see Harvey (1989)) test on the residuals of each portfolio. For those cases where the dynamic model does not converge to the OLS model, the errors for about 31% of the funds have been purged of their time series patterns. In total this means that after using the Kalman filter 29% of all funds exhibit no remaining intertemporal patterns in their residuals. This represents a substantial improvement over the results from the OLS specification which by itself produces residuals without time patterns for only 3% of the funds.

Looking across categories the model's ability to purge the errors of any time pattern varies somewhat. From a low of 18% in category 18 (large value) to a high of 45% in category 16 (large blend). A chi-squared test rejects the null hypothesis that the percentage differences across categories are due to chance. This indicates that a fund's investment objectives will impact the model's statistical performance. However, there does not appear to be a pattern across the market capitalizations of the portfolio's target firms. Rather it is those funds that invest in large and mid-cap value stocks that seem to give the model the greatest problems. Across the other size and objective categories the results are fairly uniform.

6 Empirical Decomposition of Alphas and Betas

Tables 3 through 8 show the results from the model's estimation (using the fund sample described in Section 3), as well as the breakdown of OLS estimates for fund returns into their true and dynamic components.

In many studies such as Gruber (1996), Carhart (1997), and FS the estimated alphas tend to be negative. However, those alphas include the fund's expenses and thus represent what might be called the "investor's alpha." Here, as in Grinblatt and Titman (1989b) fund expenses and performance alphas are estimated separately. Under the model, a fund incurs expenses at an estimated rate k. In exchange, the fund manager generates an informative signal F that produces excess returns by allowing trades based upon a stock's sensitivity to the signal via the parameter $\bar{\alpha}$. Table 3 shows that for most fund categories the estimated expenses are about 1.5% per annum (a monthly k of 0.0013). Given industry filings this seems to be about right, since expenses in this case include both management fees and transactions costs.

The fact that the estimated $\bar{\alpha}_P$'s are non-zero suggests that stocks in the economy have non-zero $\bar{\alpha}_i$'s. This indicates that, in general, funds choose trading strategies which are related to the instantaneous alphas of stocks in the economy. This, together with the fact that the γ_F's are non-zero, suggests that there is some hope of finding funds that are currently in an "outperformance" period (recall the discussion of Section 1). From equation (9), note that b_P measures the degree to which funds choose trading strategies that systematically profit from high frequency variation in security alphas over time. Table 3 suggests that most funds have no timing ability. The two exceptions to this are small growth funds (category 39), which have some high-frequency timing ability, and small value funds (category 40), which seem to have the unfortunate ability to systematically go long negative alpha stocks.

Table 3 also provides an estimate of the degree to which fund betas vary over time.[17] Some algebra then shows that the estimated standard deviation of beta equals $\sqrt{\sigma_F^2/(1-\gamma_F^2)}$. These values range from a low of 0.18, to a high of 0.41 per month, and average .27. For a typical fund with an intertemporal average beta of one, this implies that in any one period the 95% confidence interval for its beta lies within .5 and 1.5. Empirically then, trading appears to induce economically significant time variation in mutual fund betas. If anything, one's intuition may indicate that the variation is too large. But, consider that almost half of all funds have documented records of moving at least 20% of their assets (over the time period from 1991–1999) from stocks into bonds, and visa versa (Mamaysky and Spiegel (2001)).[18] Also, note the drift in the fund's beta is not a random walk, as the signal is assumed to mean revert. The estimated persistence parameter (γ_F) takes on values between 0.12 and 0.35 in the data. These rather low estimates indicate that a fund's informational advantage tends to be short lived (a few months at most), and thus managers tend to move their portfolios from their preferred holdings for only short periods of time.

One claim of the dynamic model is that the OLS parameter estimates are biased and that the bias can be decomposed into a number of elements related to various covariance terms. If so, then this provides a mechanism for checking the model. Assuming the dynamic model actually fits the data, then using the equations in Proposition 1 one can create "synthetic" OLS regression estimates by properly summing up the covariance of the dynamic alpha and beta estimates with the market portfolio. The resulting values can then be compared to what one obtains by actually running an OLS model on the data. If the dynamic model properly describes the data, then the synthetic and actual values should be fairly close to each other. Table 4 reports the results from this experiment. Columns six and eight show that in no category does the average absolute percentage difference between the synthetic and actual OLS parameter estimate for alpha exceed 6%, and is generally under 1%. For beta the results are even closer with every category displaying an absolute average difference under 1%.

[17]Since this is a one factor model, the value of c_P has been set to one as a normalization.

[18]Such behavior seems consistent with an attempt to implement something like Breen, Glosten, and Jagannathan's (1989) algorithm for optimally shifting between treasury bills and stocks. When done properly they show that such a strategy can potentially add as much as 2% to a fund's annual returns.

Another test of the model is the degree to which it can better explain the data relative to both a standard and rolling OLS model. The standard model produces parameter estimates based upon a fund's entire history. The rolling OLS model uses only the 48 months of data prior to any particular date. To calculate an R^2 statistic for the rolling OLS model only the final period's error term is used. This gives the OLS model a natural edge since the squared errors come from finding the set of parameter estimates that best fit only the last four years of data, while the Kalman filter parameter estimates are forced to fit the entire time series.

Columns four and five from Table 4 show that the dynamic model increases the R^2 for the both the ordinary and rolling OLS model by 0.09 to 0.30 depending upon the category. Thus, in every case the Kalman filter model, with its time varying alphas and betas, does a better job of tracking fund returns. While this is clearly not conclusive, since it applies in sample, later tables provide out of sample tests with similar results.

Tables 5 to 8 show the decomposition results for the OLS alpha and beta estimates (from equations (18) and (19)). Table 5 breaks down the OLS alphas into their constituent parts. Table 6 shows the percentage contributions of each component to the OLS alpha. Recall from (18) that the OLS alpha is composed of four components. The percent contribution for component i is given by

$$\text{Percent Contribution} \equiv \frac{|c_i|}{|c_1| + |c_2| + |c_3| + |c_4|}.$$

where c_j is the j^{th} component. Panel A reports results for first aggregating the cross-section of funds to compute each component, and then computing the percent contribution using the absolute value of the cross-sectional means of each component. Panel B computes the percent contribution at an individual fund level, and then takes an average of these for the category level numbers.

Note from Panel A that the static part of the dynamic alpha (i.e. the expected value of the dynamic alpha) can account for as little as 13% of the estimated OLS value. For categories 38 (small blend), 39 (small growth), and 40 (small value) the static part of the dynamic alpha accounts for under 50% of the estimated OLS alpha. This implies that OLS estimates for small capitalization fund alphas may be very misleading.

What seems to drive the difference between the OLS and dynamic alphas is the covariance in each fund's beta with the market, and only to a much smaller degree the covariance between the fund's alpha and the market. The beta and market covariance accounts for 21% or more of the estimated OLS alpha in half the categories. In contrast, the median contribution of alpha's correlation with the market is only 5.5%. Essentially, it appears that under the OLS model mismeasurement of the time variation in beta (due to the dynamic strategies employed by funds) then leads to erroneous conclusions about performance (alpha).

Table 6's Panel B produces similar conclusions to those from Panel A. The primary change is that looking at the component contributions this way (i.e. first computing percent contribution at the fund level, and then aggregating into categories) increases the importance of the fund beta with the squared market return. This increase generally comes at the expense of the contribution made by the covariance between a fund's beta and the market. Overall though, it is the time variation in beta that seems to induce the discrepancy between the estimated static OLS alphas and the steady-state values of the dynamic Kalman filter alphas.

Tables 7 and 8 provide similar breakdowns for the OLS beta estimates. These tables show that the OLS betas are quite close to the expected value of the dynamic beta. In no case does the OLS beta differ from the expected value of the dynamic beta by even 10%. However, this does not mean that month by month the OLS beta equals the fund's actual beta, only that the long run averages are the same. As shown previously, month by month fund betas exhibit considerable volatility and any long run average value is likely to be far from the current mark.

7 Performance of Trading Strategies

The tests presented so far have all been within sample. As Ghysels (1998) shows even if a model with time varying risk factors performs well in sample, it may not outperform simpler models out of sample. Since both the simple time invariant and more complex dynamic models are likely to be misspecified it is an empirical question as to which will work better. The out of sample tests in this section are designed to answer this question, and as will be shown the dynamic Kalman filter model does produce superior out of sample predictions.

For a fund to be included in these out of sample tests, that fund must have been alive as of December 1992 and have an ICDI objective of AG, BL, GI, IN, LG, PM, SF or UT. A fund then stays in the sample until the end of the sample period in 2000, or until the fund ceases to exist. Hence, all tests in this section control for survivorship bias. Furthermore, note that none of the results in this section use MorningStar classification data, and therefore are free of the survivorship bias that such classifications can introduce (see the discussion in Section 3).

For the beta tests, portfolios are constructed based upon three criteria. In each case, funds are classified based only upon data available prior to the evaluation year. For example, for 1993 funds are classified based on data from the start of the sample up to the end of 1992. Hence classifications are only based upon information available to the market at the time a decision had to be made. Under criteria A the OLS and Kalman filter models select among all available funds. Whenever the maximization algorithm for the Kalman filter does not converge, OLS forecasts are used instead. Under criteria B, there are two filters. First, funds are only included if the optimization algorithm for the Kalman filter model converged using data from the training period.[19] Typically, this leaves about 70% of the total funds from the original sample. This does not appear to add a systematic bias, as the distribution of the OLS model's R^2 statistic appears to be the same in both the "converged" sample

[19]This means that a fund may be available for selection by the Kalman filter model in one year, and not in another. Selection only depends upon convergence for the training sample in question, not on convergence within other training samples.

and the total population (P-value > 0.98 using the Kolmogorov-Smirnov test). Second, the 200 funds with the highest R^2 statistic for each model are then pulled. This creates two seperate, although possibly overlapping, pools. The bootstrap proceedure that follows then pulls subsamples for each model from its respective 200 fund pool. Under criteria C, the Kalman filter model selects among the 200 funds for which it produces the largest R^2 statistic relative to the OLS model. These are the funds for which the Kalman filter appears to provide the most additional information relative to the OLS model.[20]

Figure 5 displays the results from the out of sample test using criteria B. To create the graph, 40 funds are randomly selected out of each model's 200 fund pools to form equally weighted portfolios. Given 60 months of return data, the estimated coefficients of the OLS model are then used to forecast the portfolio beta in month 61.[21] Kalman forecasts are generated using the entire time history available for the fund prior to month 61.[22] These forecasted results are then used to hedge out the market factor in the 61^{st} month. After the return of the 61^{st} month has been realized, the procedure is repeated for month 62 for the same portfolio, and onward for a total of 12 months. For the test in question, Figure 5 displays the distribution of the correlation between the portfolio's return and the market return over the period 1993-2000. Perfectly hedged portfolios should exhibit no correlation with the market.

The mean correlations equal 0.0876 and 0.0453 for the OLS and Kalman filter models respectively. In years when the market excess return exceeds zero (1993, 1995 to 1999), the mean annual correlations (correlations for monthly returns within a year) are 0.0516 and -0.0379 for the OLS and Kalman models. In bear market years (1994 and 2000), the means are -0.30 and -0.01 for the OLS and Kalman models. While not displayed here, simulations based upon the two other criteria (A and C) were also conducted. Under criteria A, where all the funds are used, the mean correlations are 0.0466 and -0.0398 for the OLS and Kalman models respectively over the sample's eight year period. Under criteria C, the mean correlations come to 0.13 and 0.06 for the OLS and Kalman filter models respectively.

As Figure 5 shows, the dynamic Kalman filter model does a better job of predicting out of sample betas. Using it to construct zero beta portfolios produces portfolios whose mean correlation with the market is close to 0.05. In contrast, using the same hedging algorithm with the OLS model produces portfolios that on average have a correlation with the market close to 0.1. Thus, while both models underestimate portfolio betas to some degree, the OLS model does considerably worse. Potentially then, some of the above market returns documented in previous studies using the OLS model may be due to the OLS model's tendency to underestimate future out of sample betas.[23]

[20]The R^2 for the Kalman filter is defined as the variance of the explained portion of portfolio returns divided by the variance of portfolio returns.

[21]Similar results were obtained when the OLS estimates were conducted both with all available historical data, and with a 2 year rolling window.

[22]Estimating the Kalman filter model over the same 60 month period used in the OLS estimation produced qualitatively similar results.

[23]An underestimation of beta by 0.1, coupled with an excess market return of 6%, yields an overestimation

If the dynamic model better predicts out of sample betas, can it do so with alphas? This is perhaps the more interesting question, as it offers a test of whether it is possible to identify at least some managers that can produce abnormally high risk adjusted returns. Figure 6 and Table 9 report on several out of sample tests for predicted alphas.

For the alpha tests, fund pools are constructed based upon the same three criteria used in the beta tests. In each case the bootstrap begins by randomly selecting 50 funds from the available pool. Then from this subsample the five funds with the highest positive predicted alphas are placed into an equally weighted portfolio.[24] Using the model's predicted beta for this portfolio, the value-weighted CRSP index is hedged out to produce what the model predicts will be a zero beta fund of funds portfolio.

Figure 6 plots the return and Sharpe Ratio distributions of such zero beta portfolios under criteria B and C. Results from using all of the sample funds (irrespective of the model's R^2 statistic), as well as funds selected via criterion B and C, are displayed in Table 9, where the Kolmogorov-Smirnov test rejects the possibility that any two distributions generated by the two models are the same (all P-values are virtually zero). As Figure 6 and Table 9 show, the returns from the Kalman filter derived fund of funds portfolio first order dominate those produced with the OLS model. Generally, the Kalman fund of funds selections yield 1.5% more per annum than do the OLS selections. Importantly, the realized returns do not require investors to short a fund (a typically impossible task). All mutual fund positions are long; the only short position is in the market portfolio. Also note that the Kalman filter results are not due to inadequately hedging out the market risk. As shown in Figure 5, the OLS portfolios are the ones that retain a higher degree of market exposure, and thus have a return advantage based on some systematic factor exposure.

Table 9 displays the result from picking among funds satisfying criteria A (all funds), B (high R^2), and C (most gain from Kalman filter) respectively. Note that the criteria C funds are likely to be the "most dynamic" of the funds, and thus likely to be the ones for which the Kalman filter model can provide the greatest predictive help. In fact, allowing the Kalman filter model to select from among these funds improves its ability to predict out of sample alphas by a considerable amount. For this set of funds, 56% of the fund of funds portfolios outperform the risk free asset. This is an important result because the fund of funds portfolios have no systematic risk (to the best approximation provided by the model). Hence 56% of these fund of funds portfolios exhibit positive abnormal risk-adjusted returns. The comparable numbers for criteria A and B funds are 44% and 53% respectively. The main conclusion to draw from these results is that for funds whose returns are well explained by the Kalman model (i.e. from categories B and C), out of sample performance typically exhibits positive alphas.[25]

---

of performance equal to 60 basis points.

[24] In the rare event that there are fewer than five positive alpha funds in the subsample, an equally weighted portfolio is produced using whatever positive alpha funds are available.

[25] The bootstrapped returns assume that the investor rebalances based upon his model's predictions every month. While the transactions costs should be fairly low (as most funds no longer impose fees to enter or

Table 9 also lists the results from selecting among both B and C funds in 1994 and 2000. These years are singled out because they were years in which the market as a whole underperformed the risk free asset. (The CRSP value weighted returns equal -0.089% and 3.549% in those years respectively.) Using the high R^2 funds, the Kalman filter model produces portfolios whose returns exceed the risk free rate 62% of the time. In 2000 the results are exceptional, with the Kalman filter portfolios producing returns above the risk free rate 84% of the time. In comparison the OLS model portfolios only exceed the risk free rate 31% and 82% of the time in 1994 and 2000 respectively. Thus, the Kalman filter's performance is not just due to an ability to pick up high alpha funds in "good" years. Similar results hold for the category C funds, with the exception of 1994 when neither OLS nor Kalman portfolios yielded positive alphas on average.

Both Table 9 and Figure 6 compare the out of sample fund of funds Sharpe ratios produced by each model. The distribution of Sharpe ratios produced via the Kalman filter model first order dominates those produced by the OLS model under every test run.

Note what happens when the Kalman filter picks among the category C funds (those with the highest Kalman filter model R^2's relative to the OLS R^2's). While this restriction produces only slightly higher expected returns, it produces dramatically improved Sharpe ratios. Thus, the resulting fund of funds produces returns that have an unexpectedly low variance. This indicates that the Kalman filter is not concentrating it picks among funds with one particular strategy. Furthermore, the monthly, out of sample Sharpe ratio for the Kalman filter fund of fund portfolios is 0.14. Under the assumption of iid returns, this gives an annual Sharpe ratio of 0.48, which is higher than the average Sharpe ratio of the S&P over the last half century. Considering that the strategy yielding the 0.48 Sharpe ratio has no systematic risk, one can conclude that such a strategy offers potentially valuable diversification benefits.

7.1 Tests using a Four Factor Model

A popular alternative to the one factor market model, is the four factor model of Carhart (1997). That model uses as risk factors the three Fama-French factors and momentum. This brings up the possibility that the Kalman filter model outperforms the one factor OLS model only because the latter does not properly capture systematic risk exposures. To test this the same bootstrap procedure was conducted using the four factor model. The primary procedural difference is that there are now four factor loadings to estimate and then hedge out. The results can be found in Table 10.

The four factor results parallel those in the one factor case. Once again the Kalman filter model generally outperforms the OLS model out of sample. Similarly, the Kolmogorov-

exit) it is nevertheless a potentially labor intensive process. To test the impact of longer holding periods the results were repeated under the assumption that the investor holds the set of mutual funds for twelve months. Under criteria A the Kalman filter portfolio returns drop by about 0.5% while the OLS returns drop by about 2%. These results are not included in the tables for the sake of brevity, but are available from the authors upon request.

Smirnov test again rejects the hypothesis that any two of the displayed distributions are the same (all P-values are virtually zero). However, the results are now somewhat less dramatic. The median Kalman portfolio returns about 90 basis points more per year than the selected OLS portfolio. Similar results hold for the bear market years 1994 and 2000. As Table 10 shows the Kalman filter returns no longer first order dominates those produced by the OLS model. However, the Kalman portfolio returns still second order dominate those produced by the OLS model. The one-sided Kolmogorov-Smirnov test confirms that given any annual return value, with probability one the integral of cdf based on the Kalman forecasts is smaller than that based on the OLS forecasts.

For the Sharpe ratio first order dominance by the Kalman filter over the OLS model still holds. For the overall sample the Kalman filter produces a median monthly Sharpe ratio of 0.176, while the equivalent value for the OLS model is only 0.131. These Sharpe ratios are similar to that of the category C funds described in the previous section. If the Kalman filter model were simply loading up on funds using one specific strategy, then we would expect that to show up in a relatively low Sharpe ratio, since the fund of funds would not be very well diversified. The fact, that the Kalman produced Sharpe ratios are higher than the OLS ones indicates, at the very least, that the Sharpe portfolios are not any less diversified than the OLS ones.

Panel A1.1 shows the degree to which the performance difference between the Kalman filter and OLS selections can be explained by the Carhart factors. This test begins by subtracting the OLS fund of fund returns from those produced by the Kalman filter selections. These differenced returns are then regressed on the four factors to determine the degree to which the improved out of sample performance of the Kalman filter model can be explained by systematic misestimation of the underlying fund factor loadings. As the panel shows, this exercise indicates that the Kalman filter portfolios produce risk adjusted out of sample returns that are 1.2% better per annum than the OLS returns. Note, that this is close to the median difference of the raw returns (0.0828-0.0714 = .0114).[26] As in the single factor case the regression estimates indicate that overall the OLS model does a relatively poor job of predicting the out of sample factor loadings as the market, SMB, and momentum parameters all enter with a negative sign. In contrast the Kalman model only does an inferior job of estimating the out of sample HML loadings, and even here both the parameter estimate and t-statistic have the smallest magnitude among the four factors.

8 Comparison with FS Model

As noted earlier the dynamic model developed here has as a special case the FS model. However, so far all tests have been conducted under the restriction that the fund betas

[26] Tests were also run under the assumption that investors rebalance only every six or twelve months. (As in the single factor case the results are omitted from the tables for the sake of brevity.) The regression coefficient on the differenced returns between the Kalman and OLS portfolios comes to 0.01693 with six month holding periods, and 0.01207 with twelve month holding periods.

depend only upon some unobservable factor. This section examines the impact on the estimated model when observable conditioning information is added. The tests conducted here use the lagged treasury bill rate, and the dividend yield on the CRSP value weighted index. Thus, the equation for $\beta_{P,t}$ becomes

$$\beta_{P,t} = \beta + F_t + k_1 z_{1,t-1} + k_2 z_{2,t-1}. \tag{20}$$

If the observable information improves the model's predictive ability then k_1 and k_2 should differ from zero. To test this the model was run with and without the conditioning variables on the monthly returns of 437 mutual funds during the period of 1994 to 1998. Asymptotically, the likelihood ratio under the null should follow a chi-square distribution with two degrees of freedom. In Figure 7, the bars represent the cross-sectional distribution of the likelihood ratio while the dashed line traces out a chi-square distribution with two degrees of freedom.

Overall, the null hypothesis that k_1 and k_2 are zero cannot be rejected at the traditional 1%, 5% or 10% levels. However, for individual funds, the fraction that reject the null hypothesis at the 1%, 5% or 10% levels are 10.8%, 15.7% and 20.3%, respectively. These numbers are somewhat higher than might be expected by chance, which implies that for some funds the conditioning information appears to improve the model's fit.

Table 11 provides further evidence about the richness of the dynamic coefficient model used in this paper. This table shows the R^2's of fund return regressions on a market index using OLS, the FS two factor conditional beta model, and the Kalman model of this paper.[27] As can be seen, across all fund categories and for the entire sample, the FS model provides an improved fit relative to the OLS model. Consider however how the R^2 statisic as one moves across models. The increase when one goes from the OLS to FS model is approximately a tenth as large as the increase obtained when moving from the FS to the Kalman model. Based upon this, it appears that the Kalman model can account for a considerably larger portion of fund return fluctuations than either the OLS or the FS models.

Between this paper, FS, and GT there is now considerable evidence that mutual fund managers produce portfolios with time varying betas, and possibly alphas too. Thus, it is clear that portfolio managers are altering there portfolios in response to some set of economic variables. How then are k_1 and k_2 statistically indistinguishable from zero for most funds? The model has two ways of fitting a fund's alphas and betas. One way is to use the observable conditioning variables in some manner. Another is to use the estimated lagged values of alpha and beta, and then let them change according to an estimated relationship with an unobserved factor following an AR(1) process. Figure 7 indicates that the latter prediction method often dominates, at least when using the lagged treasury bill rate, and dividend yield on the CRSP value weighted index. One conclusion may be that a few funds use treasury bill rates and the market dividend yield to help manage their assets, while most do not.

[27]The Kalman model used in these tests does not use the FS conditioning variables, and looks only at the return series of funds and of the market index.

More practically, one can potentially use the model to both identify those funds with a more macro based approach to asset allocation and the variables they concentrate on.

9 Conclusion

Even if security returns are well described by a factor model with time invariant loadings, the same will not be true of an actively traded portfolio holding these securities. This point goes back to at least Admati and Ross (1985), and Dybvig and Ross (1985). Papers by GT and FS have then gone on to produce empirical models for such portfolios. This paper develops an empirical specification of which the FS setting can be thought of as a special case. In FS time varying factor loadings are estimated via the use of observable macro economic factors. By contrast, this paper assumes that in addition the portfolio holdings may vary in response to some unobservable variable that follows an AR(1) process.

In terms of both fitting the historical data and making out of sample predictions the empirical generalizations presented here can provide many potential advantages. By assuming an unobservable variable with a known stochastic process drives portfolio holdings, the econometrician can effectively use past changes in a portfolio's alpha and betas to predict future changes. If the underlying assumptions are even approximately true, then one expects the model to better fit the data. In fact, a number of tests presented here show that it does.

Looking at the historical data, the empirical results show that the dynamic model developed here does a much better job of capturing mutual fund portfolio returns than does a static OLS model. A decomposition of the difference shows that the improvement comes from a better estimate of the time variation in the portfolio alphas. This opens up the possibility that managerial overperformance, even if it exists, may be difficult to detect with a static model.

Out of sample the dynamic model also outperforms the static OLS model. This is tested by using each model to form portfolios that they predict should have zero betas and positive alphas. The results are quite telling. The average correlation of the portfolios formed by the dynamic model are quite close to zero but those of the OLS model are found to be positive. This indicates that the static model systematically underestimates betas, which may positively influence the conclusions one might draw about fund managers. In terms of the alphas, again the results are very strong. The out of sample portfolios formed via the dynamic model first order dominate those formed by the OLS model. Thus, the dynamic model is able to identify funds that will over the next year outperform the market on a risk adjusted basis. This result holds whether one uses a single factor model or the Carhart (1997) four factor model. The natural conclusion is that managers can produce predictable above market returns. What this study has not determined is for how long managers can accomplish this feat, and that is an interesting topic for future research.

Finally, the empirical work examines the degree to which observing macro economic variables such as treasury bill rates, and the dividend yield on the CRSP equally weighted index helps to fit the data. Thus, the question is: given one has assumed that a fund's trading strategy is driven by some unobservable variable following an AR(1) process, to what degree does observing these two variables help? Statistically, adding these observable variables to the model does not improve its overall fit, and the estimated coefficients are statistically indistinguishable from zero. However, for some funds the conditional information is helpful, and this may indicate that while some fund managers trade on the macro economic variables included in the estimation process, most do not.

Overall, describing a mutual fund's trades as relying on an unobservable factor following an AR(1) process offers an economically significant improvement over past estimators. From a pragmatic perspective, the results show that the model can identify funds that will exhibit positive alphas out of sample, which indicates that managerial talent does matter.

Appendix

A Proof of Proposition 1

The data generating process is given by equation (16), reproduced here for convenience

$$y(t) = \alpha(t) + \beta(t)\,x(t) + \epsilon(t),$$

where $y(t) \equiv r_P(t) - r(t)$, $x(t) \equiv r_m(t) - r(t)$, and where we assume that $\mathbb{E}[\epsilon(t)] = 0$ and $\mathbb{E}[\epsilon(t)|x(t)] = 0$. Consider the an OLS estimate of the following constant-coefficient equation:

$$y(t) = \hat{\alpha} + \hat{\beta}\,x(t) + \hat{\eta}(t).$$

Define $X \in \mathbb{R}^{T\times 2}$ and $Y \in \mathbb{R}^T$ as

$$X \equiv \begin{bmatrix} 1 & x(1) \\ \vdots & \vdots \\ 1 & x(T) \end{bmatrix} \qquad Y \equiv \begin{bmatrix} y(1) \\ \vdots \\ y(T) \end{bmatrix}.$$

Then the OLS estimates $\hat{\alpha}$ and $\hat{\beta}$ are given by

$$\begin{aligned} \begin{bmatrix} \hat{\alpha} \\ \hat{\beta} \end{bmatrix} &= \left(\frac{1}{T}X'X\right)^{-1}\left(\frac{1}{T}X'Y\right) \\ &= \frac{1}{v}\begin{bmatrix} \frac{1}{T}\sum x(t)^2 & -\frac{1}{T}\sum x(t) \\ -\frac{1}{T}\sum x(t) & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{T}\Big(\sum \alpha(t) + \sum \beta(t)x(t) + \sum \epsilon(t)\Big) \\ \frac{1}{T}\Big(\sum x(t)\alpha(t) + \sum \beta(t)x(t)^2 + \sum x(t)\epsilon(t)\Big) \end{bmatrix}, \end{aligned}$$

where

$$v \equiv \frac{1}{T}\sum x(t)^2 - \left(\frac{1}{T}\sum x(t)\right)^2.$$

Note all the summations are over $t = 1, \ldots, T$. We then find that

$$\begin{aligned} \hat{\alpha} &= \frac{1}{v}\Bigg[\frac{1}{T}\sum x(t)^2 \frac{1}{T}\Big(\sum \alpha(t) + \sum \beta(t)x(t) + \sum \epsilon(t)\Big) \\ &\qquad -\frac{1}{T}\sum x(t)\frac{1}{T}\Big(\sum x(t)\alpha(t) + \sum \beta(t)x(t)^2 + \sum x(t)\epsilon(t)\Big)\Bigg] \\ &\to \frac{1}{\mathrm{Var}(x(t))}\Big[\mathbb{E}[x(t)^2]\Big(\mathbb{E}[\alpha(t)] + \mathbb{E}[\beta(t)x(t)]\Big) - \mathbb{E}[x(t)]\Big(\mathbb{E}[\alpha(t)x(t)] + \mathbb{E}[\beta(t)x(t)^2]\Big)\Big] \\ &= \frac{1}{\mathrm{Var}(x(t))}\Big[\mathbb{E}[\alpha(t)]\mathrm{Var}(x(t)) - \mathbb{E}[x(t)](\mathrm{Cov}(\alpha(t), x(t)) + \mathrm{Cov}(\beta(t), x(t)^2)) \\ &\qquad + \mathbb{E}[x(t)^2]\mathrm{Cov}(\beta(t), x(t))\Big]. \end{aligned}$$

where the limit is in probability, and follows from an application of the law of large numbers. Also we have that

$$\begin{aligned}
\hat{\beta} &= \frac{1}{v}\Big[-\frac{1}{T}\sum x(t)\frac{1}{T}\Big(\sum \alpha(t) + \sum \beta(t)x(t) + \sum \epsilon(t)\Big) \\
&\quad + \frac{1}{T}\Big(\sum x(t)\alpha(t) + \sum \beta(t)x(t)^2 + \sum x(t)\epsilon(t)\Big)\Big] \\
&\to \frac{1}{\mathrm{Var}(x(t))}\Big[-\mathbb{E}[x(t)]\Big(\mathbb{E}[\alpha(t)] + \mathbb{E}[\beta(t)x(t)]\Big) + \mathbb{E}[\alpha(t)x(t)] + \mathbb{E}[\beta(t)x(t)^2]\Big] \\
&= \frac{1}{\mathrm{Var}(x(t))}\Big[\mathbb{E}[\beta(t)]\mathrm{Var}(x(t)) + \mathrm{Cov}(\alpha(t), x(t)) + \mathrm{Cov}(\beta(t), x(t)^2) \\
&\qquad - \mathbb{E}[x(t)]\mathrm{Cov}(\beta(t), x(t))\Big].
\end{aligned}$$

Again the limit is in probability, and follows from an application of the law of large numbers. Q.E.D.

References

Admati, A., and S. Ross, 1985, "Measuring Investment Performance in a Rational Expectations Equilibrium Model," *Journal of Business*, 58, 1-26.

Baks, K., A. Metrick, and J. Wachter, 2001, "Should Investors Avoid All Actively Managed Funds? A Study in Bayesian Performance Evaluation," *Journal of Finance*, 56(1), 45-86.

Blake, D., B. Lehmann, and A. Timmermann, 2002, "Performance Clustering and Incentives in the UK Pension Fund Industry," working paper University of California at San Diego.

Breen, W., L. Glosten, and R. Jagannathan, 1989, "Economic Significance of Predictable Variations in Stock Index Returns," *Journal of Finance*, 44(5), 177-1189.

Blake, D., B. Lehmann, and A. Timmermann, 1999, "Asset Allocation Dynamics and Pension Fund Performance," *Journal of Business*, 72(4), 429-461.

Blake, C., and M. Morey, 2000, "Morningstar Ratings and Mutual Fund Performance," *Journal of Financial and Quantitative Analysis*, 35(3), 451-483.

Brown, S., and W. Goetzmann, 1995,"Performance Persistence," *Journal of Finance*, 50(2), 679-698.

Carhart, M., 1997, "On Persistence in Mutual Fund Performance," *Journal of Finance*, 52(1), 57-82.

Chen, H., N. Jegadeesh, and R. Wermers, 2000, "The Value of Active Mutual Fund Management: An Examination of the Stockholdings and Trades of Fund Managers," *Journal of Financial and Quantitative Analysis*, 35(3), 343-368.

Chevalier, J., and G. Ellison, 1999, "Are Some Mutual Fund Managers Better than Others? Cross-Sectional Patterns in Behavior and Performance," *Journal of Finance*, 54(3), 875-899.

Christopherson, J., W. Ferson, and D. Glassman, 1998, "Conditioning Manager Alphas on Economic Information: Another Look at the Persistence of Performance," *Review of Financial Studies*, 11, 111-142.

Dahlquist, M., S. Engström, and P. Söderlind, 2000, "Performance and Characteristics of Swedish Mutual Funds," *Journal of Financial and Quantitative Analysis*, 35(3), 409-423.

Dybvig, P., and S. Ross, 1985, "Differential Information and Performance Measurement Using a Security Market Line," *Journal of Finance*, 40, 383-399.

Elton, E., M. Gruber, and C. Blake, 2001, "A First Look at the Accuracy of the CRSP Mutual Fund Database and a Comparison of the CRSP and Morningstar Mutual Fund Databases," *Journal of Finance*, 56, 2415-2430.

Ferson, W., and C. Harvey, 1991, "The Variation of Economic Risk Premiums," *Journal of Political Economy*, 99, 385-415.

Ferson, W., and C. Harvey, 1993, "The Risk and Predictability of International Equity Returns," *Review of Financial Studies*, 6, 527-566.

Ferson, W., and K. Khang, 2002, "Conditional Performance Measurement Using Portfolio Weights: Evidence for Pension Funds." *Journal of Financial Economics*, 65, 249-282.

Ferson, W., and R. Schadt, 1996, "Measuring Fund Strategy and Performance in Changing Economic Conditions," *Journal of Finance*, 51(2), 425-462.

Ghysels, E., 1998, "On Stable Factor Structures in the Pricing of Risk: Do Time-Varying Betas Help or Hurt?," *Journal of Finance*, 53(2), 549-573.

Grinblatt, M., and S. Titman, 1989a, "Portfolio Performance Evaluation: Old Issues and New Insights," *Review of Financial Studies*, 2(3), 396-422.

Grinblatt, M., and S. Titman, 1989b, "Mutual Fund Performance: An Analysis of Quarterly Portfolio Holdings," *Journal of Business*, 62, 394-415.

Grinblatt, M., and S. Titman, 1994, "A Study of Monthly Mutual Fund Returns and Performance Evaluation Techniques," *Journal of Financial and Quantitative Analysis*, 29(3), 419-444.

Gruber, M., 1996, "Another Puzzle: The Growth in Actively Managed Mutual Funds," *Journal of Finance*, 51(3), 783-810.

Hamilton, J., 1994, *Time Series Analysis*, Princeton University Press, Princeton, New Jersey.

Hendricks, D., J. Patel, and R. Zeckhauser, 1993, "Hot Hands in Mutual Funds: Short-Run Persistence of Relative Performance, 1974-1988," *Journal of Finance*, 48(1), 93-130.

Harvey, A., 1989, *Forecasting, Structural Time Series Models and the Kalman Filter*, Cambridge University Press, Cambridge, U.K..

Lehmann, B., and D. Modest, 1987, "Mutual Fund Performance Evaluation: A Comparison of Benchmarks and Benchmark Comparisons," *Journal of Finance*, 42, 233-265.

Mamaysky, H. and M. Spiegel, 2002, "A theory of mutual funds: Optimal fund objectives and industry organization," Yale School of Management working paper.

Pástor, Ľ., and R. Stambaugh, 2002a, "Mutual Fund Performance and Seemingly Unrelated Assets," *Journal of Financial Economics*, 63(3), 315-350.

Pástor, Ľ., and R. Stambaugh, 2002b, "Investing in Equity Mutual Funds," *Journal of Financial Economics*, 63(3), 351-380.

Teo, M., and S. Woo, 2001, "Persistence in Style-Adjusted Mutual Fund Returns," working paper, Harvard University, Department of Economics.

Wermers, R., 2000, "Mutual Fund Performance: An Empirical Decomposition into Stock-Picking talent, Style, Transactions Costs, and Expenses," *Journal of Finance*, 55(4), 1655-1695.

Table 1: **Descriptive statistics.** The statistical analysis uses CRSP mutual fund monthly return data from 1970 to 2000. To be included in this table a fund must have more than 48 months of valid return data, and located in one of nine MorningStar categories as of 1999. Reported below are the 9 MorningStar categories, the total number of funds within each category, the mean excess return and Sharp ratio for funds within each category, and the number of funds estimated to be "dynamic" in each category. A fund is said to be dynamic if the Kalman filter estimates do not converge to the static OLS estimates.

Category	Category Name	Total Funds	Return	Sharp Ratio	Dynamic Funds
16	Large Blend	112	0.0096	0.2334	96
17	Large Growth	107	0.0129	0.2559	85
18	Large Value	105	0.0077	0.1961	93
22	Mid-Cap Blend	66	0.009	0.1959	57
23	Mid-Cap Growth	48	0.0133	0.2137	28
24	Mid-Cap Value	36	0.0069	0.1589	31
38	Small Blend	29	0.0085	0.1734	28
39	Small Growth	50	0.0136	0.1938	36
40	Small Value	19	0.0075	0.1662	19
Summary		572			473

Table 2: **Convergence of Kalman filter and the CUSUMSQ residual test.** This table reports the convergence properties of the Kalman filter estimates and the CUSUMSQ residual test. Each entry lists the number of funds. For funds in the OLS columns the Kalman filter estimates converge to the OLS parameters. For funds under the DYN column the Kalman filter estimates diverge from the OLS estimates. The last column lists the total number of dynamic funds within each category (the sum of the two DYN columns). A "1" indicates that the CUSUMSQ test accepts the null that the residual has no time structure, at the 5% level. A "2" indicates that the CUSUMSQ test rejects the null that the residual has no time structure, at the 5% level.

Cate	funds	OLS,$Q = 0^1$	OLS,Q=1^2	DYN,Q=0^1	DYN,Q=1^2	Dynamic Funds
16	112	2	14	43	53	96
17	107	3	19	33	52	85
18	105	0	12	17	76	93
22	66	4	5	21	36	57
23	49	2	19	12	16	28
24	36	2	3	6	25	31
38	29	0	1	6	22	28
39	50	3	11	8	28	36
40	19	0	0	4	15	19
SUM	573	16	84	150	323	473
Percentage	100	3	15	26	56	83

Table 3: **Estimation of a one-factor model.** This table reports the parameters estimated from the following model: $r_P(t) - r(t) = -k + b_P F(t-1)^2 + \bar{\beta}_P(r_m(t) - r(t)) + (r_m(t) - r(t) + \bar{\alpha}_P)F(t-1) + \epsilon_P(t)$, where $F(t) = \gamma_F F(t-1) + \eta_F(t)$. Here $r_m(t)$ is the market portfolio's return. $\bar{\beta}_P$ is the static exposure of the portfolio to the market excess returns. σ_ϵ^2 is the variance for $\epsilon_P(t)$, and σ_F^2 is the variance for $\eta_F(t)$. For each category of funds, the first line reports the cross sectional mean for the estimated parameters while the second line reports the cross sectional T-ratio for the mean. The whole system is estimated using an extended Kalman Filter.

Category		$-k$	$\bar{\beta}_P$	γ_F	σ_ϵ^2	σ_F^2	$\bar{\alpha}_P$	b_P
16	mean	-0.0008	0.9332	0.3446	0.0001	0.0348	0.0327	0.0746
	T ratio	-3.2913	67.6583	8.7938	7.9700	5.1765	3.8792	0.7357
17	mean	-0.0008	1.0909	0.3251	0.0002	0.0295	0.0778	-0.0236
	T ratio	-3.2203	65.0044	7.9177	7.8602	7.2951	8.9236	-0.1902
18	mean	-0.0013	0.8467	0.3475	0.0001	0.0316	0.0696	-0.2073
	T ratio	-3.9877	63.9289	8.8629	8.7819	7.6943	7.8703	-1.8951
22	mean	-0.0022	0.9279	0.3486	0.0003	0.0750	-0.0026	-0.0155
	T ratio	-5.1357	40.6962	6.4198	11.7703	3.7868	-0.1993	-0.1168
23	mean	0.0004	1.1303	0.3294	0.0004	0.0702	0.0538	0.2342
	T ratio	0.7450	23.4796	5.1599	6.6534	4.7233	3.3583	1.2607
24	mean	-0.0016	0.8574	0.4041	0.0004	0.0517	0.0399	-0.2657
	T ratio	-2.6351	31.5863	4.9125	5.5452	3.4231	2.7608	-0.8768
38	mean	-0.0013	0.8516	0.2099	0.0006	0.1194	-0.0279	-0.1612
	T ratio	-1.7792	28.8316	3.2067	13.1113	3.2952	-1.1432	-0.9159
39	mean	0.0002	1.1997	0.1272	0.0008	0.1640	0.0817	0.3336
	T ratio	0.2154	23.6649	2.3737	4.9963	4.5008	3.2419	2.1897
40	mean	-0.0013	0.7885	0.3351	0.0006	0.0897	-0.2185	-0.2708
	T ratio	-0.9082	18.1309	3.7387	6.4321	4.2417	-0.8279	-2.8581

Table 4: R^2 **increment and decomposition error of the OLS Alpha and Beta.** Kalman filter estimates are used to calculate predicted OLS alphas and betas according to the following formula $\hat{\alpha} = E(\alpha_t) - \frac{\mathbb{E}(x_t)}{Var(x_t)} cov(\alpha_t, x_t) + (1 + \frac{\mathbb{E}(x_t)^2}{Var(x_t)}) cov(\beta_t, x_t) - \frac{\mathbb{E}(x_t)}{Var(x_t)} cov(\beta_t, x_t^2)$ and $\hat{\beta} = E(\beta_t) + \frac{1}{Var(x_t)} cov(\alpha_t, x_t) - \frac{\mathbb{E}(x_t)}{Var(x_t)} cov(\beta_t, x_t) + \frac{1}{Var(x_t)} cov(\beta_t, x_t^2)$, where the α_t and b_t are dynamic portfolio alphas and betas, and x_t the market excess return. Asymptotically $\hat{\alpha}$ and $\hat{\beta}$ should converge to the OLS estimates. The table also lists the R^2_{OLS} from the OLS regression, and the R^2 improvement from estimating the dynamic Kalman filter model. Consistent with the OLS model, the R^2_{Kal} for the Kalman filter equals $1 - \mathbb{E}(\epsilon_P(t)^2)/\mathbb{E}((y_t - \bar{y})^2)$, where y_t is the excess portfolio return and $\epsilon_P(t)$ is the residual from the dynamic model. The variable $\bar{y}$ equals the mean value of y_t, and $\triangle R^2 = R^2_{Kal} - R^2_{OLS}$. The $\triangle R^2_2$ measure compares a rolling OLS model with the Kalman filter. Here the rolling OLS model estimates employ data from the previous 48 months, with the current month as the 48th. A residual from the 48th month is then calculated and used to compute the R^2 statistic, labeled R^2_{ROLS}. Finally, the time period is incremented by one and the process repeated until the end of the data set has been reached. The $\triangle R^2_2$ variable equals $R^2_{Kal} - R^2_{ROLS}$. The numbers in the corresponding columns report the absolute and average percentage errors between $\hat{\alpha}$ and α_{OLS}, and the corresponding statistics regarding the fund betas.

category		R^2_{OLS}	$\triangle R^2$	$\triangle R^2_2$	$\lvert\%err\alpha\rvert$	$\%err\alpha$	$\lvert\%err\beta\rvert$	$\%err\beta$
16	mean	0.8502	0.0980	0.1046	1.8375	-0.7134	0.1491	0.0115
	T ratio		8.3080	7.0693	4.3327	-1.5392	3.9289	0.2793
17	mean	0.7900	0.1590	0.1687	1.9367	-1.0208	0.1472	-0.0400
	T ratio		13.3937	10.6376	3.4011	-1.6993	3.3324	-0.8493
18	mean	0.7523	0.1971	0.2531	3.2518	-1.2980	0.2660	-0.0834
	T ratio		14.4164	11.1611	3.2965	-1.2474	4.0752	-1.1742
22	mean	0.7262	0.1612	0.1606	1.2465	-0.1311	0.1924	0.0234
	T ratio		7.8809	7.9059	3.4214	-0.3236	2.5423	0.2912
23	mean	0.6909	0.2240	0.2233	1.7470	-0.9109	0.2501	-0.0195
	T ratio		7.6710	7.1438	2.2648	-1.1014	3.2108	-0.2105
24	mean	0.6749	0.1859	0.1792	1.9382	0.9353	0.2505	0.0245
	T ratio		8.2108	6.3536	2.7805	1.2063	3.2131	0.2651
38	mean	0.5717	0.2170	0.2185	2.4482	1.7071	0.5077	0.3839
	T ratio		8.2048	7.5761	2.3324	1.5357	1.9069	1.3956
39	mean	0.5525	0.2859	0.3018	0.8575	-0.1833	0.0257	0.0068
	T ratio		7.7169	7.5502	2.3994	-0.4544	1.6708	0.4149
40	mean	0.5640	0.2095	0.1878	5.5118	3.4770	0.6355	0.4031
	T ratio		6.0037	6.0649	1.8530	1.1037	1.9330	1.1526

Table 5: **Decomposition of OLS Alpha.** This table shows each of the four components for the following decomposition of the OLS alpha $\hat{\alpha} = E(\alpha_t) - \frac{\mathbb{E}(x_t)}{Var(x_t)} cov(\alpha_t, x) + (1 + \frac{\mathbb{E}(x_t)^2}{Var(x_t)}) cov(\beta_t, x_t) - \frac{\mathbb{E}(x_t)}{Var(x_t)} cov(\beta_t, x_t^2)$, where α_t and b_t are dynamic alpha and beta, and x_t denotes the market excess return. For each category of funds, the first line reports the mean while the second line reports the cross sectional T-ratio for the null hypothesis that the component equals zero.

Category		α_{OLS}	$E(\alpha_t)$	cov(α_t,x_t)	cov(β_t,x_t)	cov(α,x_t^2)
16	mean	-0.0008	-0.0008	-0.0000	-0.0000	-0.0000
	T ratio	-4.0256	-3.7697	-0.8837	-0.2670	-0.8418
17	mean	-0.0002	-0.0004	-0.0001	0.0002	0.0001
	T ratio	-0.8277	-1.6248	-1.4517	3.4613	2.2654
18	mean	-0.0018	-0.0017	-0.0000	-0.0001	-0.0000
	T ratio	-6.3687	-6.3907	-0.0196	-0.9528	-0.3409
22	mean	-0.0017	-0.0017	0.0000	0.0001	-0.0001
	T ratio	-4.9560	-5.1258	0.0981	0.6472	-1.4891
23	mean	0.0010	0.0012	0.0003	-0.0004	-0.0000
	T ratio	1.9970	2.2095	3.3088	-2.7784	-0.3545
24	mean	-0.0025	-0.0020	-0.0001	-0.0003	-0.0001
	T ratio	-4.4783	-3.6393	-1.3809	-2.0696	-1.0917
38	mean	-0.0025	-0.0011	-0.0001	-0.0010	-0.0004
	T ratio	-3.9895	-1.6027	-1.4530	-6.9325	-4.9655
39	mean	-0.0001	0.0002	0.0004	-0.0008	0.0001
	T ratio	-0.1151	0.2178	2.0645	-4.5043	0.8668
40	mean	-0.0018	-0.0005	-0.0001	-0.0008	-0.0004
	T ratio	-2.9564	-0.8172	-1.6062	-5.2917	-3.6075

Table 6: **Components of OLS Alpha: absolute percentage.** This table reports the results for the decomposition, $\hat{\alpha} = E(\alpha_t) - \frac{\mathbb{E}(x_t)}{Var(x_t)} cov(\alpha_t, x_t) + (1 + \frac{\mathbb{E}(x_t)^2}{Var(x_t)}) cov(\beta_t, x_t) - \frac{\mathbb{E}(x_t)}{Var(x_t)} cov(\beta_t, x_t^2)$, where and x_t denotes the market excess return. Panel A columns three through six report the percentage contribution of each component to the value of $\hat{\alpha}$. To arrive at these values the average value of each component across all funds is computed. Then the absolute value of the average is calculated. The resulting numbers are then added together to produce column seven, which then serves as the denominator for the percentage contributions reported in columns three through six. Panel B repeats the analysis in Panel A, except that absolute values are taken fund by fund prior to calculating the mean value of each component.

Category	α_{OLS}	α_{static}	cov(α_t,x)	cov(β_t,x)	cov(β_t,x^2)	Sum($\lvert\alpha(components)\rvert$)
A						
16	-0.0008	90.9497	2.4515	2.5525	4.0464	0.0008
17	-0.0002	53.8183	10.2323	27.8133	8.1361	0.0007
18	-0.0018	95.4011	0.0650	3.7367	0.7972	0.0018
22	-0.0017	88.6364	0.1510	5.1362	6.0765	0.0019
23	0.0010	62.1663	14.8368	21.4123	1.5846	0.0019
24	-0.0025	80.0956	3.8869	12.4355	3.5819	0.0025
38	-0.0025	42.0451	5.5257	38.6425	13.7868	0.0025
39	-0.0001	12.7093	27.2640	53.5571	6.4696	0.0014
40	-0.0018	28.9745	6.1534	44.3250	20.5472	0.0019
B						
16	-0.0008	62.8109	7.3571	20.0651	9.7668	
17	-0.0002	62.1060	10.3851	17.5283	9.9806	
18	-0.0018	65.6543	7.6162	15.7693	10.9602	
22	-0.0017	67.6874	4.1612	17.3352	10.8162	
23	0.0010	59.2356	9.4512	20.7004	10.6128	
24	-0.0025	60.9653	5.8977	22.8405	10.2965	
38	-0.0025	55.5984	5.0968	27.3056	11.9992	
39	-0.0001	53.9725	10.4055	21.8664	13.7556	
40	-0.0018	46.3407	6.8832	31.6218	15.1543	

Table 7: **Decomposition of OLS Beta.** This table shows each of the four components for the following decomposition of OLS beta, $\hat{\beta} = E(\beta_t) + \frac{1}{Var(x_t)} cov(\alpha_t, x_t) - \frac{\mathbb{E}(x_t)}{Var(x_t)} cov(\beta_t, x_t) + \frac{1}{Var(x_t)} cov(\beta_t, x_t^2)$ where α_t and b_t are dynamic part of alpha and beta, and x_t denotes the market excess return. For each category of funds, the first line reports the mean while the second line reports the cross sectional T-ratio for the null hypothesis that the component is zero.

Category		β_{OLS}	$E(\beta_t)$	cov(α_t,x_t)	cov(β_t,x_t)	cov(β,x_t^2)
16	mean	0.9373	0.9361	0.0016	0.0001	-0.0003
	T ratio	64.0177	70.2978	0.9286	0.1841	-0.0777
17	mean	1.0897	1.0923	0.0061	-0.0012	-0.0081
	T ratio	65.9604	65.9432	1.7305	-2.9941	-3.1629
18	mean	0.8496	0.8488	0.0010	0.0006	-0.0015
	T ratio	65.7914	67.5370	0.2499	1.1252	-0.4364
22	mean	0.9339	0.9276	0.0006	-0.0004	0.0066
	T ratio	35.1753	39.7614	0.2191	-0.4607	1.0119
23	mean	1.1105	1.1327	-0.0238	0.0025	-0.0003
	T ratio	26.3939	23.9411	-3.4784	2.4814	-0.0416
24	mean	0.8807	0.8647	0.0083	0.0012	0.0067
	T ratio	31.6878	32.4088	1.3961	1.4033	0.7859
38	mean	0.8997	0.8606	0.0104	0.0070	0.0253
	T ratio	37.6198	30.7002	1.4812	6.2558	4.4721
39	mean	1.1669	1.2001	-0.0270	0.0056	-0.0117
	T ratio	26.0820	23.7304	-2.2857	4.1762	-1.4276
40	mean	0.8392	0.7925	0.0103	0.0048	0.0345
	T ratio	20.8500	18.7460	1.6194	5.7729	3.7118

Table 8: **Components of OLS Beta: absolute percentage.** This table reports the results for the decomposition, $\hat{\beta} = E(\beta_t) + \frac{1}{Var(x_t)}cov(\alpha_t, x_t) - \frac{\mathbb{E}(x_t)}{Var(x_t)}cov(\beta_t, x_t) + \frac{1}{Var(x_t)}cov(\beta_t, x_t^2)$. Panel A columns three through six report the percentage contribution of each component to the value of $\hat{\beta}$. To arrive at these values the average value of each component across all funds is computed. Then the absolute value of the average is calculated. The resulting numbers are then added together to produce column seven, which then serves as the denominator for the percentage contributions reported in columns three through six. Panel B repeats the analysis in Panel A, except that absolute values are taken fund by fund prior to calculating the mean value of each component.

Category	β_{OLS}	β_{static}	cov(α_t,x_t)	cov(β_t,x_t)	cov(β_t,x_t^2)	Sum($\lvert\beta(components)\rvert$)
A						
16	0.9380	99.7886	0.1707	0.0107	0.0300	0.9375
17	1.1077	98.6071	0.5536	0.1108	0.7285	1.0891
18	0.8519	99.6382	0.1180	0.0652	0.1785	0.8489
22	0.9352	99.1872	0.0598	0.0423	0.7106	0.9344
23	1.1594	97.7010	2.0570	0.2159	0.0261	1.1111
24	0.8809	98.1551	0.9467	0.1356	0.7626	0.8809
38	0.9032	95.2782	1.1528	0.7712	2.7978	0.9032
39	1.2444	96.4370	2.1723	0.4527	0.9379	1.1670
40	0.8421	94.1103	1.2238	0.5732	4.0927	0.8421
B						
16	0.9380	96.6476	0.9253	0.3032	2.1239	
17	1.1077	96.3535	1.7856	0.2303	1.6306	
18	0.8519	94.5962	2.4769	0.3397	2.5872	
22	0.9352	94.8573	1.0829	0.4115	3.6483	
23	1.1594	94.5373	2.0243	0.2570	3.1813	
24	0.8809	93.6710	2.2537	0.4080	3.6673	
38	0.9032	93.2985	2.1374	0.7951	3.7690	
39	1.2444	94.6653	2.4042	0.5168	2.4137	
40	0.8421	92.3934	2.2299	0.6316	4.7451	

Table 9: **Out of Sample Portfolio Returns.** For all domestic equity mutual funds that existed at the end of year 1992 and had at least 5 years of monthly return data, both the Kalman and the OLS models are used to forecast a fund's alpha and beta. These forecasts are then used to construct zero-beta portfolios using the five funds with the highest (positive) estimated alphas. In Panel A, all funds are pooled. Whenever the maximization algorithm for the Kalman filter model does not converge, OLS forecasts are used instead. In each simulation, fifty funds are selected randomly from the pool. From Jan 1993 to Dec 2000, for each model the five funds with the highest predicted alphas are chosen out of the fifty funds to form an equally weighted portfolio. If a fund dies, a random fund from the pool replaces it. The portfolio's predicted market risks are hedged out and its realized annual returns are calculated in each month. The simulations are repeated one thousand times for each date. Panel B, restricts the sample to funds for which the maximization algorithm for the Kalman filter model converged. Within this pool the two hundred funds with highest R^2 are selected separately based upon the OLS and Kalman models. The simulation process described above is then applied separately to each group. Panel C reports simulation results for those funds for which the Kalman filter model appears to provide the greatest marginal information relative to the OLS model. These funds are defined as those where the Kalman filter's R^2's differ the most from those produce via the OLS model. Panel C1 lists the pooled annual returns and Sharpe ratios for years 1993 to 2000, while Panels C2 and C3 list the same measures for the bear market years 1994 and 2000. All panels report the mean, the cross-sectional standard deviation, and the 5, 10, 50, 90 and 95 percentage quantiles for the distribution (cdf) of the pooled annual returns (R_{OLS} or R_{kal}) and the Sharp Ratio of the monthly excess returns (SP_{OLS} or SP_{kal}). The last column reports the fraction of the simulations that can generate annual returns higher than the mean risk free rate. Bold faced numbers are those in excess of the risk free rate. The Kolmogorov-Smirnov test rejects the hypothesis that the same probability distribution produced any two sets of figures (all P-values virtually zero).

	mean	Std Dev	5%	10%	50%	90%	95%	$>R_f$
A. All funds 1993-2000								
R_{OLS}	1.0278	0.0507	0.9421	0.9643	1.0293	**1.0911**	**1.1053**	0.2979
R_{kal}	1.0485	0.0691	0.9379	0.9638	1.0457	**1.1314**	**1.1626**	0.4449
SP_{OLS}	-0.1111	0.0516	-0.1925	-0.1778	-0.1125	-0.0454	-0.0239	
SP_{kal}	-0.0314	0.0761	-0.1528	-0.1282	-0.0302	0.0646	0.0949	
B1. 200 highest R^2 funds 1993-2000								
R_{OLS}	1.0384	0.0651	0.9204	0.9508	1.0435	**1.1185**	**1.1348**	0.4245
R_{kal}	1.0606	0.0733	0.9431	0.9708	**1.0585**	**1.1553**	**1.1841**	0.5269
SP_{OLS}	-0.0724	0.0606	-0.1673	-0.1472	-0.0733	0.0079	0.0320	
SP_{kal}	0.0212	0.0846	-0.1311	-0.0951	0.0250	0.1282	0.1521	
B2. 200 highest R^2 funds 1994								
R_{OLS}	1.0408	0.0392	0.9779	0.9909	1.0415	1.0962	**1.1107**	0.3190
R_{KAL}	1.0655	0.0296	1.0176	1.0276	1.0651	**1.1026**	**1.1141**	0.6240
B3. 200 highest R^2 funds 2000								
R_{OLS}	1.0893	0.0366	1.0292	1.0426	**1.0903**	**1.1338**	**1.1491**	0.8200
R_{KAL}	1.1118	0.0561	1.0159	1.0390	**1.1132**	**1.1847**	**1.2027**	0.8440
C1. 200 most dynamic funds 1993-2000								
R_{OLS}	1.0135	0.0604	0.9140	0.9355	1.0145	**1.0890**	**1.1097**	0.2883
R_{kal}	1.0885	0.1026	0.9729	0.9913	**1.0582**	**1.2558**	**1.3112**	0.5613
SP_{OLS}	-0.1214	0.0675	-0.2335	-0.2090	-0.1228	-0.0348	-0.0142	
SP_{kal}	0.1392	0.0654	0.0301	0.0565	0.1412	0.2186	0.2456	
C2. 200 most dynamic funds 1994								
R_{OLS}	1.0019	0.0480	0.9267	0.9393	0.9992	**1.0649**	**1.0849**	0.1730
R_{kal}	1.0315	0.0343	0.9772	0.9872	1.0303	**1.0770**	**1.0894**	0.3220
SP_{OLS}	-0.2257	0.2926	-0.6988	-0.5978	-0.2233	0.1508	0.2481	
SP_{kal}	-0.0964	0.3289	-0.6669	-0.5279	-0.0687	0.2934	0.3945	
C3. 200 most dynamic funds 2000								
R_{OLS}	1.0232	0.0621	0.9251	0.9454	1.0223	**1.1032**	**1.1280**	0.3490
R_{kal}	1.2692	0.0801	**1.1371**	**1.1616**	**1.2706**	**1.3686**	**1.3939**	0.9980
SP_{OLS}	-0.0647	0.1524	-0.3258	-0.2639	-0.0533	0.1123	0.1718	
SP_{kal}	0.4049	0.1558	0.1924	0.2354	0.3856	0.6016	0.6769	

Table 10: **Out of Sample Portfolio Returns: The Four Factor Case.** Table 10 expands the dynamic model to a multi-factor environment, with the systematic risk factors taken to be the Fama-French three factors plus the momentum factor. A same set of equity funds are used as described in Table 9. In Panel A1, all funds are pooled. Kalman and OLS models are used to forecast a fund's alphas and betas from Jan 1993 to Dec 2000. Whenever the maximization algorithm for the Kalman filter model does not converge, OLS forecasts are used instead. The same process described in Table 9 Panel A1 is then used to simulate most-positive-alpha and zero-beta hedging portfolios. The only difference is in this case all four factors are forecasted and hedged out. Panel A1 reports the mean, the cross-sectional standard deviation, and the 5, 10, 50, 90 and 95 percentage quantiles for the distribution (cdf) of the pooled annual returns (R_{OLS} or R_{kal}) and the Sharp Ratio of the monthly excess returns (SP_{OLS} or SP_{kal}), as well as the fraction of the simulations that can generate annual returns higher than the mean risk free rate. Bold faced numbers are those in excess of the risk free rate. Panel A2 to A3 report the same statistics for year 1994 and 2000. In Panel A1.1, the difference for each simulation (used to create Panel A1) is first calculated as the Kalman fund of funds return minus the OLS fund of funds return. The differenced return is then regressed against the market, SMB, HML and momentum factors. Panel A1.1 reports the cross-sectional mean and T ratio for these regression paremeters. Finally, the Kolmogorov-Smirnov test rejects the hypothesis that the same probability distribution produced any two sets of figures (all P-values virtually zero).

	mean	Std Dev	5%	10%	50%	90%	95%	$>R_f$
A1. All funds 1993-2000								
R_{OLS}	1.1018	0.1579	0.9216	0.9556	**1.0714**	**1.3421**	**1.4856**	0.5700
R_{kal}	1.1139	0.1587	0.9470	0.9767	**1.0828**	**1.3465**	**1.5066**	0.6328
SP_{OLS}	0.1262	0.0684	0.0008	0.0391	0.1309	0.2071	0.2293	
SP_{kal}	0.1687	0.0585	0.0588	0.0894	0.1760	0.2379	0.2530	
A2. Year 1994								
R_{OLS}	1.0830	0.0439	1.0096	1.0237	**1.0862**	**1.1386**	**1.1542**	0.5190
R_{kal}	1.1052	0.0358	1.0441	1.0608	**1.1057**	**1.1513**	**1.1636**	0.7430
SP_{OLS}	0.0309	0.3691	-0.5408	-0.4270	0.0241	0.5083	0.6640	
SP_{kal}	0.2012	0.3436	-0.3543	-0.2204	0.1899	0.6350	0.7629	
A3. Year 2000								
R_{OLS}	**1.4509**	**0.1278**	**1.2426**	**1.2858**	**1.4485**	**1.6229**	**1.6728**	1.0000
R_{kal}	**1.4724**	**0.1493**	**1.2306**	**1.2749**	**1.4697**	**1.6649**	**1.7334**	1.0000
SP_{OLS}	0.6849	0.2438	0.3984	0.4392	0.6225	1.0075	1.1686	
SP_{kal}	0.6716	0.2005	0.4028	0.4442	0.6503	0.9273	1.0187	
A1.1. Factor loadings for Kalman minus the OLS returns, All funds 1993-2000								
	alpha	b(MKT)	b(SMB)	b(HML)	b(MOM)			
Mean	0.0121	-0.0174	-0.0396	0.0080	-0.0163			
T	21.4147	-9.9304	-11.1596	2.7987	-4.2122			

Table 11: **A Comparison with Conditional Model.** The sample used for this table includes all available funds in the CRSP database that contain 60 months of monthly return data from 1994 to 1998. Model parameters are estimated for the unconditional CAPM model, the conditional beta model, and the Kalman filter model. The variables R^2_{CAPM}, R^2_{Cond} and R^2_{Kal} represent R^2 statistics for each of the models respectively. Consistent with the OLS model, R^2_{Kal} is defined as $1 - \mathbb{E}(\epsilon_P(t)^2)/\mathbb{E}((y_t - \bar{y})^2)$, where y_t is the excess portfolio return and $\epsilon_P(t)$ is the residual from the Kalman filter model. The variable $\bar{y}$ equals the mean value of the y_t. Also reported are the cross-sectional means and T-ratios for the improvements of R^2, $\triangle R^2_1 = R^2_{cond} - R^2_{CAPM}$ and $\triangle R^2_2 = R^2_{Kal} - R^2_{cond}$.

Category		fund No.	R^2_{CAPM}	R^2_{Cond}	R^2_{Kal}	$\triangle R^2_1$	$\triangle R^2_2$
16	mean	92	0.8994	0.9013	0.9382	0.0031	0.0369
	T ratio					7.07	3.17
17	mean	81	0.861	0.8673	0.9146	0.0063	0.0473
	T ratio					8.74	6.97
18	mean	75	0.8676	0.8734	0.9287	0.0059	0.0553
	T ratio					7.52	6.67
22	mean	46	0.7618	0.7709	0.9095	0.0090	0.1386
	T ratio					5.56	6.51
23	mean	36	0.7501	0.7601	0.8638	0.0100	0.1036
	T ratio					2.94	4.97
24	mean	31	0.7408	0.7477	0.8484	0.0069	0.1007
	T ratio					7.11	5.65
38	mean	23	0.6479	0.6536	0.8614	0.0056	0.2079
	T ratio					3.37	4.76
39	mean	35	0.6413	0.6448	0.7011	0.0035	0.0563
	T ratio					5.12	3.15
40	mean	18	0.6527	0.6581	0.8007	0.0055	0.1426
	T ratio					5.44	6.35
All	mean	437	0.8047	0.8098	0.8877	0.0059	0.0779
	T ratio					14.03	13.80

Figure 1: **Benchmark factor model.** The economy contains 20 stocks that do not pay any dividends. Each stock is randomly assigned a fixed $\bar{\alpha}$ and β drawn from normal distributions (designated N(mean, standard deviation)) with parameters N(0, .005) and N(1, .8). Next, asset excess returns are generated via: $r_i(t) = \alpha_i(t) + \beta_i x(t) + \epsilon_i(t)$, where $\alpha_i(t) = \bar{\alpha}_i F(t)$, $x(t)$ is the market excess return from Jan 1994 to Dec 2000 and $\epsilon_i(t) \sim$ iid N(0, .001). Portfolio weights are driven by $f_i(t) = \bar{f}_i + l_i F(t) + \epsilon_f(t)$, where $\bar{f}_i = .05$, $l_i \sim$ iid N(0,1), $\epsilon_f \sim$ iid N(0, .0001). Finally, $F(t) = \gamma_F F(t-1) + \eta_F(t)$ with $F(t=1) = 0$, $\gamma_F = 0.80$ and $\eta(t) \sim$ N(0, .1). Two types of simulations are conducted labeled "cross sectional" and "time series." In the cross sectional case each run draws a new time series for $F(t)$, and the $\epsilon_i(t)$'s. In addition, the realized market returns from January 1994 to December 1998 are drawn randomly without replacement to produce a run specific sequence of $r_m(t)$ terms. After this portfolios are constructed, and the Kalman filter model estimated. In the time series case the same sequence of values for $F(t)$ is used across all runs, and the market return is the realized sequence (in order) of the actual market returns from January 1994 to December 1998. The only random variables that differ across runs are the $\epsilon_i(t)$'s. This proceedure has the advantage of producing a single time series for the true portfolio alphas and betas, thus allowing one to determine how well the model is likely to fit any one realization of the economy. Panels A and B plot the cross sectional standard deviations for the true portfolio alphas and betas (solid lines) and the standard deviations for estimation errors (dotted lines). Panels C and D plot the cross sectional mean of the estimation errors (solid lines) and their 10% and 90% intervals (dotted lines). Panels E and F report results from 200 time series simulations. Plotted are both the true portfolio alphas and betas (solid lines) and the 10% and 90% intervals for the estimated alphas and betas (dotted lines).

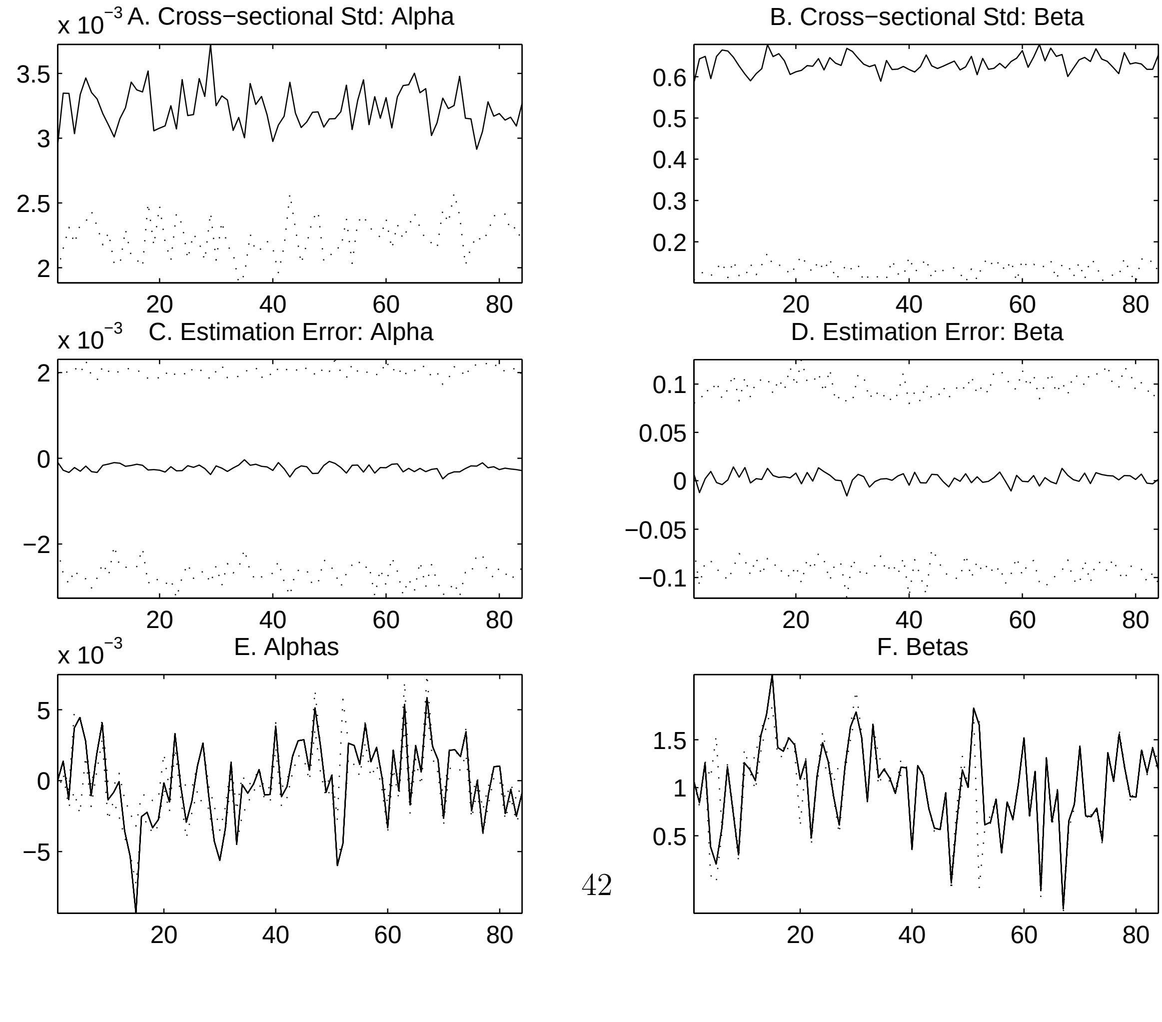


Figure 2: **Switching between two groups of stocks.** The simulation procedure is identical to the one described in Figure 1. The trading strategies are as follows: $\bar{f}_i = .05$ for all 20 stocks, and $l_i = 0.1$ for first ten assets and -0.1 for the second ten.

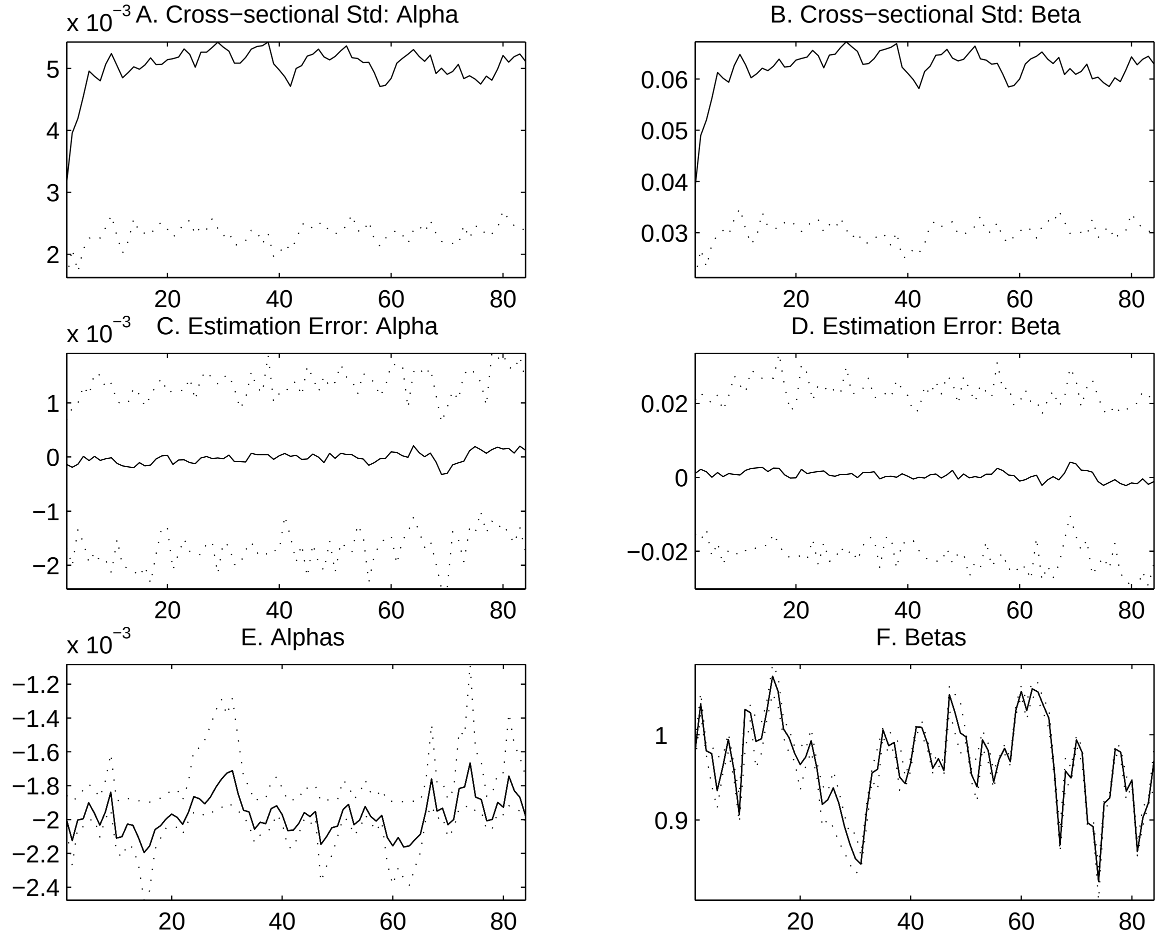


Figure 3: **Random strategy.** The simulation procedure is identical to the one described in Figure 1. The trading strategies are as follows: $f_i(t) = .05 + \eta_f(t)$, where $\eta_f(t) \sim$ iid N(0, .2). First, different $\eta_f(t)$ time series and a bootstrapped market factor are simulated for 500 times (different portfolio alpha and beta time series).

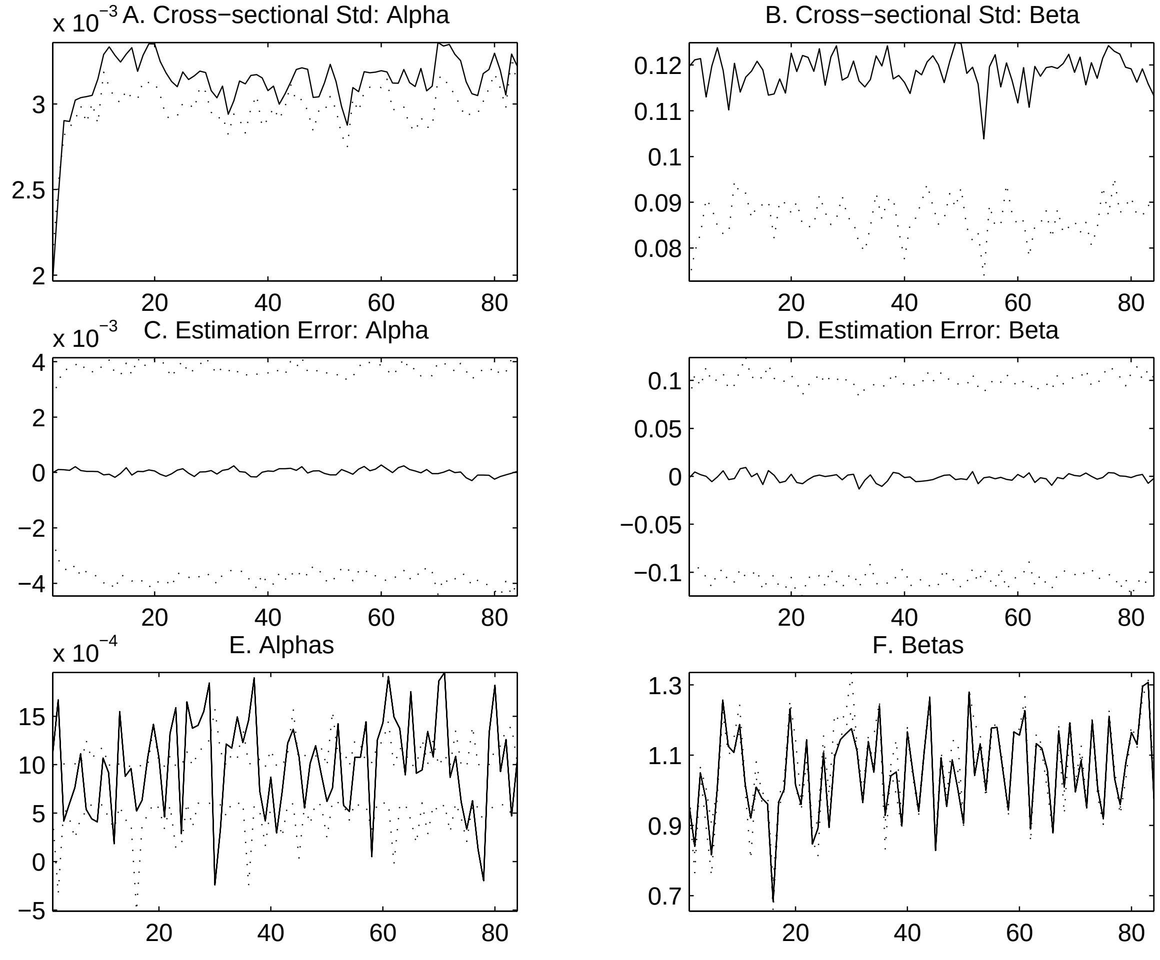


Figure 4: **Buy and hold.** The simulation procedure is identical to the one described in Figure 1. In each case the portfolio buys an equally weighted portfolio of all the stocks at the beginning of the simulation and holds it for 84 months.

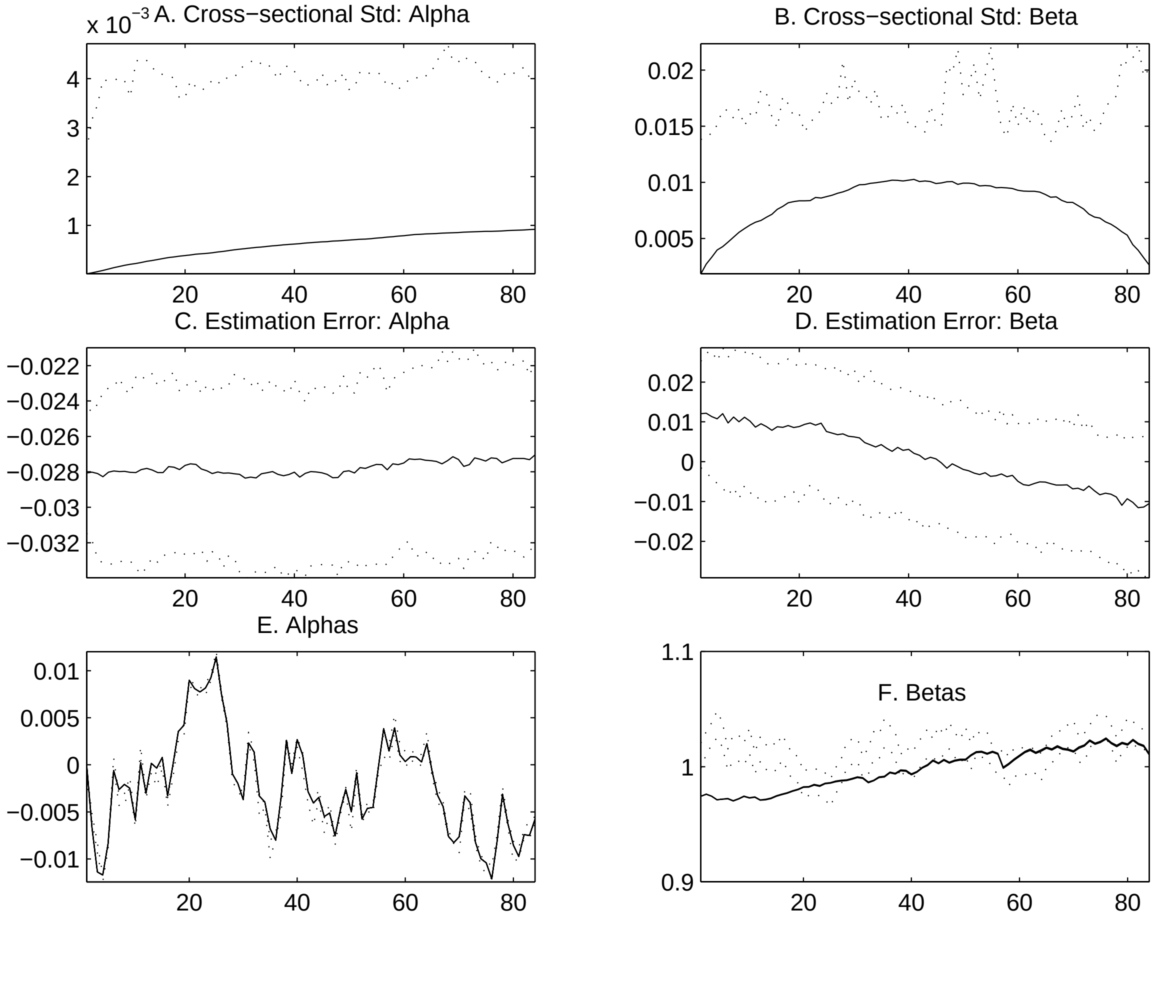


Figure 5: **Out of Sample Zero-Beta Portfolio Correlations with the Market.** Both the Kalman filter and OLS models are estimated on the set of domestic equity mutual funds (ICDI-OBJ: AG, BL, GI, IN, LG, PM, SF and UT) that existed at the end of 1992 and had at least five years of monthly return data. This figure displays the results for funds for which the maximization algorithm for the Kalman filter model converged using data up to the date in question. This leaves about 70% of the total funds from the original sample. Within the pooled funds, the two hundred funds with the highest R^2 statistics are selected separately for the OLS and Kalman groups. One thousand simulations are then produced. For each simulation, forty funds are selected randomly from the 200-fund pools. Funds with a positive forecasted alpha are then used to form equally weighted zero beta portfolios. When a fund dies and drops out of the forty fund sample, a random fund from the positive alpha pool replaces it. This figure plots the bootstrapped distribution of the correlations between the monthly excess returns of the zero-beta portfolio and that of the market for the eight-year period from 1993 to 2000.

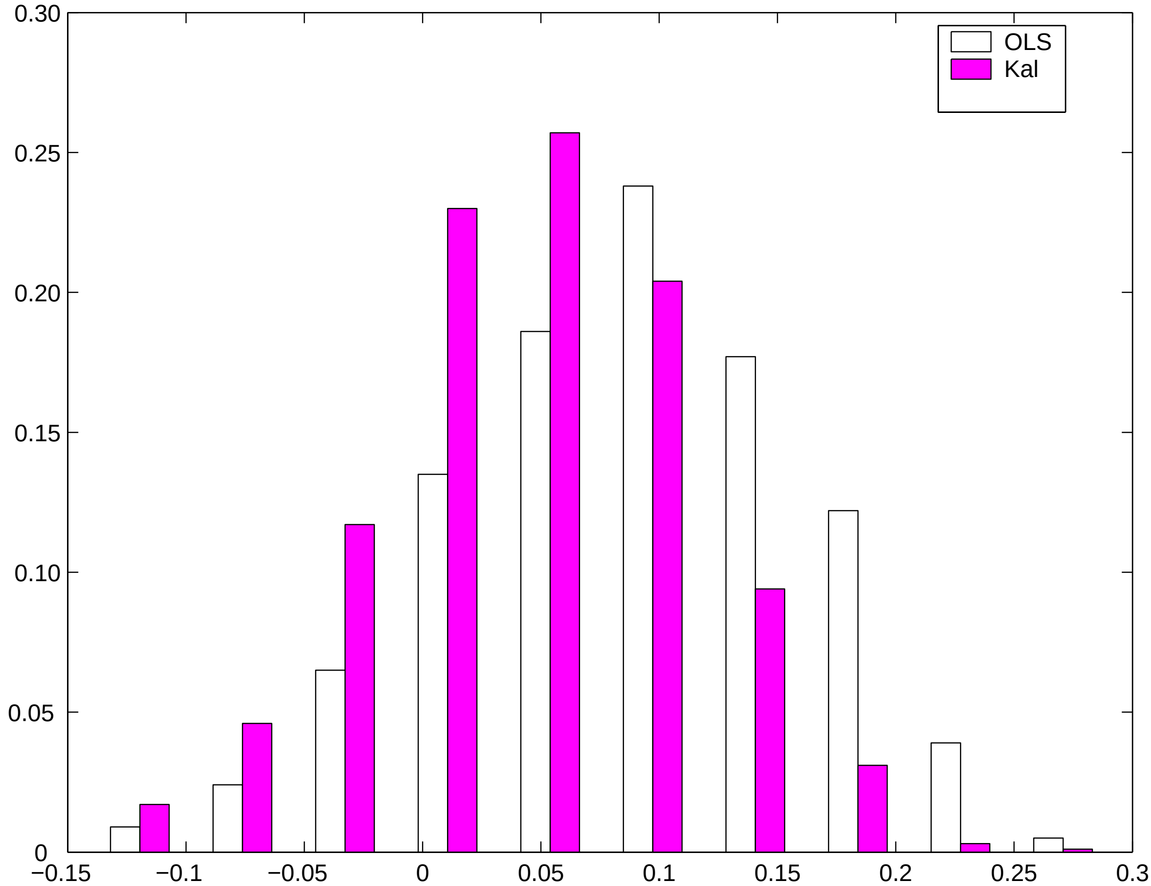
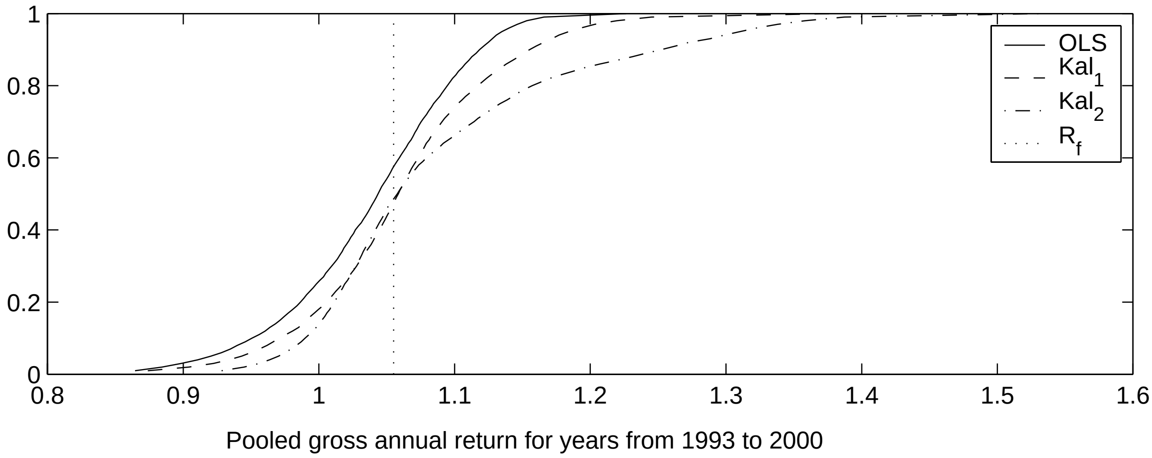


Figure 6: **Bootstrapped Cumulative Out of Sample Return Distributions.** The initial data base consists of all domestic equity mutual funds that existed at the end of year 1992 and had at least 5 years of monthly return data. Funds are included if the maximization algorithm for the Kalman filter model converges using data up to the date in question. To generate the OLS and Kal_1 curves within the pooled funds, the two hundred funds with the highest R^2 statistics are selected separately for each model. One thousand simulations are then produced for each date. For each simulation, fifty funds are selected randomly from each group's pool. Next the five funds with highest alphas (predicted by the model in question) are chosen to form an equally weighted fund of funds portfolio and predicted market risks are hedged out. Finally, the out of sample returns are calculated. If a fund dies, a random fund from the positive alpha pool replaces it. Line Kal_2 derives from a pool containing the funds for which the Kalman filter appears to provide the most information relative to the OLS model. These funds are selected by taking the 200 funds with the greatest difference between the Kalman filter and OLS R^2 statistics. The top graph shows the distributions of the pooled annual returns when the OLS (Line OLS) and Kalman filter (Line Kal_1 and Kal_2) models are used. The bottom graph shows the distributions of the Sharpe ratios of the monthly excess returns.



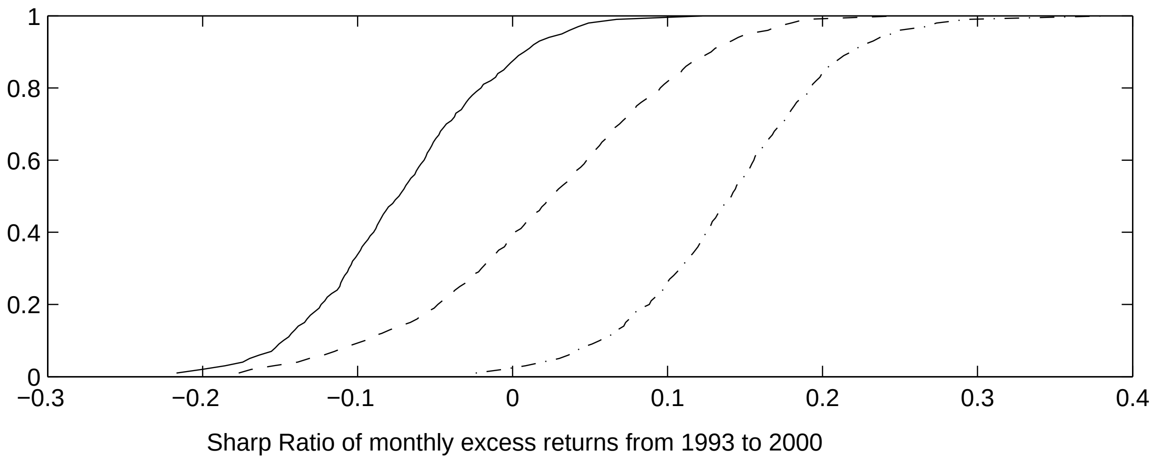


Figure 7: **Likelihood ratio tests.** This figure shows the results from a likelihood ratio test in which the Kalman filter model includes conditional information similar to that of Ferson and Schadt (1996). As in their application, lagged macroeconomic information drives the portfolio weights. However, here portfolio weights are also assumed to vary from some unobserved factor following an AR(1) process. As a first order approximation, the portfolio weights become $f_t^i = \bar{f}^i + L^i F(t) + D^i Z(t-1)$, where $Z(t-1)$ is the lagged information. This model can be estimated via extended Kalman filter. For two macro instruments, the estimated system of equations is given by $r_P(t) - r(t) = \alpha_P(t) + \beta_P(t)(r_m(t) - r(t)) + \varepsilon_t$, where $\beta_P(t) = \beta + F(t) + k_1 z_1(t-1) + k_2 z_2(t-1)$ and $\alpha_P(t) = \alpha + a_P F(t) + b_P F(t)^2$ and $F(t) = \gamma_F F(t-1) + \eta(t)$. Here z_1 and z_2 are instruments for the lagged T-Bill rate, and CRSP value weighted index's dividend yield. The null hypothesis is that $k_1 = k_2 = 0$. The constrained and unconstrained models are estimated using monthly returns from 437 mutual funds during the period of 1994 to 1998. Asymptotically, the likelihood ratio test under the null should follow a chi-square distribution with two degrees of freedom. In Figure 7, the bars represent the cross-sectional distribution of the likelihood ratio while the dashed line displays the mathematical values for a chi-square distribution with two degrees of freedom. The fraction of funds that reject the null hypothesis at the 1%, 5% or 10% levels are 10.8%, 15.7% and 20.3%, respectively.