

Full-Scale Verification of Dynamic Properties from Short Duration Records

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ABSTRACT

This study examines the dynamic properties extracted from a suite of reinforced concrete buildings in South Korea, with emphasis on the performance of system identification approaches applied to short duration ambient records. The resulting extracted properties are then analyzed to identify trends relating damping and frequency to parameters characterizing the structural system and building geometry to highlight underlying behaviors and establish predictive tools suitable for use in the design stage. Comparisons between finite element models and in-situ values are provided in select cases to demonstrate challenges associated with modeling reinforced concrete structures.

INTRODUCTION

Despite the advances in modeling and design, the in-situ properties of many structures deviate significantly from design predictions, particularly with respect to damping values that are largely assumed based on limited apocryphal data. However, even natural frequencies related directly to mass and stiffness properties have also shown similar deviations due to their heavy dependence on modeling assumptions, especially for reinforced concrete structures. This has led to structures that meet serviceably limit states in design but not ultimately in practice. This issue becomes even more important given the increasing height and complexity of modern designs that are generally governed by this limit state. Researchers have sought to address this need through predictive models (e.g., [Jeary 1986, Lagomarsino 1993]) and the establishment of more comprehensive full-scale databases (e.g., [Satake et al. 2003, Lagomarsino and Pagnini 1995]). Inspection of these vital resources reveals that there is still significant scatter in the data due in part to the difficulty in extracting dynamic properties from limited amounts of ambient data, as well as need for more full-scale observations to diversify the structural systems represented and to encompass all the variables that effect dynamic properties. This study focuses on a specific class of structural system and extracts the dynamic properties within a framework appropriate for short duration data, parameterizing the results based on critical attributes of the structural system, thus addressing both of the underlying issues with current databases.

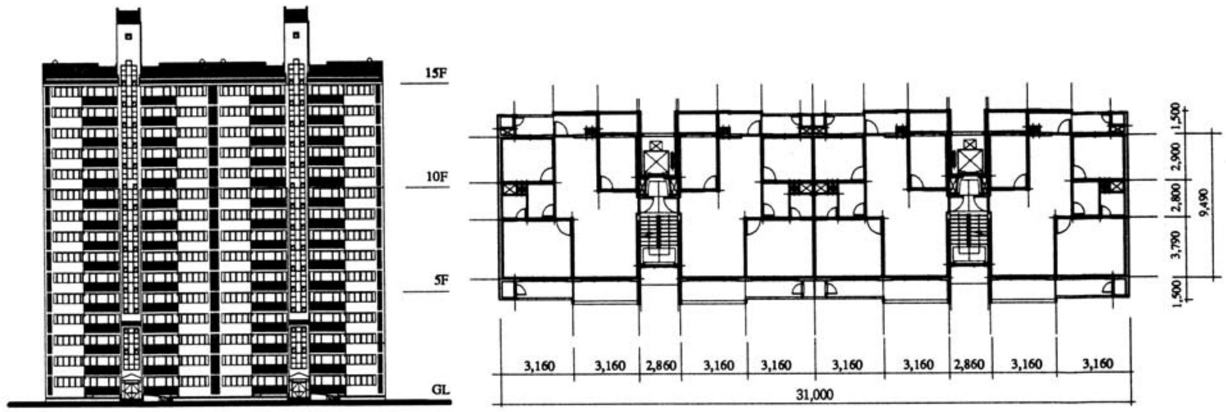

FIGURE 1 - ELEVATION AND PLAN VIEW OF TWO MODULE UNIT

OVERVIEW OF KOREAN DATASET AND STAGE 1 ANALYSES

In response to the aforementioned concerns, an expansive full-scale database of 66 buildings in South Korea was recently established [Yoon et al. 2003] in order to determine dynamic properties specific to Korean-style construction. This study focuses on a subset of the database: 22 reinforced concrete (RC) apartment buildings ranging from 9 to 25 stories.

Obviously dynamic properties are also sensitive to the structural system employed. The apartment buildings considered here are very modular in nature, with each module supported by a service core, as shown in Figure 1. These buildings often have very long aspect ratios and multiple cores. For the buildings considered herein, aspect ratios range from 2.3 to 10.7, with up to 6 modules (cores). The main structural system consists dominantly of reinforced concrete slabs and shear walls with some link beams. The shear walls provide the primary lateral resistance in the short direction with the cross sectional area of shear walls to slabs being as high as 6%, in contrast with 3% in the long direction [Chun 1998].

The ambient vibrations of the buildings were measured by four servo accelerometers: two placed orthogonally at the approximate centerline of the building and two placed out on the perimeter, as shown in Figure 2 [Yoon et al. 2003]. Note that several buildings within this dataset were also excited by a synchronized human excitation (SHE) at the resonant frequency to establish a more reliable free vibration decay.

Initial system identification on this dataset was performed by the third author; these results will be referred to as the *Stage 1* analyses [Yoon and Ju 2004]. The half-power bandwidth (HPBW) was applied to all the response power spectral densities (PSD), which were treated by Hanning windows, while the random decrement technique (RDT) [Vandiver et al. 1982] was applied only to select responses. Variance errors were of concern since response time histories were relatively short in duration, as low as 1720 seconds, thus prompting the project's Stage 2.

STAGE 2 SYSTEM IDENTIFICATION PROGRAM

The Stage 2 analyses, detailed in Erwin et al. [2006], also applied RDT and HPBW but with refinements to improve performance. First, the spectral resolutions were increased as needed to ensure spectral bias errors of -2%, and the Hanning window treatment was not used, so that bandwidths (damping estimates) would not be artificially inflated. For the second refinement, a

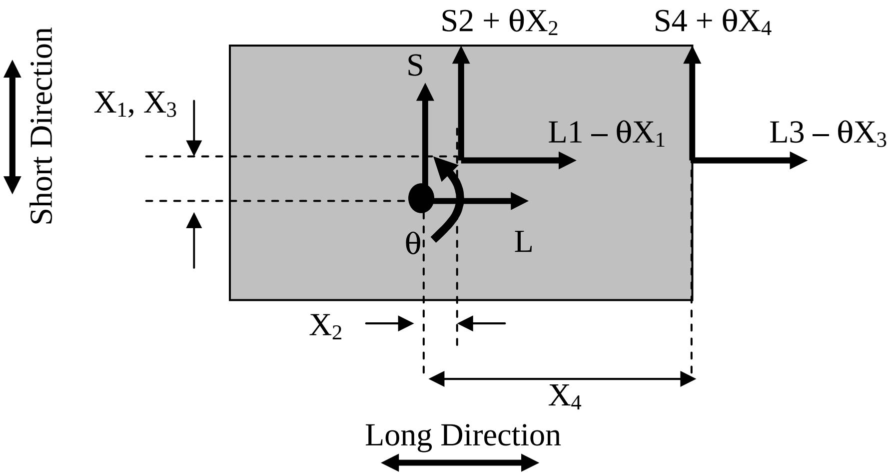

FIGURE 2 - APPROXIMATE INSTALLATION POINTS AND RESPONSE COMPONENTS

local averaging scheme was introduced on the RDT trigger to minimize trigger sensitivity and a least squares minimization was applied to the Hilbert-transformed random decrement signature (RDS) analytic signal for more accurate extraction of dynamic properties [Kijewski-Correa et al. 2006]. In this process, the desired trigger level (local extrema) was set to the root mean square acceleration level of the record (σ) and the array of additional triggers used in the local averaging was defined as: [0.85, 0.9, 1.0, 1.1, 1.15]σ. As the variance of the RDS is known to increase with each cycle of oscillation [Vandiver et al. 1982], best parameter extraction was executed over cycles 1-4.

It should be noted that stationarity of all time histories was verified using the reverse arrangement test [Bendat and Piersol 2000]. As detailed in Erwin et al. [2006], time histories considered were, on average, 93.2% stationary. Furthermore, repeatability of estimated parameters was established by comparing the frequency and damping estimates of a given mode to that of every sensor output in which that mode was observed, and mean values are reported.

MAXIMUM LIKELIHOOD ESTIMATOR

While the refinements proposed in Stage 2 did reduce the bias of the spectra, they result in a higher variance error. To overcome this, the Maximum Likelihood Estimator (MLE) is introduced. This approach has been shown to be effective in extracting the dynamic properties from PSDs generated from limited ambient data [Montpellier 1997]. MLE is a statistical method used to determine the parameters of a target distribution function that best fit empirical data. The governing equation for a set of data, $y = (y_1, y_2 \ldots y_n)$, is:

$$L(y,\theta) = \prod_{i=1}^{n} p(y_i,\theta) \tag{1}$$

where θ is a set of parameters to be estimated (in this case natural frequency f_n and damping ξ), L is the likelihood function, and p is the probability density function (PDF) of known form [Montpellier 1997]. As often is the case, L is a differentiable function of θ; therefore, L must be maximized by taking partial derivates.

As PSDs of ambient response share many mathematical similarities with PDFs, this approach can be extended to the problem at hand, as shown in Figure 3. Therefore, for the single-degree-of-freedom (SDOF) oscillator analogy, p is replaced with a normalized and weighted frequency response function (FRF):

$$p(y,(f_n,\xi)) = \left(\left(\frac{4\xi}{\pi f_n - 4 f_n \xi + 2\Delta\xi} \right) \bullet \left(\frac{1}{\left(1-\left(\frac{f_i}{f_n}\right)^2\right)^2 + \left(2\xi \frac{f_i}{f_n}\right)^2} \right) \right)^{\frac{S_R(f_i)}{A_{peak}}} \tag{2}$$

where Δ is the frequency range associated with the resonant peak, A_{peak} is the area under the resonant peak, S_R is the spectral value, and f_i is the frequency value associated with the point y_i [Montpellier 1997]. The normalization here is necessary to insure that this expression has unit area, as necessary for any PDF. Given that the input process is assumed to be white noise within the bandwidth of this oscillator, the response PSD will be proportional to the FRF, thus allowing the expression in (2) to be applied to output-only observations.

The major limitation of MLE is that fact that the likelihood function must be fit to the spectra over a finite frequency range, (f_{min}, f_{max}), whose definition is critical, particularly for multimode responses. Previously, these limits were set somewhat arbitrarily. For example, Montpellier [1997] set the minimum frequency (f_{min}) for a specified mode of interest as the frequency at which the normalized spectrum first exceeds unity. f_{max} was then defined as $2f_n - f_{min}$. Erwin et al. [2006] proposed and applied herein a more intuitive measure less sensitive to noise, basing this frequency range on the second spectral moment with respect to the natural frequency, μ_2:

$$(f_{min}, f_{max}) = (f_n - 2\sqrt{\mu_2}\ , f_n + 2\sqrt{\mu_2}\ ) \tag{3}$$

where $$\mu_2 = \int_{-\infty}^{\infty} (f - f_n)^2 \hat{S}(f) df \tag{4}$$

and $\hat{S}(f)$ is the normalized power spectrum.

MLE Proof of Concept: Simulation Study

To verify the performance of the MLE method, the response of a SDOF system, with known natural frequency of 1.43 Hz and damping ratio of 1.3% critical, is simulated under Gaussian white noise. The spectral resolution was selected to achieve -2% bias. By varying the duration of the simulation, variance errors associated with limited spectral averages (1, 5 and 10) are evaluated. MLE results are compared to conventional HPBW analyses and differences in mean parameter estimates and their CoVs over 10 repeated trials are graphically depicted in Figure 3. The performance assessment herein shall focus on damping, the most difficult parameter to identify. Although the bias in the spectra is identical for both the MLE and HPBW result, it is important to note that the resulting damping estimates by MLE are far less biased from the actual value of 1.3% critical. The variance in the damping estimates diminishes as the number of spectral averages increases. While this is an expected result, note that the variance in the MLE results is considerably smaller than the HPBW. This indicates that when there are at least 5 averages in the power spectral estimate, the MLE method can provide relatively accurate results.

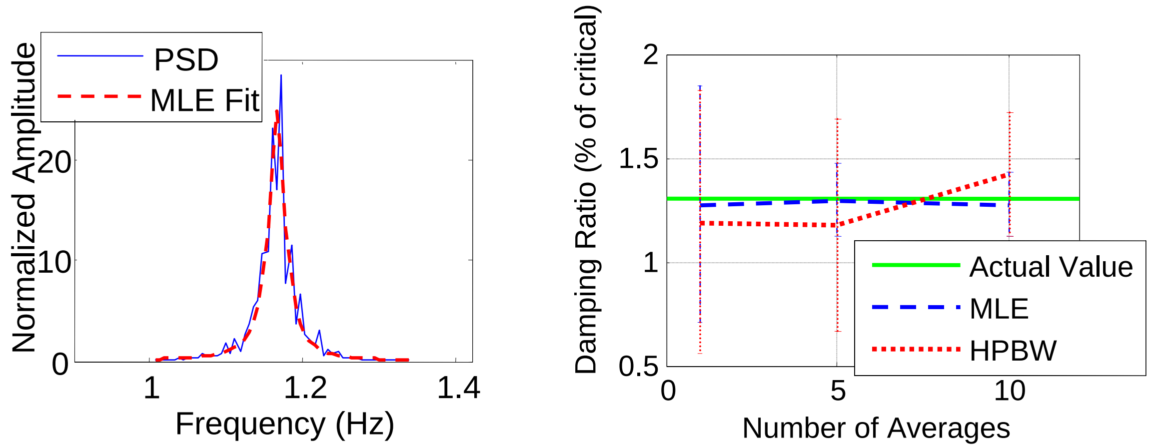

FIGURE 3 – NORMALIZED PSD AND MLE FIT (LEFT) AND COMPARISON OF HPBW AND MLE DAMPING ESTIMATES (RIGHT)

Stage 1 Compared to Stage 2	Long	Short	Torsion
Stage 1 HPBW and Stage 2 HPBW	-2.4%	-18.6%	
Stage 1 RDT and Stage 2 RDT	-50.7%	-23.6%	
Stage 1 SHE and Stage 2 MLE	-4.9%	-0.7%	
Stage 2 Method Comparison			
Stage 2 RDT and Stage 2 HPBW	21.2%	11.4%	37.7%
Stage 2 RDT and Stage 2 MLE	24.9%	26.6%	51.2%
Stage 2 MLE and Stage 2 HPBW	-1.6%	-2.3%	-5.6%
Note: % difference = (method 2 - method 1)/method 1			

TABLE 1 – PERCENT DIFFERENCE BETWEEN METHODS

APPLICATION TO KOREAN DATASET: PREDICTIVE TOOLS

The Stage 2 analyses by HPBW, RDT and MLE are now presented and compared to the Stage 1 analyses also by HPBW, RDT (select buildings) and SHE (select buildings). Recall that the SHE results are assumed to be more reliable as they impart controlled excitations to generate free vibration curves. The focus herein will be on the damping estimates. Though the comparisons between methods vary from building to building and are provided in Erwin et al. [2006], the average percent differences are summarized in Table 1, with the following observations:

- Stage 1 HPBW damping estimates are larger than their Stage 2 counterparts, which is expected as the Stage 1 estimates tend to overestimate damping due to larger spectral bias and the use of a Hanning window;
- RDT comparisons show a similar trend, but to lesser known reason. This may be a result of the breakdown of the RDS in the later cycles, leading to a larger damping estimate; restricting the number of cycles used in the estimation may have alleviated this in Stage 2;
- MLE results in Stage 2 agree well with SHE in Stage 1, further confirming that MLE yields the most reliable parameter estimate, particularly with limited data;

- The MLE and HPBW estimates tend to be larger than the RDT estimates by 20% or more. This may be due to inherent spectral bias and is consistent with observations in AIJ [2000]. It is unclear why this affect is more pronounced for torsion;
- Stage 2 MLE and HPBW results compare well, which indicates on average that the amount of data available for most buildings in the suite was sufficient to mitigate variance issues, even when increased spectral resolutions were used.

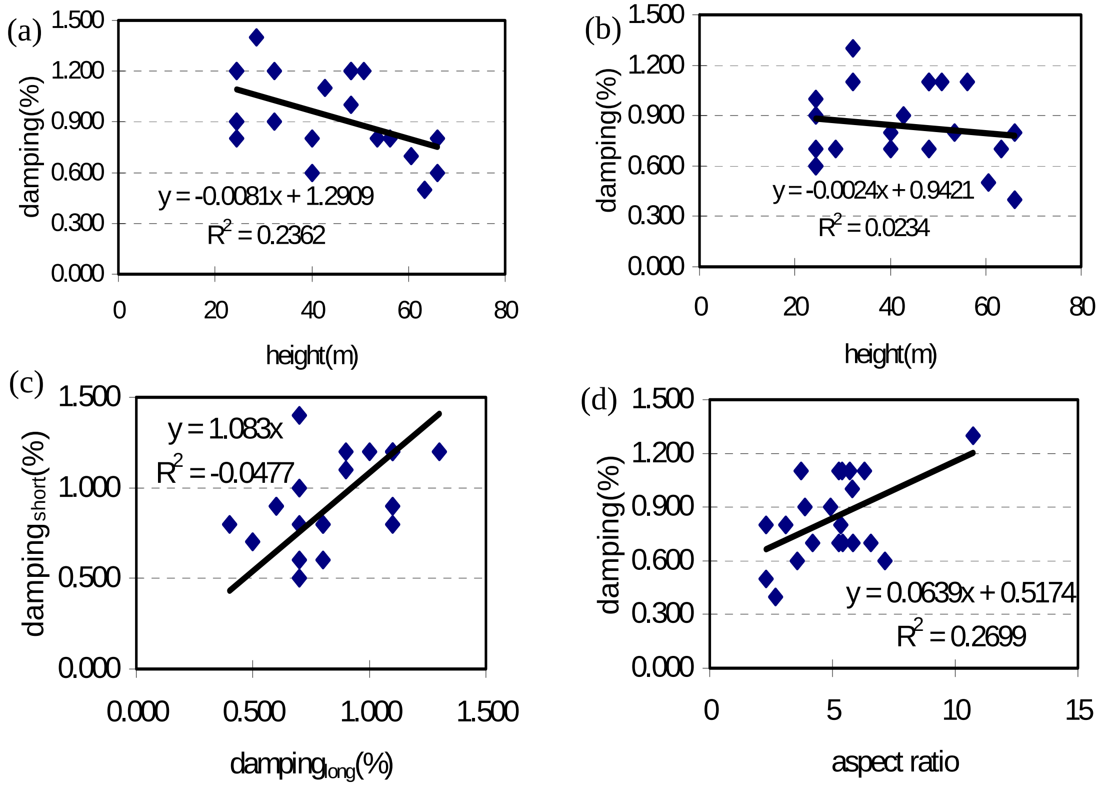

FIGURE 4 – STAGE 2 RDT DAMPING (A) WITH HEIGHT IN SHORT DIRECTION, (B) WITH HEIGHT IN LONG DIRECTION, (C) COMPARISON BETWEEN DIRECTIONS, AND (D) WITH ASPECT RATIO IN LONG DIRECTION

Percent Critical Damping

Erwin et al. [2006] explored the influence of a number of parameters on the damping and frequency values extracted in the Stage 2 analysis. Since the buildings are all of the same construction materials, usage, structural system and foundation type, this provides an excellent opportunity to isolate the influences of specific variables on energy dissipation. The discussion herein will first focus on trends in damping, using the Stage 2 RDT results, which are expected to be unbiased. For the RC apartments considered herein, the plan depth in the short direction is 9.9 to 12.7 m; thus, the influence of aspect ratio should be negligible in this direction, making height the logical parameter to explore. Despite the scatter, a decrease of damping with height is noted in the short direction (Figure 4a), consistent with the findings of Satake et al. [2003]. This result can be contrasted with the damping observed in the long direction, which reflects similar scatter and a less pronounced dependence on height (Figure 4b). A comparison of the y-intercepts in Figures 4a and 4b and the slope in Figure 4c indicates that independent of height, the damping is on average 8% larger in the short direction. This is in contrast with the findings of Satake et al. [2003] who found that the damping ratio in the short direction was 75% of the long; however, the reason for this is again tied to the features of the structural system. The short direction for the Korean RC building suite has a greater contribution of shear walls to its lateral resistance, providing more pronounced energy dissipation due to shear wall deformations. This underscores the importance of considering structural system type and even regional design practice (e.g., Korean vs. Japanese) in the development of any predictive model for damping.

The lack of a decisive trend in the long direction (Figure 4b) may further be explained by the fact that plan depth for this suite of buildings in the long direction varies considerably between 29 m and 118 m. Thus while the parameterization of dynamic properties such as damping has considered solitary variables such as height [Satake et al. 2003, Yoon and Ju 2004], the impact of aspect ratio on damping can be noteworthy, as shown in Figure 4d. The trend indicates that damping increases with aspect ratio; this may be rationalized in part by the fact that the long direction relies heavily on the slab-action and longer aspect buildings will experience more participation of the slab in lateral resistance and understandably more energy dissipation due to the increased out-of-plane deformations. Perhaps more importantly, in residential buildings, such as those considered here, as the aspect ratio increases, so does the number of partitions. These non-structural elements can have a considerable impact on energy dissipation, as also noted by Satake et al. [2003]. Unfortunately, additional data on buildings with aspect ratios of 7 or greater will be required to verify this finding.

To further the discussion, the observed damping values from Stage 2 are averaged over all methods considered and are displayed with respect to aspect ratio and height, as shown for the long direction in Figure 5. A best-fit plane is cast over the space to highlight the major trend in the data. In order to generate a predictive model, a variety of aspect ratios were selected and the corresponding linear trends with height are extracted from this plane (Table 2).

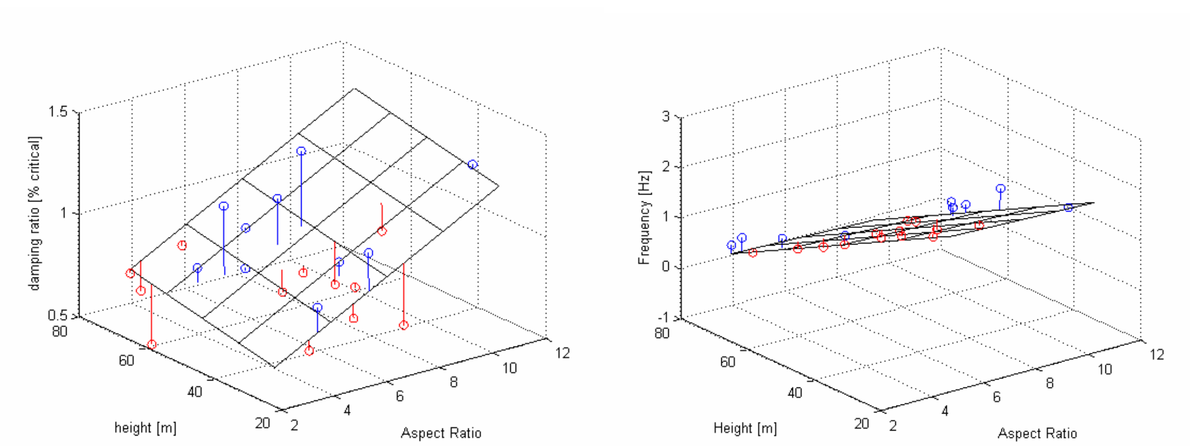

FIGURE 5 - EXAMPLE 3-DIMENSIONAL PLOT OF DAMPING IN THE LONG DIRECTION (LEFT) AND FREQUENCY (RIGHT) FOR LONG DIRECTION

A few observations are immediately evident:

- The slopes in Table 2 indicate that damping shows a more substantial reliance on height for the short direction. Increases in height lead to more slender shear walls providing lateral support in this direction. These more slender shear walls are increasingly dominated by cantilever action and dissipate less energy as their height increases, consistent with the findings for steel buildings [Kijewski-Correa et al. 2006];

- The y-intercepts indicate that damping is largest in the short direction (due to the increased shear walls) and slightly increases with aspect ratio by 7%;
- The long direction shows a more dramatic increase in damping with aspect ratio (42%), for reasons noted earlier;
- Torsional damping shows only slight increases with aspect ratio and modest decreases with height; therefore, it can be assumed to be generally constant for this class of structural system.

Aspect Ratio	Long Direction Equation	Short Direction Equation	Torsion Equation
1	ξ (%) = 0.0036H+0.4864	ξ (%) = -0.0088H+1.3949	ξ (%) = -0.0043H+1.1269
2	ξ (%) = 0.0036H+0.5562	ξ (%) = -0.0088H+1.4136	ξ (%) = -0.0043H+1.1361
4	ξ (%) = 0.0036H+0.6958	ξ (%) = -0.0088H+1.4510	ξ (%) = -0.0043H+1.1545
6	ξ (%) = 0.0036H+0.8354	ξ (%) = -0.0088H+1.4884	ξ (%) = -0.0043H+1.1729
where H is in meters			

Aspect Ratio	Long Direction Equation	Short Direction Equation	Torsion Equation
1	f_n (Hz) = -0.0462H+3.8605	f_n (Hz) = -0.0526H+4.1294	f_n (Hz) = -0.0575H+5.1296
2	f_n (Hz) = -0.0462H+3.7615	f_n (Hz) = -0.0526H+4.0427	f_n (Hz) = -0.0575H+4.9686
4	f_n (Hz) = -0.0462H+3.5633	f_n (Hz) = -0.0526H+3.8692	f_n (Hz) = -0.0575H+4.6465
6	f_n (Hz) = -0.0462H+3.3651	f_n (Hz) = -0.0526H+3.6957	f_n (Hz) = -0.0575H+4.3244
where H is in meters			

TABLE 2 - TRENDS FOR DAMPING (TOP) AND FREQUENCY (BOTTOM), AVERAGED OVER ALL STAGE 2 METHODS

Natural Frequency

Predictive models have been developed for frequency, particularly for use in preliminary design or in codes/standards (e.g., [ASCE 7-05]). As demonstrated by Table 2, the frequency values averaged over all Stage 2 analyses show decreases of frequency with height, as expected and observed elsewhere [Satake et al. 2003, Yoon and Ju 2004]. However, this study also underscores the importance of other structural parameters including aspect ratio. Table 2 indicates that natural frequency also decreases with aspect ratio by 11 to 19% for this structural system. The finding is not surprising due to enhanced diaphragm flexibility and the associated difficulties with fully engaging structural elements over elongated floor plans. As demonstrated in Figure 5, the noted trends in frequency demonstrate considerably less scatter than damping, as expected.

Since final frequency values are generally estimated by finite element models (FEM) for most engineered buildings, a comparision of the predicted and in-situ values is also provided for three of the RC buildings analyzed. The buildings were represented by FEMs in the commercial software MIDAS-ADS using common Korean design assumptions. The slab thickness was modeled as 135 mm; perimeter/shear walls were 180 mm thick; partitioning walls were 150 mm thick, except for building RC16, which used 180 mm. RC compressive strengths were 24 MPa for floors 1-6 and 21 MPa for floors above this level. The flexural stiffness of link beams between vertical wall components was reduced to one-fifth of the elastic stiffness, as is customary in Korean design practice. Out-of-plane slab stiffness was set to 0%, 25% and 100% to explore its influence on the natural frequency of the buildings [Yoon et al. 2006].

By neglecting the out-of-plane stiffness of the slab, natural frequencies in the short direction can be captured relatively well by FEM (Table 3). This is not the case for the long direction. In the short direction, lateral resistance is again primarily offered by the shear walls, without considerable reliance on the out-of-plane stiffness of the floor slab (diaphragm action); however, the role of the slab diaphragm is more important in the long direction, which is characterized by a smaller shear wall to slab ratio, as discussed previously. Thus the complete modeling of the out-of-plane stiffness of the floor slab (up to 100% stiffness) is required in this direction (Table 3). Similar observations were made by Yoon et al. [2006] based on Stage 1 HPBW results. This analysis underscores the importance of the modeling assumptions made in reinforced concrete buildings, particularly with respect to the participation of various elements in the overall lateral resistance.

	Aspect Ratio	In-Situ MLE Result (Hz)		FEM (Hz) [0%, 25%, 100% slab stiffness]	
		Long direction	Short direction	Long direction	Short direction
RC 7	10.7	1.43	1.74	1.28, 1.38, 1.58	1.78, 1.92, 2.08
RC 12	4.9	1.33	1.17	0.95, 1.01, 1.15	1.39, 1.47, 1.59
RC 16	3.7	1.07	1.06	0.62, 0.69, 0.84	1.09, 1.14, 1.22

TABLE 3 - NATURAL FREQUENCIES BY FEM AND IN-SITU MLE RESULTS

CONCLUSIONS

This study discusses the frequency and damping values associated with a suite of Korean reinforced concrete apartment buildings highlighting three areas: performance of various identification methods, trends in dynamic properties, and the comparison between in-situ and predicted frequency values. First, the performance of various identification methods was assessed, demonstrating the superior performance of Maximum Likelihood Estimators in situations where limited data is available, though still acknowledging the presence of spectral bias. This method was shown to produce low bias, low variance estimates of damping when at least five spectral averages are available. Trend analyses producing predictive models as a function of height and aspect ratio revealed that damping in the short direction was 8% larger due to the presence of more shear walls. This damping contribution was most sensitive to height, decreasing as height increases due to growing cantilever action in the shear walls. This deformation mechanism had been previously hypothesized to dissipate less energy than frame action, so findings reported here are consistent. On the other hand, long direction damping is derived primarily from slab diaphragm action and as such is directly proportional to the aspect ratio. This underscores the importance of parameterizing damping databases using variables appropriate for the primary structural system of each response component. In the case of torsion, there was no significant influence of aspect ratio or height. Overall, the damping observed in this suite of reinforced concrete buildings was less than the customary 2% serviceability value, but it should be noted that the damping values presented here were extracted under mild ambient vibrations and may increase under larger wind events associated with serviceability limit states. The final evaluation demonstrated the impact of underlying assumptions in the finite element modeling of reinforced concrete systems, specifically the participation of floor systems in the overall lateral resistance. This was shown to be far more critical in the long direction, more reliant on slab action, than in the short direction characterized by more shear walls. In-situ frequency values demonstrated the expected trends of decreasing value with height and aspect ratio.

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