Parameter estimation for in situ observation bias with an ensemble Kalman filter

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General aim: Improve Numerical Weather Prediction (NWP) over complex terrain.

Strategies :

1. Improve NWP models => e.g., parameterizations
2. Improve initial conditions => e.g., data assimilation (DA)

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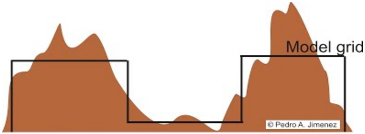
1. Improve NWP models => e.g., parameterizations
   Idea: semi-empirical or physical models that resolve some atmospheric processes that dynamic core can't because they are sub-grid processes, too complex processes or non-well know processes. Ex.: planetary boundary layer, surface fluxes, etc.
   Especially relevant for complex terrain is the **sub-grid scale orography** parameterization (e.g Jimenez and Dudhia, 2012).



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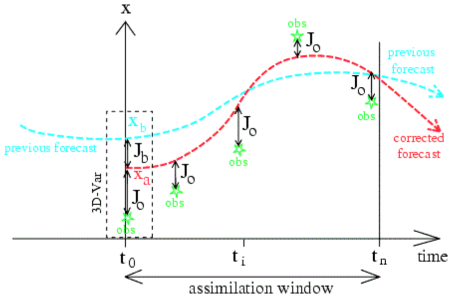
General aim: Improve Numerical Weather Prediction (NWP) over complex terrain.

Strategies :

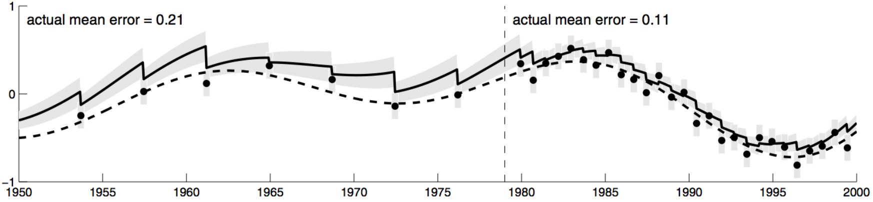
1. Improve NWP models => e.g., parameterizations
2. Improve initial conditions => e.g., data assimilation (DA)
   Idea: Observations are used to correct the errors in a model-generated background estimation.



source: ECMWF

Hypothesis: model and observations have **random errors** (zero mean).
In reality: model and observations have **systematic errors** (non-zero mean).

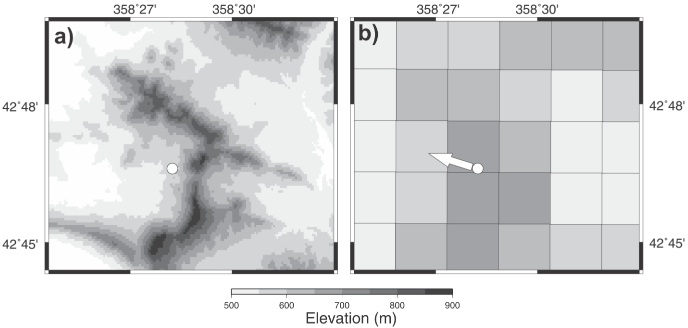
Example of assimilation of unbiased observations in a biased model.



The dashed curve represents the true state evolution, observations are indicated by the dots, and the solid curve is the product of the assimilation cycle (Source: Dee, 2005).

A cause of bias over complex terrain:

- Representativeness errors that account for processes that are captured by observations but the model is unable to represent.
- Tautological concept. Is the error in the model or it is in the observations? In DA, in the observations.
- Especially important in the assimilation of **surface observations**.



Source: Jimenez and Dudhia, 2012.

The aim of this work is to improve the assimilation of surface observations.

- We propose a method to **estimate and correct observations systematic errors**.
- We focus on surface **in situ observations**: bias is dependent on the location and uncorrelated in space.
- Methodology is inspired by satellite radiances: the parameter bias is included in the forward operator and the augmented space estate.

Bias aware method

The observations bias are location dependent.

A biased observations ($\mathbf{y}$) can be modeled by:

$$\mathbf{y} = h(\mathbf{x^t}) + \boldsymbol{\epsilon}' + \boldsymbol{\beta} \tag{1}$$

where

- $\mathbf{x^t}$ is the true model state
- $\boldsymbol{\epsilon}'$ is gaussian noise with zero mean and standard deviation σ^2
- $\boldsymbol{\beta}$ is bias parameter
- $h(\mathbf{x^t})$ is the forward operator that linearly interpolates $\mathbf{x^t}$ to the observational space considering the nearest grid-point

Augmented state

State space augmentation $\mathbf{z} = [\mathbf{x}, \beta]^\top$ is used for observations bias.
The statistical analysis equations are then given by:

$$\mathbf{z}^\mathbf{a} = \mathbf{z}^\mathbf{f} + \mathbf{K}(\mathbf{y} - \mathbf{H}\mathbf{z}^\mathbf{f}) \quad (2)$$

$$\mathbf{K} = \mathbf{P}^\mathbf{f}\mathbf{H}^\top(\mathbf{H}\mathbf{P}^\mathbf{f}\mathbf{H}^\top + \mathbf{R})^{-1} \quad (3)$$

$$\mathbf{P}^\mathbf{a} = (\mathbf{I} - \mathbf{K}\mathbf{H})\mathbf{P}^\mathbf{f} \quad (4)$$

where:
$\mathbf{R}$ error covariances for observations.
$\mathbf{P}^{\mathbf{f},\mathbf{a}}$ error covariances obtained through the ensemble perturbations for the background or analysis augmented state.

$$\mathbf{P}^{\mathbf{f},\mathbf{a}} = \begin{bmatrix} \mathbf{P}^{\mathbf{f},\mathbf{a}}_{\mathbf{x}} & \mathbf{P}^{\mathbf{f},\mathbf{a}}_{\mathbf{x}\beta} \\ \mathbf{P}^{\mathbf{f},\mathbf{a}}_{\mathbf{x}\beta} & \mathbf{P}^{\mathbf{f},\mathbf{a}}_{\beta} \end{bmatrix} \quad (5)$$

Cross-correlations are not negligible!

Lorenz 2005 model

In Lorenz's model (version III) the governing equation of variables Z_n is:

$$\frac{dZ_n}{dt} = [X, X]_{K,n} + b^2[Y, Y]_{1,n} + c[Y, X]_{1,n} - X_n - bY_n + F \quad (6)$$

where

- the small-scale (short waves, X_n) and large scale (long waves, Y_n) are superposed to Z_n
- the advection terms are defined to increase the spatial variability between grid-points
- F represents the forcing term => tuned to introduce **model error**

Experimental design

Lorenz's parameters:

- 960 grid points.
- External forcing:
  - F=15 => Perfect model
  - F=17 => Imperfect model

Synthetic observations:

- 240 observations spatially randomly distributed.
- Modeled by Eq. 1 ($\mathbf{y} = h(\mathbf{x^t}) + \boldsymbol{\epsilon}' + \boldsymbol{\beta}$):
  - ϵ' normal noise with mean 0 and variance 0.5.
  - $\boldsymbol{\beta}$ bias parameter such as $\beta_{\mathbf{i}} = 0.3$ for i=1,240 observations.

Assimilation strategies:

- Ensemble Kalman filter (Anderson, 2001) which was fine tuned for this specific application.
- Observations are assimilated every 6 h (50 time steps).

Sensitivity Experiments

The sensitivity of the bias correction and estimation is tested for:

- **Model error**. Tuning the forcing term in Lorenz model.

Experiment	Bias source	Estimated parm.
DFT-NOBIAS	-	-
DFT-BIAS	$\beta_{o,M}$	-
AUG-BIAS-FC	$\beta_{o,M}$	β_M
AUG-BIAS-OBS	$\beta_{o,M}$	β_o
AUG-BIAS-FCOBS	$\beta_{o,M}$	$\beta_o + \beta_M$
AUG-NOBIAS-FC	β_M	β_M

β_o = observations bias and β_M = model bias
DFT = Default or no augmented state
AUG = Augmented state

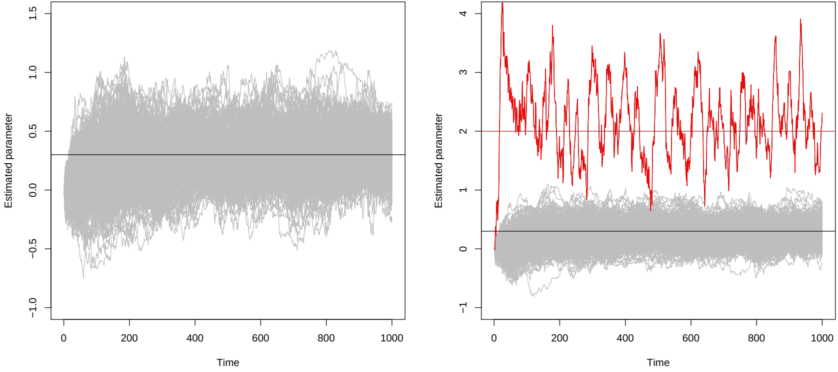
- **Ensemble size** VS. number of **parameters**.

Performance: bias estimation

Time evolution of the estimated 0.3 bias for 240 observations with and without model error. For the model error, time evolution of the estimated model bias.

PERFECT MODEL (F=15) β_O

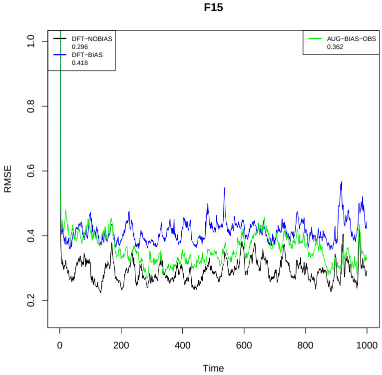
IMPEFECT MODEL (F=17) β_O & β_M



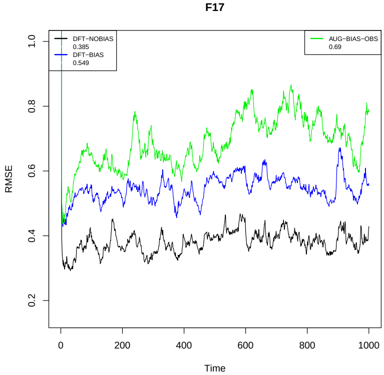
Performance: bias correction

Temporal evolution of Prior RMSE for different parameter estimation.

PERFECT MODEL (F=15)



IMPERFECT MODEL (F=17)



DFT-NOBIAS	DFT-BIAS	AUG-BIAS-OBS
-	$\beta_{o,M}$	$\beta_{o,M}$
-	-	β_o

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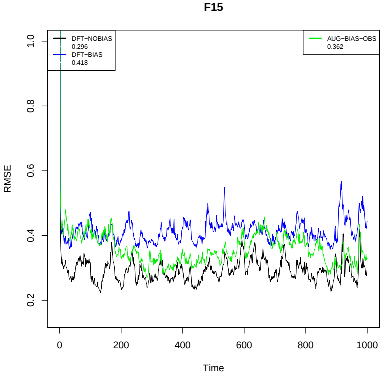

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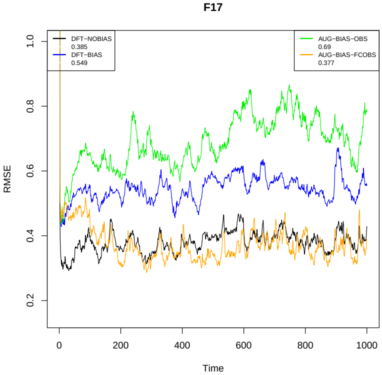

DFT-NOBIAS	DFT-BIAS	AUG-BIAS-OBS	AUG-BIAS-FCOBS
-	$\beta_{o,M}$	$\beta_{o,M}$	
-	-	β_o	$\beta_{o,M}$

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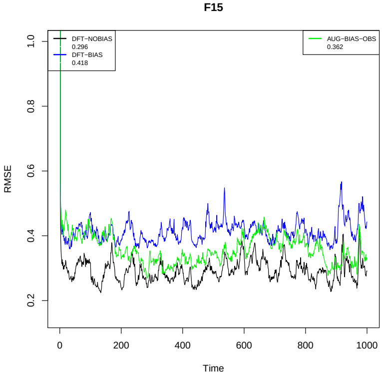

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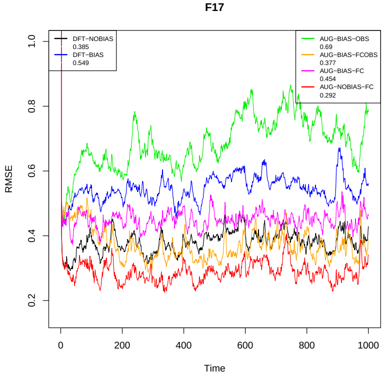
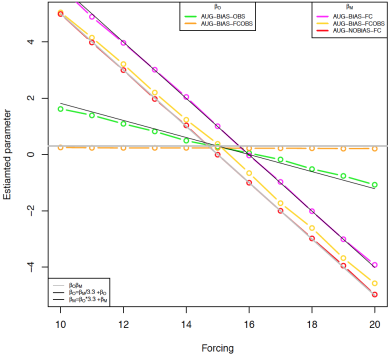


DFT-NOBIAS	AUG-NOBIAS-FC	DFT-BIAS	AUG-BIAS-FC	AUG-BIAS-OBS	AUG-BIAS-FCOBS
-	β_M	$\beta_{o,M}$	$\beta_{o,M}$	$\beta_{o,M}$	$\beta_{o,M}$
-	β_M	-	β_M	β_o	β_o

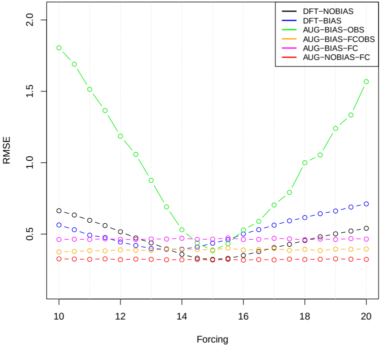
Model error

Parameter estimation and RMSE for different parameter estimation, varying by the forcing term F in the assimilation. The correct (true) F=15.

Bias estimation (β_O and β_M)



Bias correction

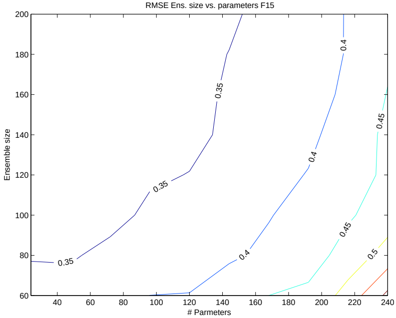


DFT-NOBIAS	AUG-NOBIAS-FC	DFT-BIAS	AUG-BIAS-FC	AUG-BIAS-OBS	AUG-BIAS-FCOBS
-	β_M	$\beta_{o,M}$	$\beta_{o,M}$	$\beta_{o,M}$	$\beta_{o,M}$
-	β_M	-	β_M	β_o	β_o

Ensemble size vs. parameters

Prior RMSE (colors) for different ensemble size and number of estimated parameters.

PERFECT MODEL (F=15)



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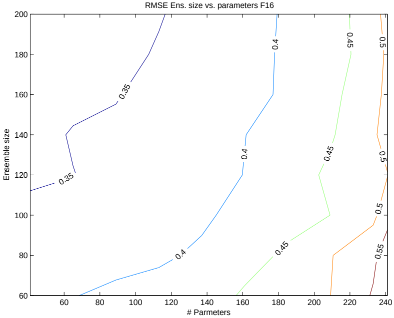


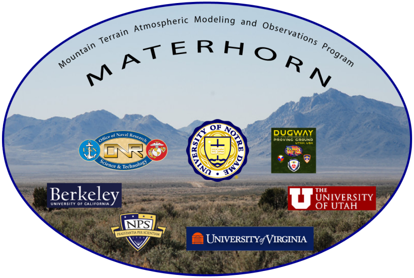
Summary and conclusions

A method to estimate and correct bias for in situ observations have been developed.

- For **perfect model**, the proposed approach is able to estimate and correct observations bias.
- For **imperfect model**, observation bias is corrected and estimated if model bias is also estimated.
- As long as the **ensemble size** and the number of parameters is large enough, errors are independent of them.
- There is an optimum **parameter variance** that minimizes the errors.

Future work: To implement this method in WRF model.

Gracias!

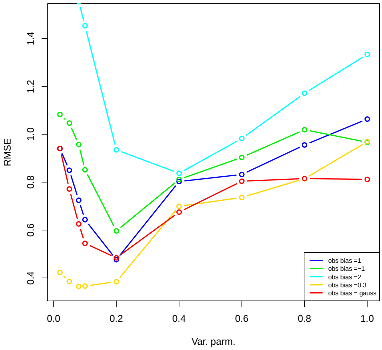


Isaiah 40:4 (ESV) Every valley shall be lifted up, and every mountain and hill be made low; the uneven ground shall become level, and the rough places a plain.

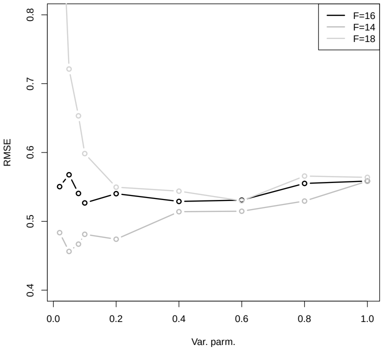
Parameter variance

Prior RMSE for different parameter variances with different observations bias (color lines) and different model bias (grey lines).

OBS. PARM. VAR.



MODEL PARAM. VAR.



Error covariances matrix

The matrix of error covariances are obtained through $\mathbf{P}^{\mathbf{f},\mathbf{a}} = \mathbf{Z}^{\mathbf{f},\mathbf{a}}(\mathbf{Z}^{\mathbf{f},\mathbf{a}})^{\top}$ where $\mathbf{Z}^{\mathbf{f},\mathbf{a}}$ is the matrix of ensemble perturbations for the background or analysis augmented state.

$$\mathbf{P}^{\mathbf{f},\mathbf{a}} = \begin{bmatrix} \mathbf{P}^{\mathbf{f},\mathbf{a}}_{\mathbf{x}} & \mathbf{P}^{\mathbf{f},\mathbf{a}}_{\mathbf{x}\beta} \\ \mathbf{P}^{\mathbf{f},\mathbf{a}}_{\mathbf{x}\beta} & \mathbf{P}^{\mathbf{f},\mathbf{a}}_{\beta} \end{bmatrix} \tag{7}$$

Cross-correlations are not negligible!.
Scalar example with model β_M and obs. bias β_o:

$$\sigma^2_{\beta_M \beta_O a} = \frac{-\sigma^2_{\beta_O f}\sigma^2_{\beta_M f}}{\sigma^2_{TOT}} \tag{8a}$$

$$\sigma^2_{xa\beta_O} = \frac{-\sigma^2_{\beta_O f}\sigma^2_{xf}}{\sigma^2_{TOT}} \tag{8b}$$

$$\sigma^2_{xa\beta_M} = \frac{-\sigma^2_{\beta_M f}\sigma^2_{xf}}{\sigma^2_{TOT}} \tag{8c}$$

where $\sigma^2_{TOT} = \sigma^2_{\beta_M f} + \sigma^2_{\beta_O f} + \sigma^2_x f + \sigma^2_O$

Lorenz's parameters:

- N=960 (grid points), F=15 (perfect model), K=32, I=12, b=10, c=2.5

Synthetic observations (240):

$\mathbf{y^o} = \tilde{h}(\mathbf{x^t}) + \epsilon^o + \beta$

- ϵ^o normal noise with mean 0 and variance 0.5
- $\beta_\mathbf{i} = 0.3$, i=1,240 or $\beta = N(0, 0.3)$

Assimilation strategies:

- EAKF (Anderson, 2001)
- To avoid filter divergence the spatially-varying state space is "inflated" for Prior with initial value of **1.1**, initial standard deviation 0.6 and damping 0.9 (Anderson and Anderson, 1999)
- Localization: the fifth-order piecewise rational function with a **0.3** cutoff (Gaspari and Cohn, 1999)

The RMSE for the background is employed to evaluate the bias correction in the assimilation

$$RMSE = \sqrt{\frac{1}{T}\sum_{t=1}^{T}\frac{1}{N}\sum_{i=1}^{N}(x_i^t(t) - x_i^f(t))^2} \quad (9)$$

where N(=960) number of variables.
The deterioration of the parameter estimation is evaluated through RMSEpar, defined by:

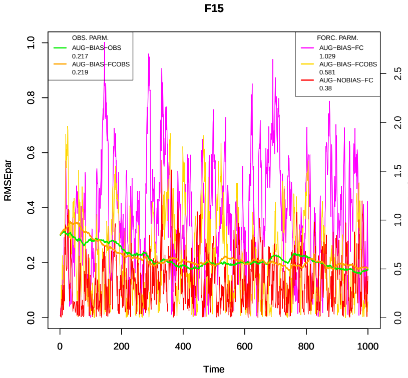
$$RMSEpar = \sqrt{\frac{1}{T}\sum_{t=1}^{T}\frac{1}{M}\sum_{j=1}^{M}(\beta_j(t) - p_j)^2} \quad (10)$$

where M(=240) number of parameters, p_i real parameter and β_i estimated parameter

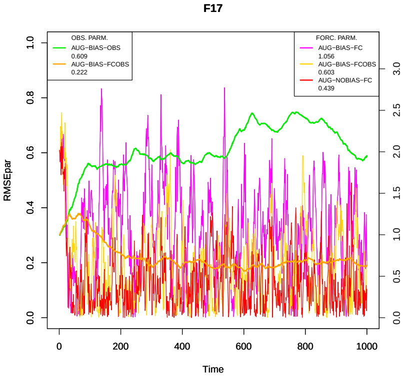
Performance: bias parameter estimation

Temporal evolution of RMSEpar for different parameter estimation.

PERFECT MODEL



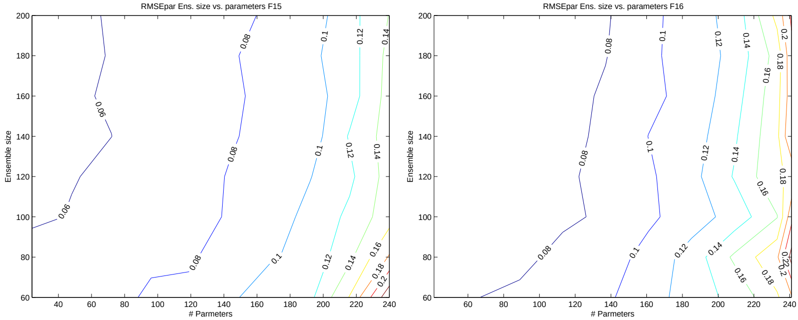
IMPERFECT MODEL



DFT-NOBIAS	AUG-NOBIAS-FC	DFT-BIAS	AUG-BIAS-FC	AUG-BIAS-OBS	AUG-BIAS-FCOBS
-	β_M	$\beta_{o,M}$	$\beta_{o,M}$	$\beta_{o,M}$	$\beta_{o,M}$
-	β_M	-	β_M	β_o	β_o

Ensemble size: Imperfect model

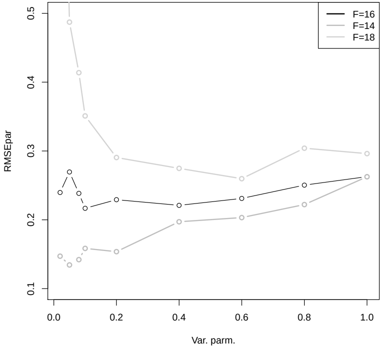
Prior RMSE and RMSEpar for different ensemble size and number of estimated parameters.

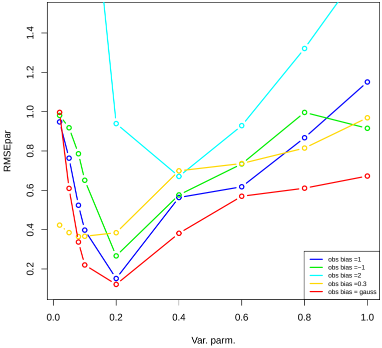


Imperfect-model assimilation experiments with F=16 and obs bias =1

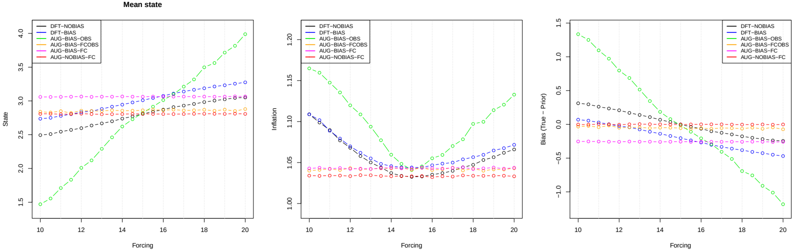
Parameter variance: model bias

RMSE and RMSEpar for different parameter variances.





For the imperfect-model assimilation experiments with different forcing and the parameter variances are varying for the model bias parameter.


Mean state
State
Forcing
Inflation
Forcing
Bias (True – Prior)
Forcing
DFT–NOBIAS
DFT–BIAS
AUG–BIAS–OBS
AUG–BIAS–FCOBS
AUG–BIAS–FC
AUG–NOBIAS–FC

General notation:

f forecast prior background, t truth, a analysis, o observation $\mathbf{x_t}$ true model state (dimension n)
$\mathbf{x_b}$ background model state (dimension n)
$\mathbf{x_a}$ analysis model state (dimension n)
y vector of observations (dimension p)
H observation operator (from dimension n to p)
B covariance matrix of the background errors $((\mathbf{x_b} - \mathbf{x_t}))$ (dimension nxn)
R covariance matrix of the observations errors $((\mathbf{y} - \mathbf{H}^{[}\mathbf{x_t}]))$ (dimension pxp)
A covariance matrix of the background errors $((\mathbf{x_a} - \mathbf{x_t}))$ (dimension nxn)

Hypothesis

- Linearized observations operator
- Non-trivial error
- Uncorrelated error
- Linear analysis: analysis is defined by corrections of the bacground which depend linearly on innovation
- Optimal analysis: the true is as closet as possible to the true state in an rms sense
- Unbiased error

This includes 'perfect model' experiments (also called Observing System Simulation Experiments - OSSEs). Essentially, the model is run forward from some state and, at predefined times, the observation forward operator is applied to the model state to harvest synthetic observations. This model trajectory is known as the 'true state'. The synthetic observations are then used in an assimilation experiment. The assimilation performance can then be evaluated precisely because the true state (of the model) is known.