Predictability in complex terrain

Investigations with ensemble sensitivity

Josh Hacker (and others)

MATERHORN investigator meeting, 17 Aug. 2012

MATERHORN-M Goals

- Determine extent to which errors can be reduced by improving ICs
- Propose and test observation strategies to reduce prediction error
- Quantify spatial and temporal scales of error growth due to initial-condition (IC) error
- Quantify and characterize model inadequacy (residual)

Tools/resources

- Ensemble data assimilation
  - WRF
  - COAMPS
- Big computers
- New observations from 2012 MATERHORN field program

Framework

- Ensemble data assimilation with WRF and/or COAMPS:
  - Short-range intrinsic error growth
  - Systematic (but temporally and spatially varying) analysis increments
  - Ensemble sensitivity analysis (ESA)
- Clustering methods

Ensemble sensitivity analysis (ESA)

- Sensitivity scales (time and space) can be used to infer predictability in time and space
- Open issues:
  - Sampling error, especially for mesoscales
  - Linearity assumptions in complex terrain

ESA and proposed observations (1)

- ESA can be used to approximate the adjoint of the forward tangent linear model, and thus used to evaluate the linear impact of hypothetical observations.
- Relationship between deterministic and ensemble adjoint first investigated by Ancell and Hakim (2008) although the idea has been around longer.
- Mathematically, and within sampling error, the only difference is that the ensemble adjoint is *defined* using an approximation to the full covariance: the diagonal.
- The approximation is not necessary when formulated as a scalar problem.

ESA and proposed observations (2)

- Sensitivity of scalar forecast metric J to analysis $\mathbf{x}^{\mathbf{a}}$ is $\dfrac{\partial J}{\partial \mathbf{x}^{\mathbf{a}}}$, and can be estimated with an adjoint of the tangent linear model.

- Using sample statistics from an ensemble, the sensitivity can be estimated probabilistically, resulting in a linear regression problem:

$$\left(\frac{\partial J}{\partial \mathbf{x}^{\mathbf{a}}}\right)^{\mathbf{T}} = \left\langle \delta J \left(\delta \mathbf{x}^{\mathbf{a}}\right)^{\mathbf{T}} \right\rangle \left(\mathbf{P}^{\mathbf{a}}\right)^{-1}$$

$$\mathbf{P}^{\mathbf{a}} = \left\langle \delta \mathbf{x}^{\mathbf{a}} \left(\delta \mathbf{x}^{\mathbf{a}}\right)^{\mathbf{T}} \right\rangle$$

ESA and proposed observations (3)

- Approximation of $\mathbf{P}^{\mathbf{a}}$ with diagonal leads to the following definition for *ensemble* sensitivity:

$$\frac{\partial J_e}{\partial \mathbf{x}^{\mathbf{a}}} = \left(\mathbf{D}^{\mathbf{a}}\right)^{-1} \mathbf{P}^{\mathbf{a}} \frac{\partial J}{\partial \mathbf{x}^{\mathbf{a}}}$$

$$\left(\frac{\partial J_e}{\partial \mathbf{x}^{\mathbf{a}}}\right)^{\mathbf{T}} = \left\langle \delta J \left(\delta \mathbf{x}^{\mathbf{a}}\right)^{\mathbf{T}} \right\rangle \left(\mathbf{D}^{\mathbf{a}}\right)^{-1}$$

ESA is simply linear regression

$$\mathbf{D}^{\mathbf{a}} = \mathrm{diag}\left(\mathbf{P}^{\mathbf{a}}\right)$$

ESA and proposed observations (4)

- Approximation of $\mathbf{P}^{\mathbf{a}}$ with diagonal leads to the following definition for *ensemble* sensitivity:

$$\frac{\partial J_e}{\partial \mathbf{x}^{\mathbf{a}}} = \left(\mathbf{D}^{\mathbf{a}}\right)^{-1} \mathbf{P}^{\mathbf{a}} \frac{\partial J}{\partial \mathbf{x}^{\mathbf{a}}}$$

$$\left(\frac{\partial J_e}{\partial \mathbf{x}^{\mathbf{a}}}\right)^{\mathbf{T}} = \left\langle \delta J \left(\delta \mathbf{x}^{\mathbf{a}}\right)^{\mathbf{T}} \right\rangle \left(\mathbf{D}^{\mathbf{a}}\right)^{-1}$$

$$\mathbf{D}^{\mathbf{a}} = \mathrm{diag}\left(\mathbf{P}^{\mathbf{a}}\right)$$

ESA and proposed observations (5)

- Question: what change in forecast metric J can we expect from introducing (an additional) hypothetical observation y^o to an analysis?

- After a few lines of math:

$$\frac{\delta J}{\delta y^o} = \frac{\delta \mathbf{x}^{\mathbf{a}}}{\delta y^o}\frac{\partial J}{\partial \mathbf{x}^{\mathbf{a}}}$$

$$\delta J = \left(\mathbf{D}^{\mathbf{a}}\frac{\partial J_e}{\partial \mathbf{x}^{\mathbf{a}}}\right)^{\mathbf{T}} \mathbf{H}^{\mathbf{T}}\left(\mathbf{H}\mathbf{P}^{\mathbf{a}}\mathbf{H}^{\mathbf{T}} + \mathbf{R}\right)^{\mathbf{-1}}\left(y^o - \mathbf{H}\mathbf{x}^{\mathbf{a}}\right)$$

ESA and proposed observations (6)

- Rewriting as a scalar problem:

$$\frac{\partial J}{\partial x_i^a} = \frac{\partial J_e}{\partial x_i^a} = \frac{\mathrm{cov}(J, x_i^a)}{\mathrm{var}(x_i^a)} = \frac{\sigma_{Jx}^2}{\sigma_{xx}^2}$$

$$\delta J = \frac{\partial J}{\partial x_i^a} \frac{\sigma_{xy}^2}{\sigma_{yy}^2 + \sigma_o^2} \left(y^o - y^a \right)$$

$$\delta J = \frac{\sigma_{Jx}^2}{\sigma_{xx}^2} \frac{\sigma_{xy}^2}{\sigma_{yy}^2 + \sigma_o^2} \left(y^o - y^a \right)$$

$$\sigma_{yy}^2 = \mathrm{var}(y^a)$$

$$\sigma_{xy}^2 = \mathrm{cov}(x_i^a, y^a)$$

- y^o is hypothetical, or additional, observation not yet assimilated
- y^a is analyzed value of hypothetical observation before proposed assimilation
- ESA is simply linear regression

Sampling error in ESA and ensemble DA (1)

- Ensemble statistics used in both the assimilation and ESA (adjoint approx.)
- Sampling error in assimilation most often handled by *covariance localization*
- Sampling error in ESA not yet addressed

Sampling error in ESA and ensemble DA (2)

- In ensemble DA, covariances are typically weighted by a function of distance with $0 \le \rho \le 1$:

$$\delta J = \frac{\sigma_{Jx}^2}{\sigma_{xx}^2} \rho \frac{\sigma_{xy}^2}{\sigma_{yy}^2 + \sigma_o^2} \left(y^o - y^a \right)$$

$$\delta J = \rho \frac{\sigma_{xy}^2}{\sigma_{xx}^2} \frac{\sigma_{Jx}^2}{\sigma_{yy}^2 + \sigma_o^2} \left(y^o - y^a \right)$$

- It is clear that localization applies to the regression between analysis and observations

Sampling error in ESA and ensemble DA (3)

- Propose a similar weighting on the ESA regressions, with $0 \leq \alpha \leq 1$:

$$\delta J = \frac{\sigma_{Jx}^2}{\sigma_{xx}^2} \rho \frac{\sigma_{xy}^2}{\sigma_{yy}^2 + \sigma_o^2} \left(y^o - y^a \right)$$

$$\delta J = \alpha \frac{\sigma_{Jx}^2}{\sigma_{xx}^2} \rho \frac{\sigma_{xy}^2}{\sigma_{yy}^2 + \sigma_o^2} \left(y^o - y^a \right)$$

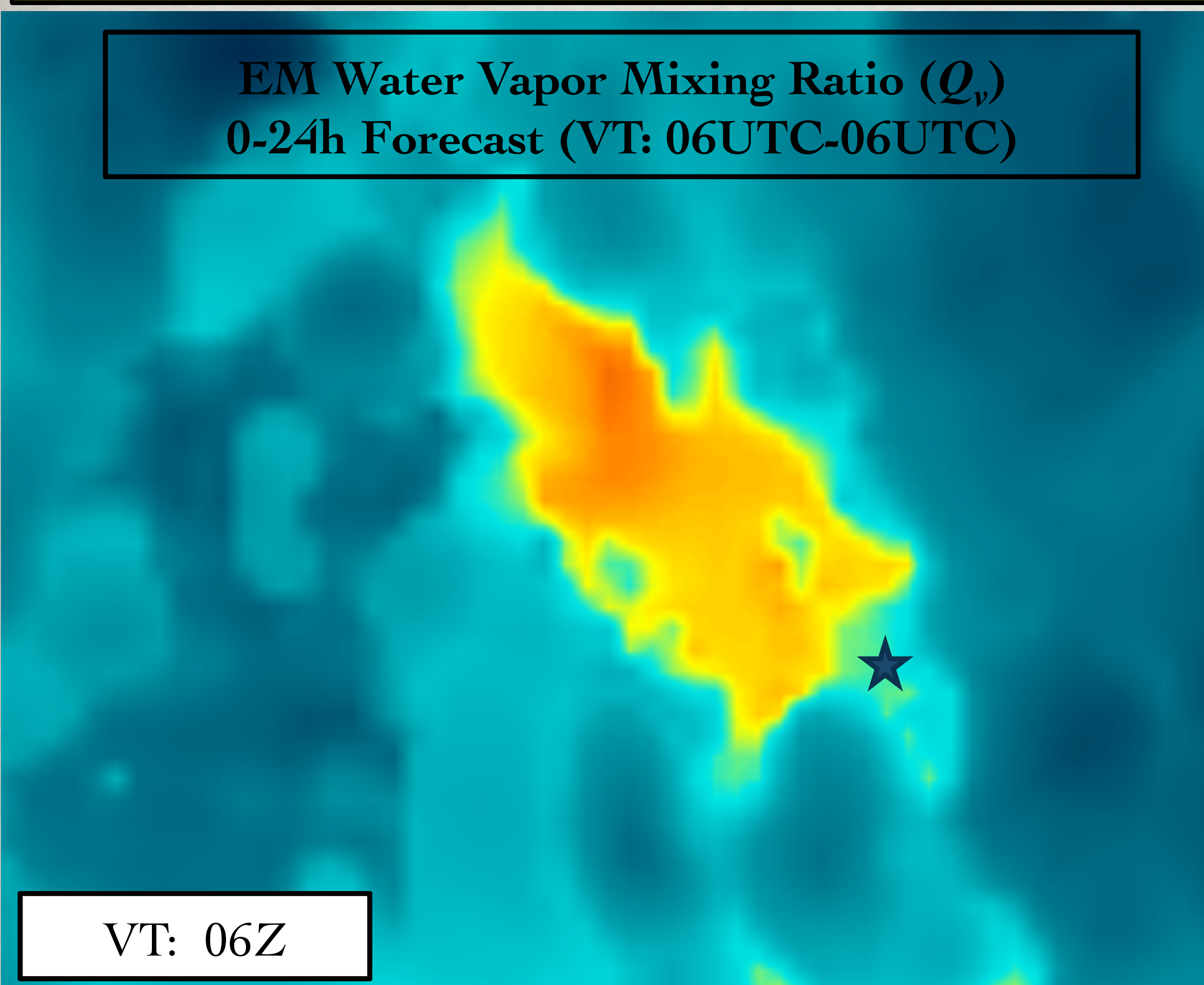
- J and x^a differ in both space and time
- J may not have a unique spatial location

Sampling error is present, but a spatial function is inappropriate for α because the regression is both in space and time (i.e. it includes linear approximation to dynamics).

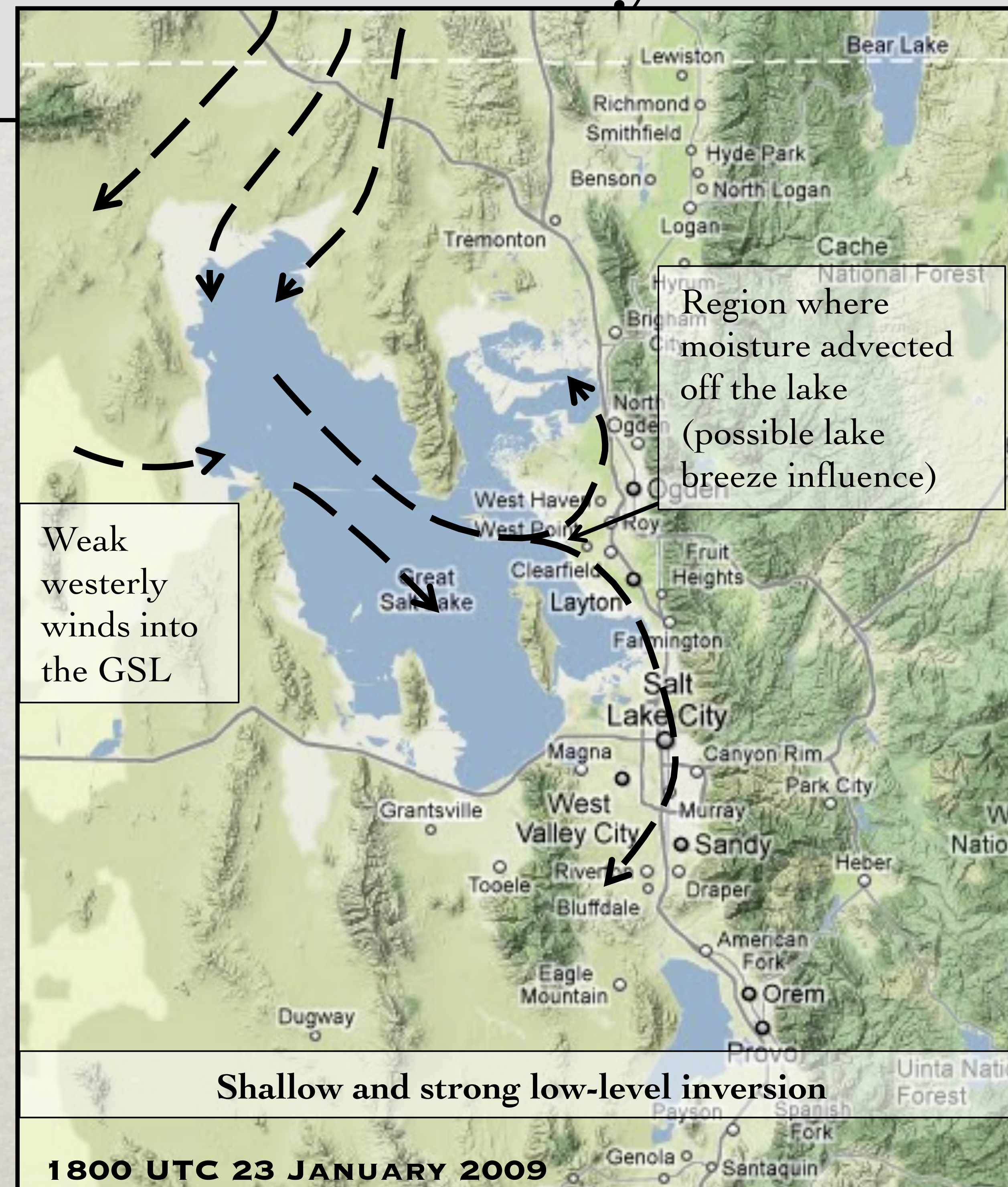
KSLC Fog Event

23 Jan 2009

- Visibilities 0-1/4SM after sunset
- Strong low-level inversion
- Light (<5kts) low-level winds (<130m)
    - Wind direction controlled by both synoptic and local effects/terrain
- Warm temps in the afternoon (49°F)
    - Possible Lake Breeze

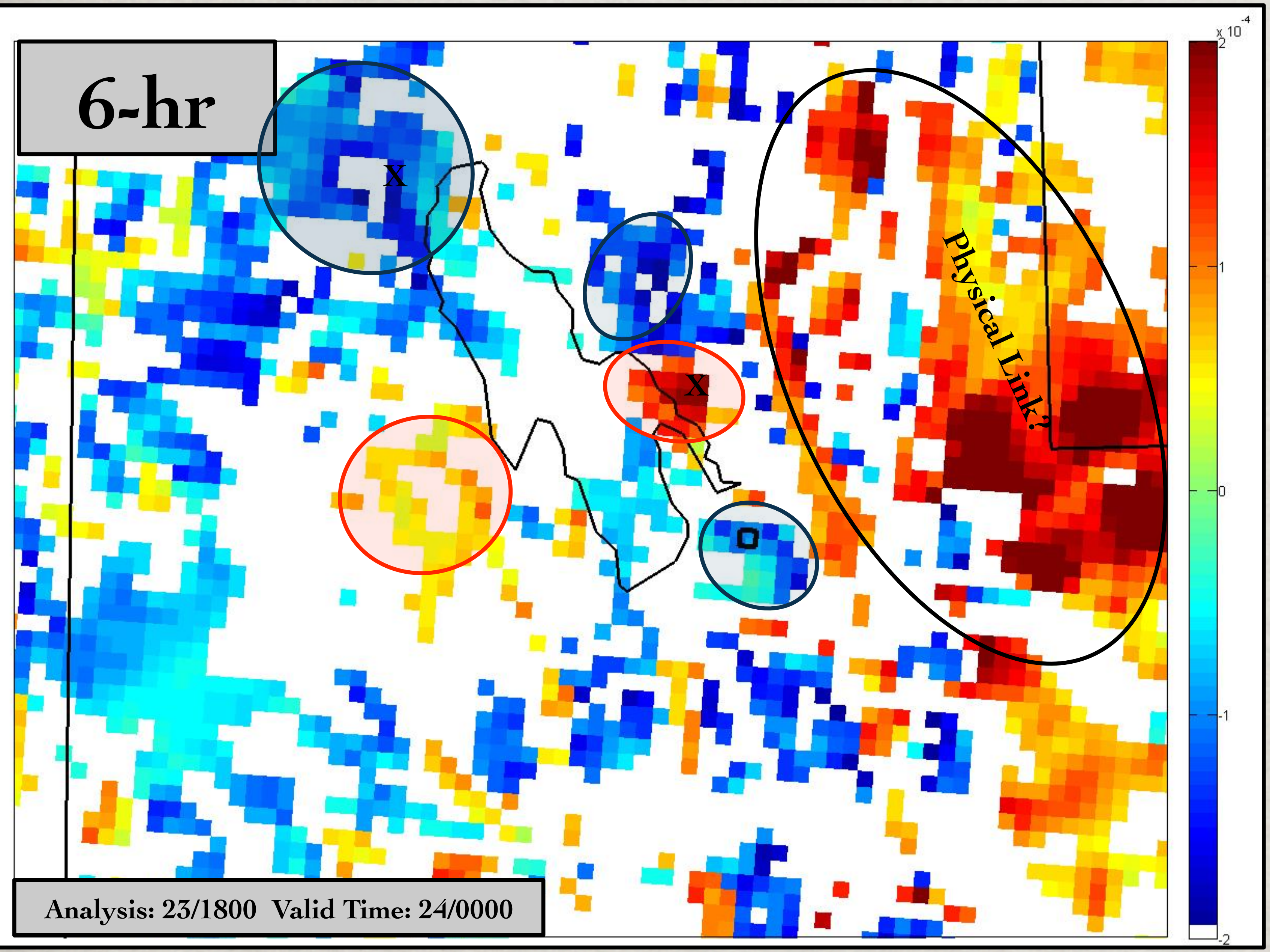


1800 UTC Analysis Low-



Sensitivity of Q_v to U winds

U-winds (x) and Q_v (J) in a 8-km x 8-km box over KSLC



Units: kg kg^{-1} m s-1

Sensitivity

Region 1:
±0.09 m s^{-1} ($1\sigma_u$) at X = ±2.2x10^{-5} kg kg^{-1} change in $\boldsymbol{Q_v}$ 6 hours later at the airport

Region 2:
±0.07 m s^{-1} ($1\sigma_u$) at X = ±1.6x10^{-5} kg kg^{-1} change in $\boldsymbol{Q_v}$ 6 hours later at the airport

- Tests to region 1

Linearity tests simulating assimilation of a perfect observation

- Choose a perturbation point – typically located at a sensitivity maximum
- Perturb ensemble mean with $\pm 1\sigma_x$, $2\sigma_x$, $3\sigma_x$
- Use analysis ensemble to regress perturbation onto model state (ensemble mean)
- Run perturbed ensemble; look for:
  - Agreement between predicted forecast δJ and actual
  - Range of perturbations that give forecast δJ lying on the same line

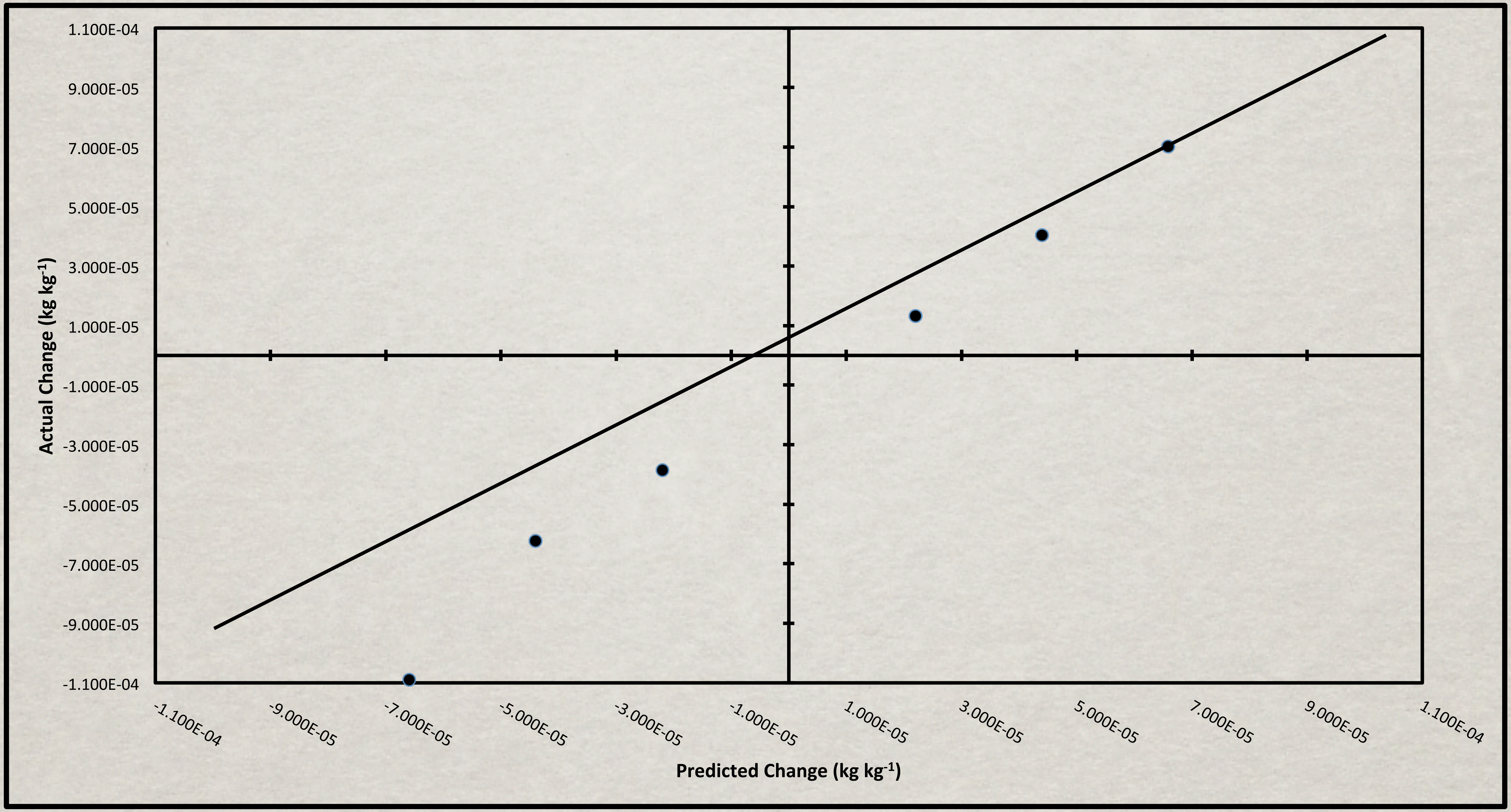
Sampling error in linearity tests

- Sensitivity estimates
- Regression to create perturbed ICs
- Possible treatments and outcomes, assuming linearity:

Treatment	Effect
Localize IC perturbations to account for sampling error in analysis covariances	Reduced IC perturbations and actual δJ response
Account for sampling error in sensitivity estimates	Reduces predicted δJ
No accounting for sampling error	Systematic over/under prediction results from more/less sampling error in

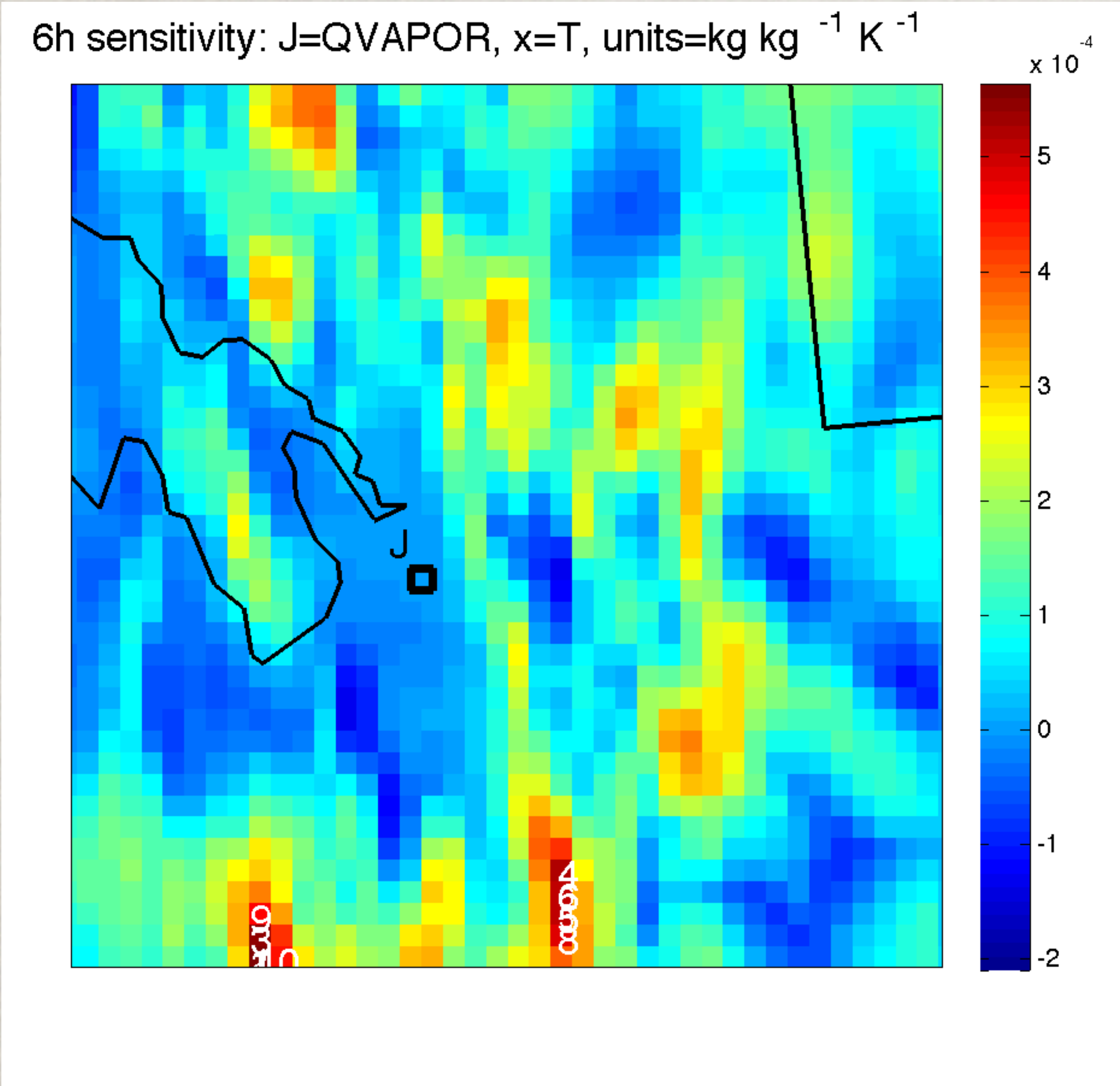
Linearity Test

1800 UTC Analysis and 6-hr Forecast



- Q_v responses to U-wind perturbation are mostly linear, but weak nonlinearity evident at $\pm 3\sigma_x$.
- Actual response greater magnitude that predicted response → sampling error in assimilating perfect observation, without localization, produces systematically large initial perturbation

Same case, different sensitivity

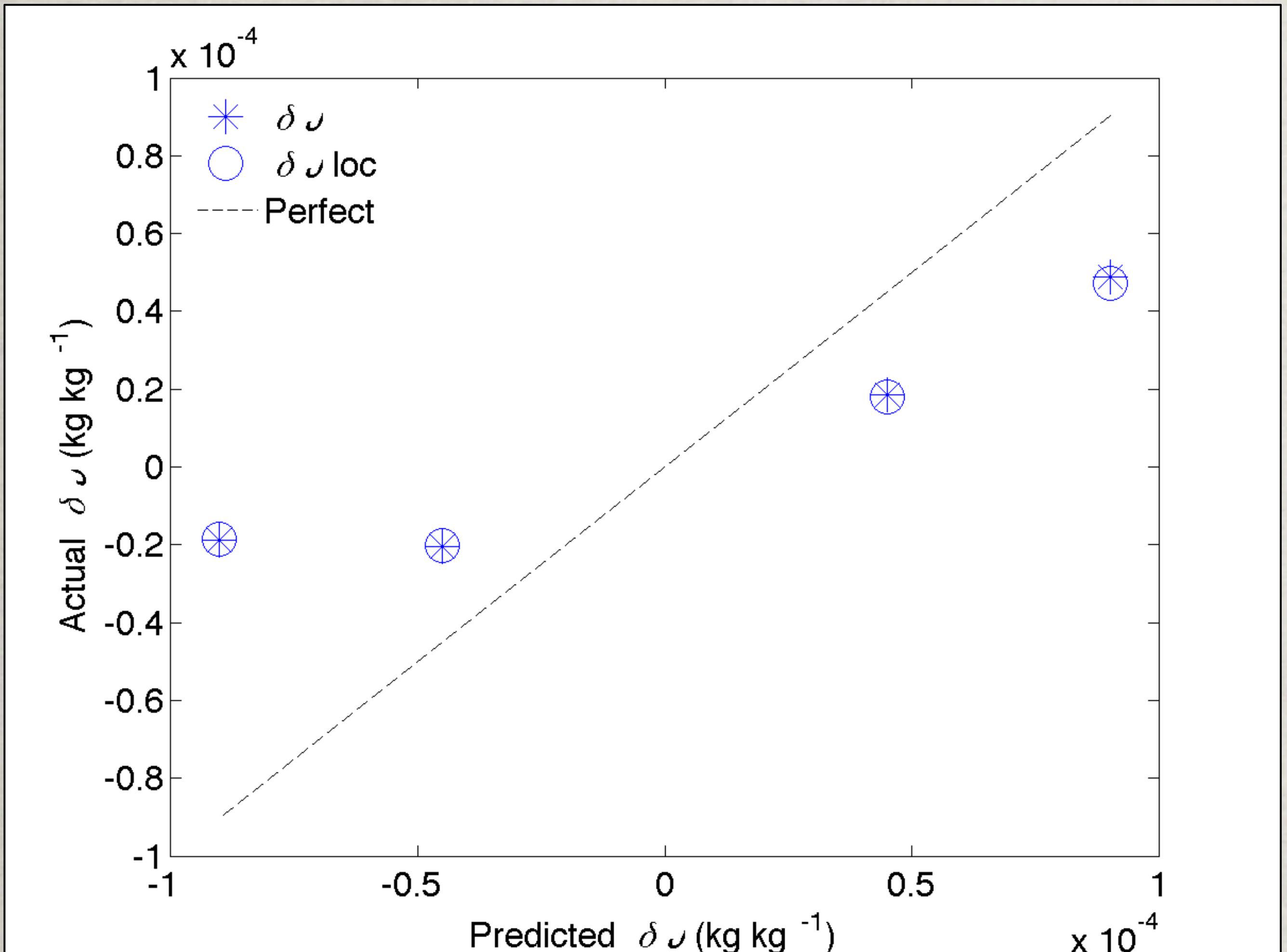


- More realistic observation network; updated version of WRF
- Qualitatively similar sensitivity to *U* winds as seen in earlier experiments
- Examine sensitivity to temperature *T*

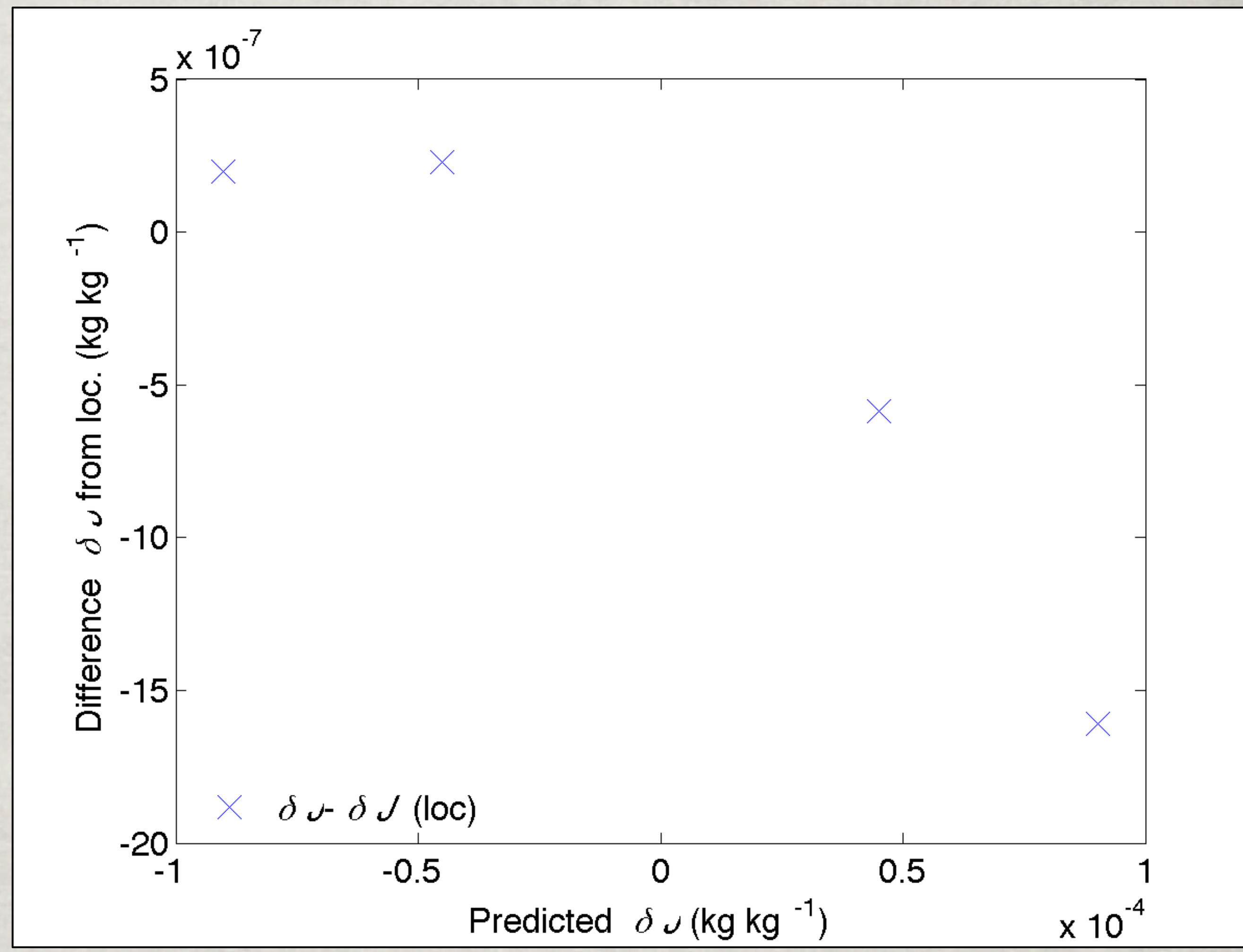
Linearity Test

With and without horizontal localization on inner-most domain

Responses

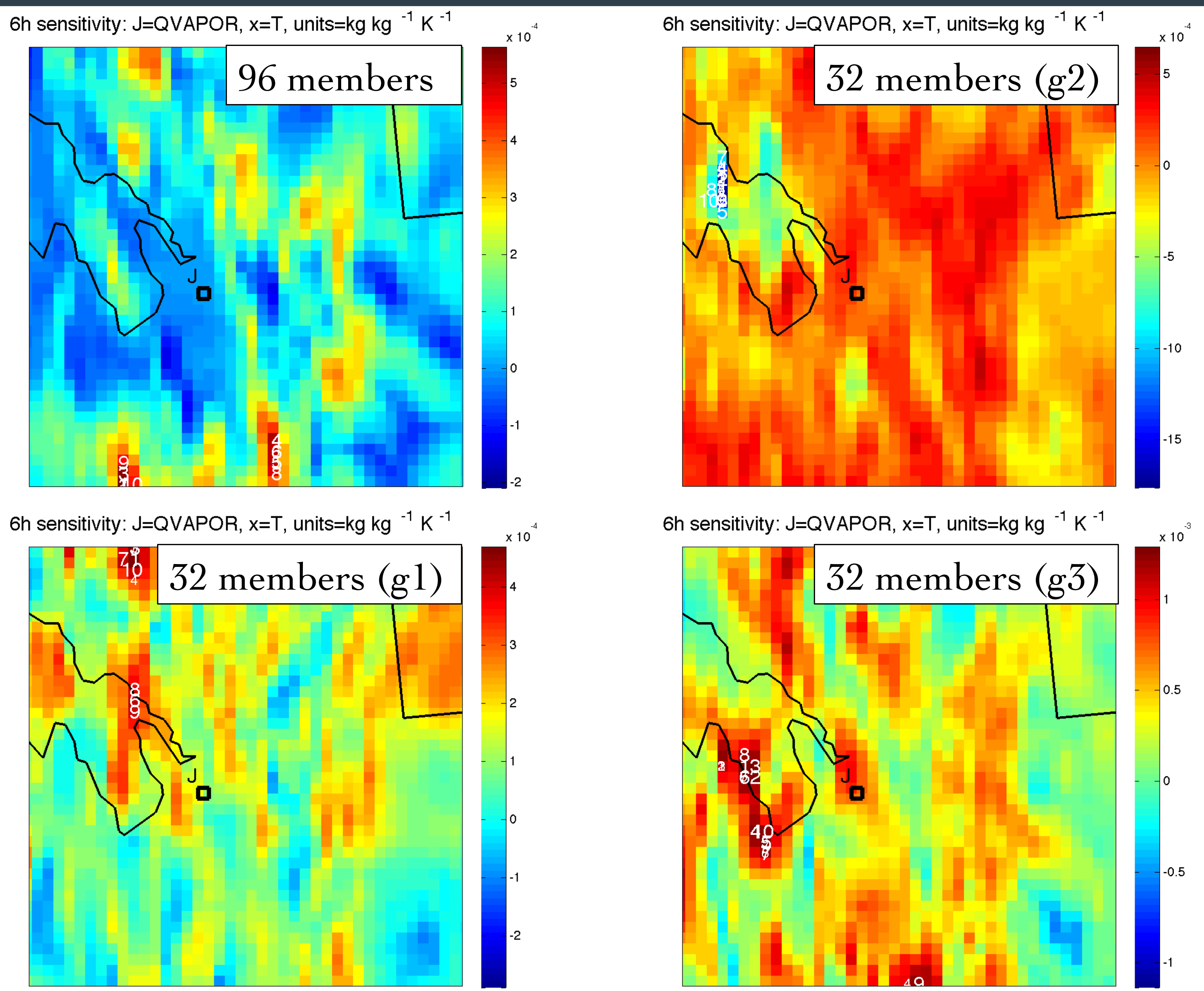


Responses with – without localization



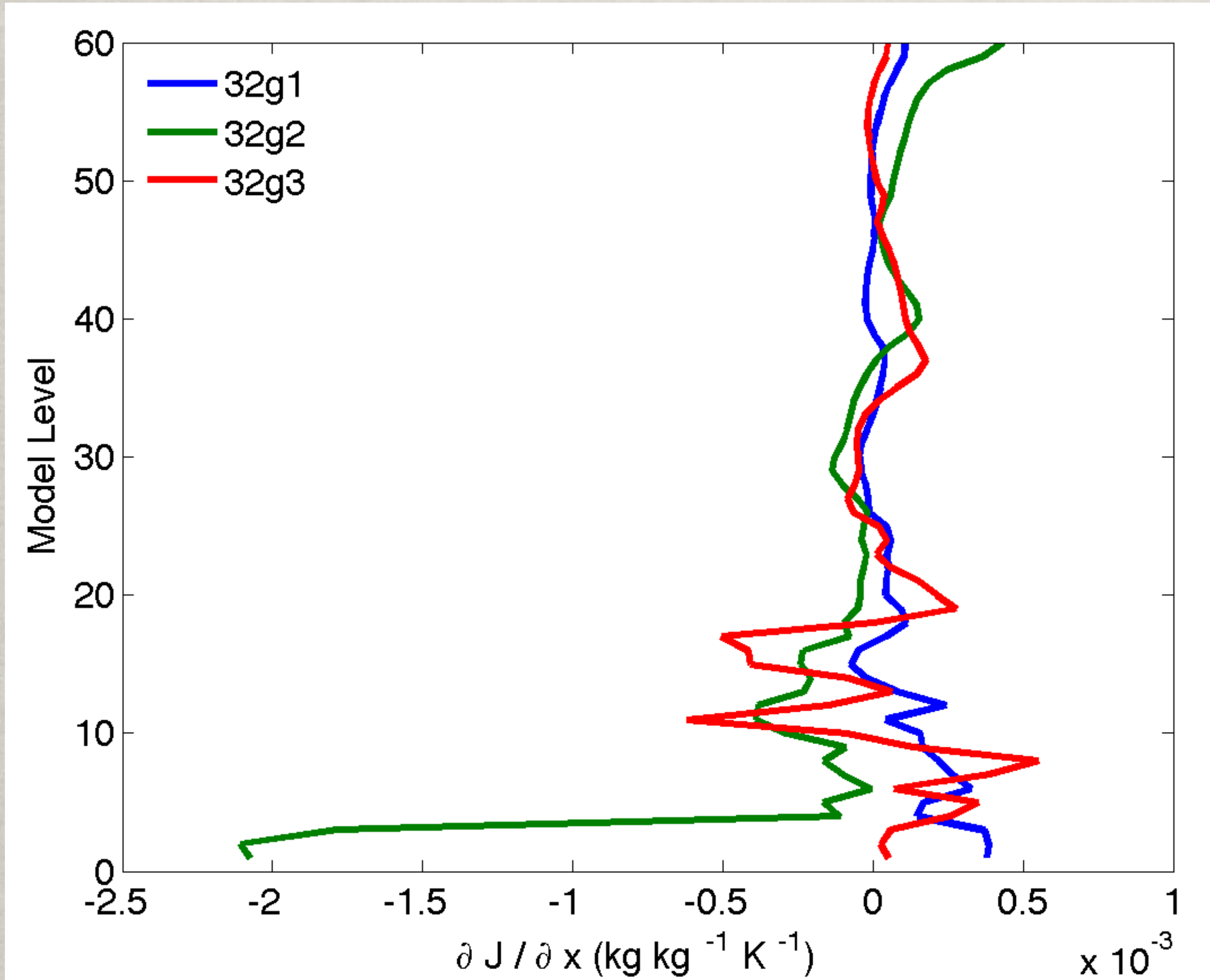
- Q_v prediction response to T perturbation weakly nonlinear
- Systematic over-prediction of response magnitude → sampling error in estimating sensitivity produces systematically large δJ predictions
- Effect of localization as expected – actual response magnitude smaller
- Expect greater effect with localization in vertical and on more domains

Ex: sampling error in $\partial J / \partial x$



All 6-h forecast valid at 00Z 24 Jan 2009.

Sensitivity Profiles

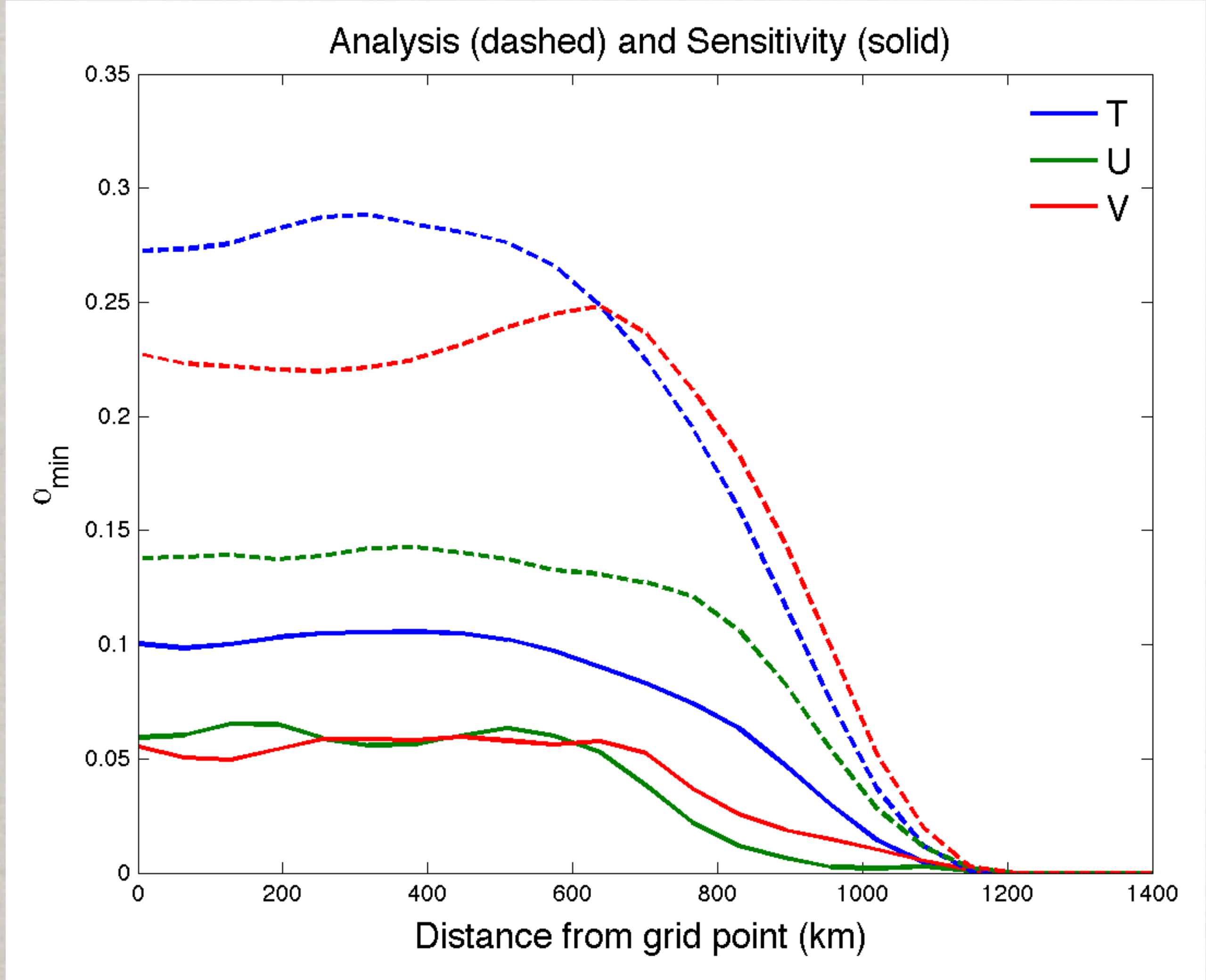


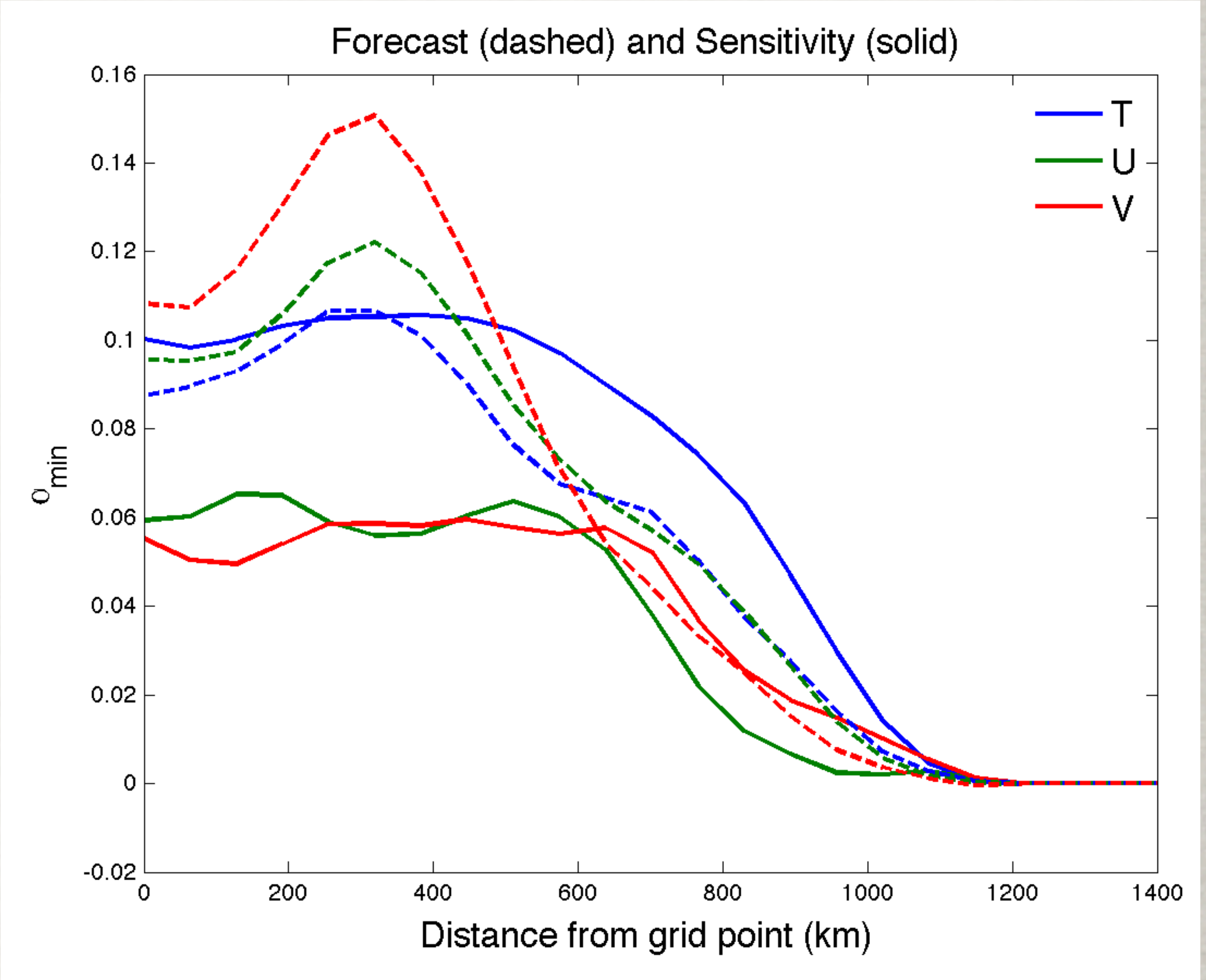
- Sensitivities confined to lowest 2 model layers
- Broad disagreement between groups
- Suggests large sampling error in sensitivities

Sampling error in $\partial J / \partial x$

- Estimate the sampling error directly from multiple ensemble predictions
- Rather than 1 ensemble of 96 members, use 3 ensemble of 36 members (cycled independently)
  - CAVEAT: this estimates sampling error in 32-member ensemble, not the 96-member ensemble
- Follows Anderson (2006, *Physica D*), who proposed estimating error in regressions from a sample of regression coefficients

Horizontal weights on T-Q_v regressions, 32-member groups





- Analysis regressions appear to be estimated more accurately
- Accounting for both, using these weights, would decrease predicted δJ relative to actual δJ (i.e. a correction)

Summary

- ESA appears promising
- Sampling error enters into both ESA calculations and linearity tests
- Mathematical treatment of sampling error in ESA and predicted observation impact is straightforward, if sampling error known
- Severity of sampling error is variable
- Current: attempt to account for sampling error in different parts of process

Personnel

- Hacker
- Students:
  - Capt. Hank Chilcoat (M.S. 2011)
  - Maj. Paul Homan (Ph.D. Candidate)
  - Capt. Sean Wile (M.S. Student)
- Post-doc:
  - Dr. Jared Lee (arrives Oct 2012)
- Collaborators
  - Dr. Lili Lei (ASP post-doc)
  - Dr. Dorita Rostkier-Edelstein (Israel Institute for Biological Research)

Contributions from collaborators

A look at systematic model error

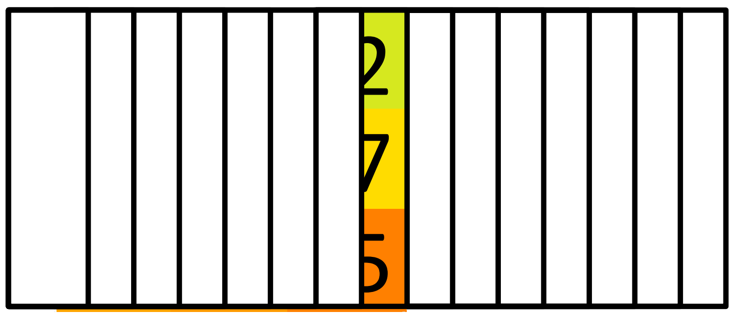
- Analysis increments are opposite both random and systematic error

- Systematic components of analysis increments provide a “map” of systematic model error (scaled by ratio of analysis to observation error variances)

- Looking for ways to characterize time and space scales of systematic model errors: Self-Organizing Maps (SOMS)

Self-Organizing Maps

Data (forecasts, analyses, etc.)

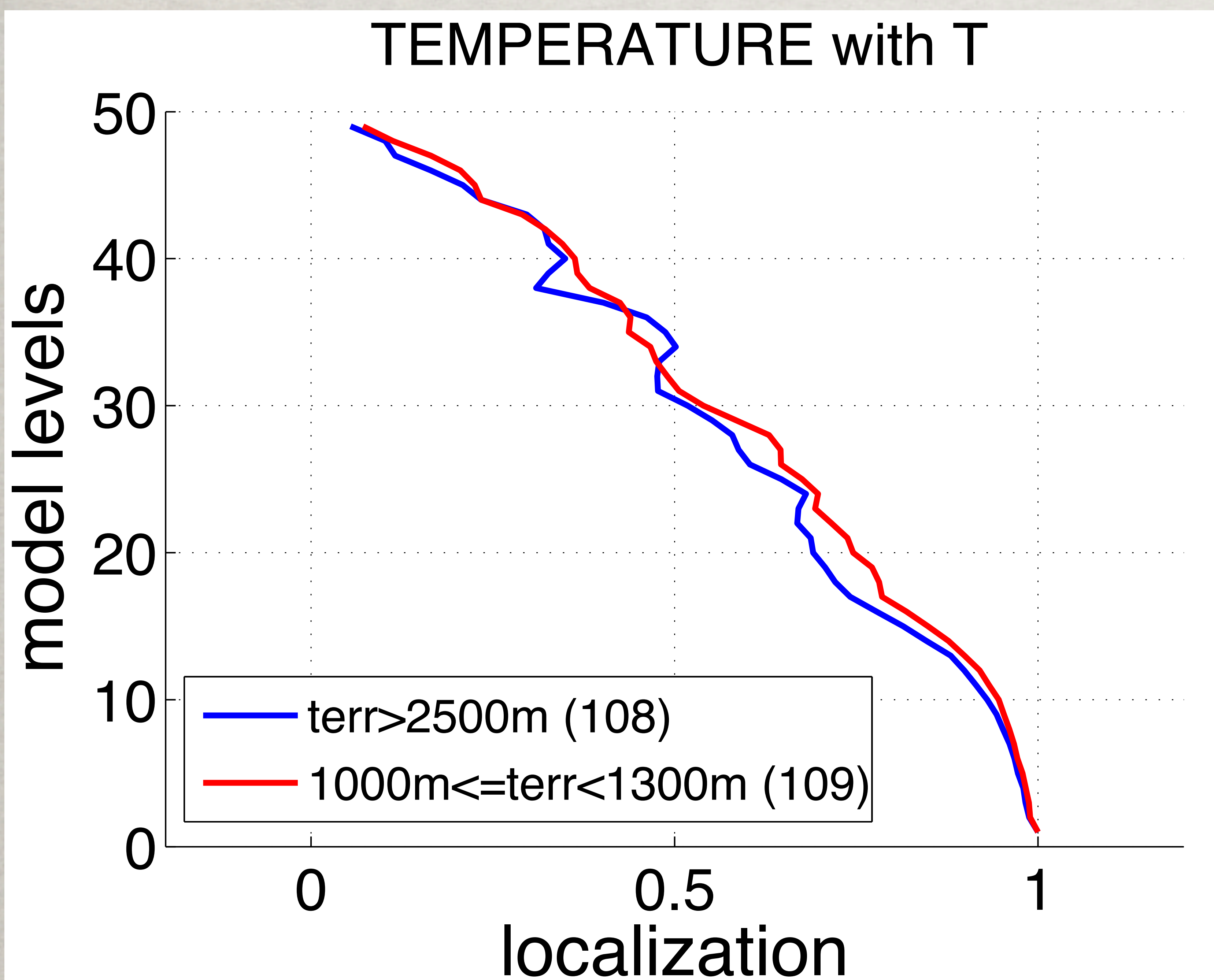


- Nodes resemble plausible data states
- Nearby nodes are similar to each other
- Data state can include multiple variables at once
- Useful for identifying spatial patterns in complex data
- Commonly thought of as a non-linear analogue of Principal Component Analysis

1. Create array of nodesand initialize
2. Select single state from data at random
3. Identify the node closest to selected state
4. Update that node and nearby nodes to look more like selected state
   - Smaller changes farther away from identified node
5. Repeat, making smaller updates each iteration

32	22	11	30	27	22	24	28	30	20	37	44
44	35	27	42	36	33	34	41	52	45	50	60
65	60	55	63	62	60	59	64	69	57	67	70
45	42	32	45	45	45	39	48	51	38	44	50
64	55	48	60	61	60	57	63	70	55	65	72
77	68	62	76	78	77	72	74	79	72	79	86
71	65	60	70	66	63	60	67	69	60	61	65
83	77	70	81	77	69	70	72	77	67	72	80
99	90	84	93	94	90	87	90	92	82	87	95

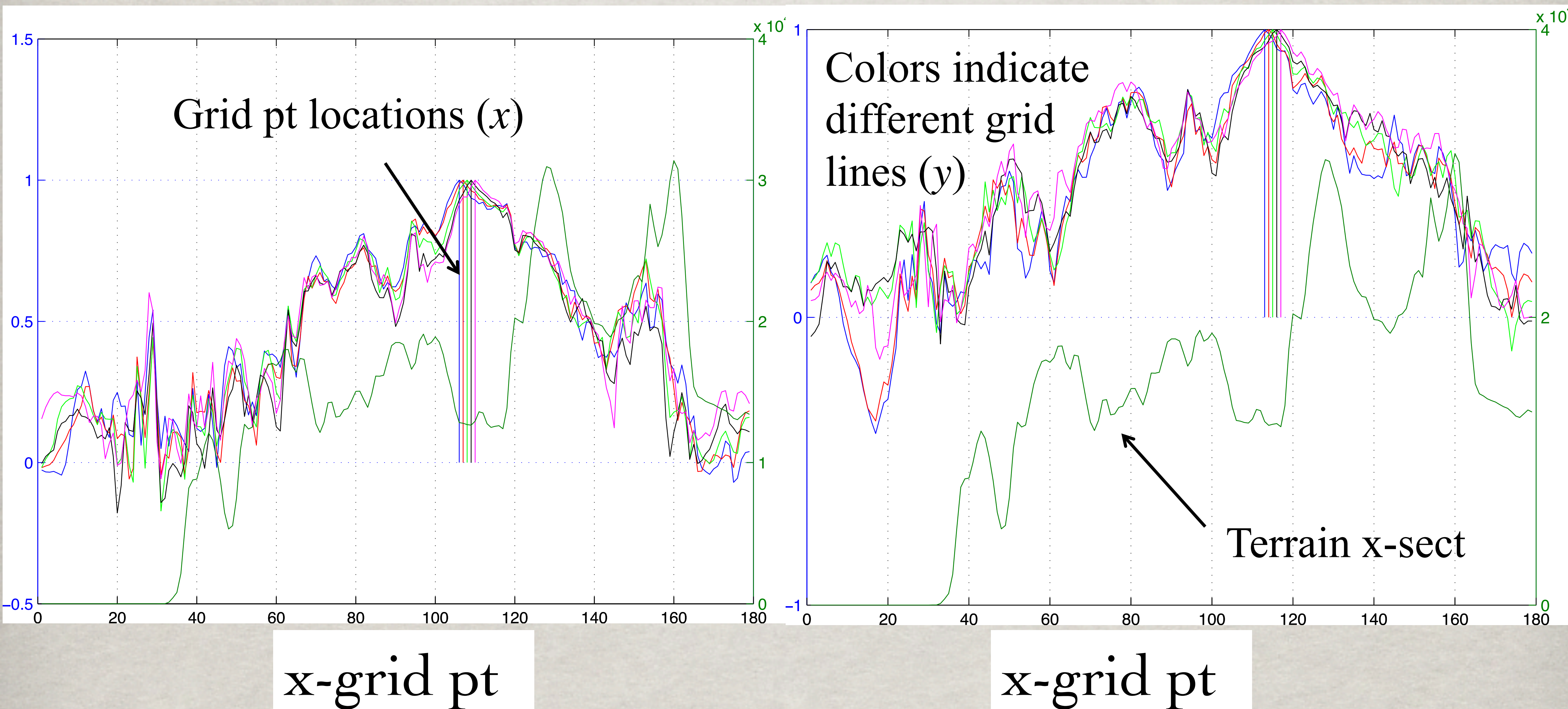
DA regression weights: effect of terrain



- Shallower structures over terrain
- Would result in smaller profile increment from near-surface observation

Image from Lili Lei

DA regression weights varying by spatial location



- *T-T* regression weights across grid on lowest model layer
- Regression weights anti-correlated with terrain
- Regression weights anisotropic

Image from Lili Lei

Near-surface bias in observations and models

- A fundamental assumption of standard Kalman filter methods is that observations and forecasts are unbiased.
- In reality, and in particular in the surface and boundary layers:
  - Bias in observations result from:
    - Instruments inaccuracy
    - Representativeness errors
    - Inaccuracy of forward operators
  - Forecast biases result from:
    - Model structure, parameterization and discretization
    - Bias in initial conditions.

Rationale, contd.

- Near the surface, the fine vertical variability due to complex terrain and canopy extend beyond the first or even first few grid layers.
- This is not accounted for by forward operators used to assimilate surface observations; these usually rely on similarity theories which assume constant-flux layer, horizontal homogeneity, and local equilibrium.
- Model-grid-to-observation-site vertical and horizontal interpolation steps usually disregard the difference between model and true elevation which becomes more acute in areas of complex terrain.
- Covariances used to regress surface increments onto the column aloft may result from biased PBL parameterizations.

Slide from Dorita Rostkier-Edelstein

Example: assimilation of surface observations with an SCM and ensemble filter

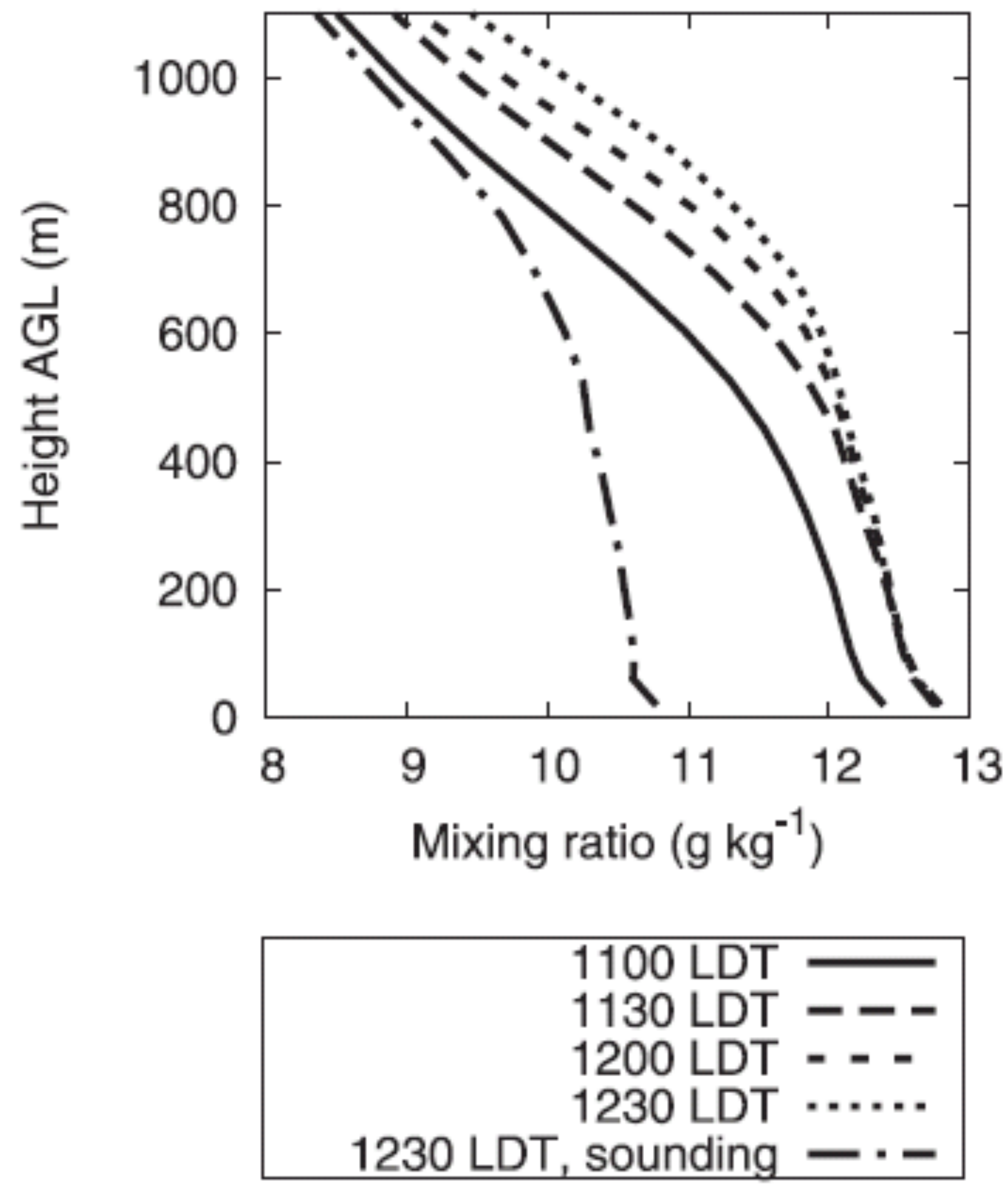


FIG. 16. The 30-min Q_v-forecast profiles and the observed profile during our daytime experiments for the assimilation-only model configuration.

Rostkier-Edelstein and Hacker, WAF 2010

Research plan

1. Investigate the sources and the diurnal variability of bias at the surface and in the PBL profile through "perfect model" simulations with an SCM and EnKF assimilation of surface observations. These will enable a separation analysis of the factors responsible for bias.

2. Investigate the correlation between surface and PBL bias to model physical quantities along the diurnal cycle.

3. Run real model and real observations numerical experiments.

4. Use insight gained in 1, 2 and 3 to develop a bias correction algorithm to be run on-line with the EnKF. Algorithm would be based on simple bias model (e.g. persistence) and/or statistical predictors.

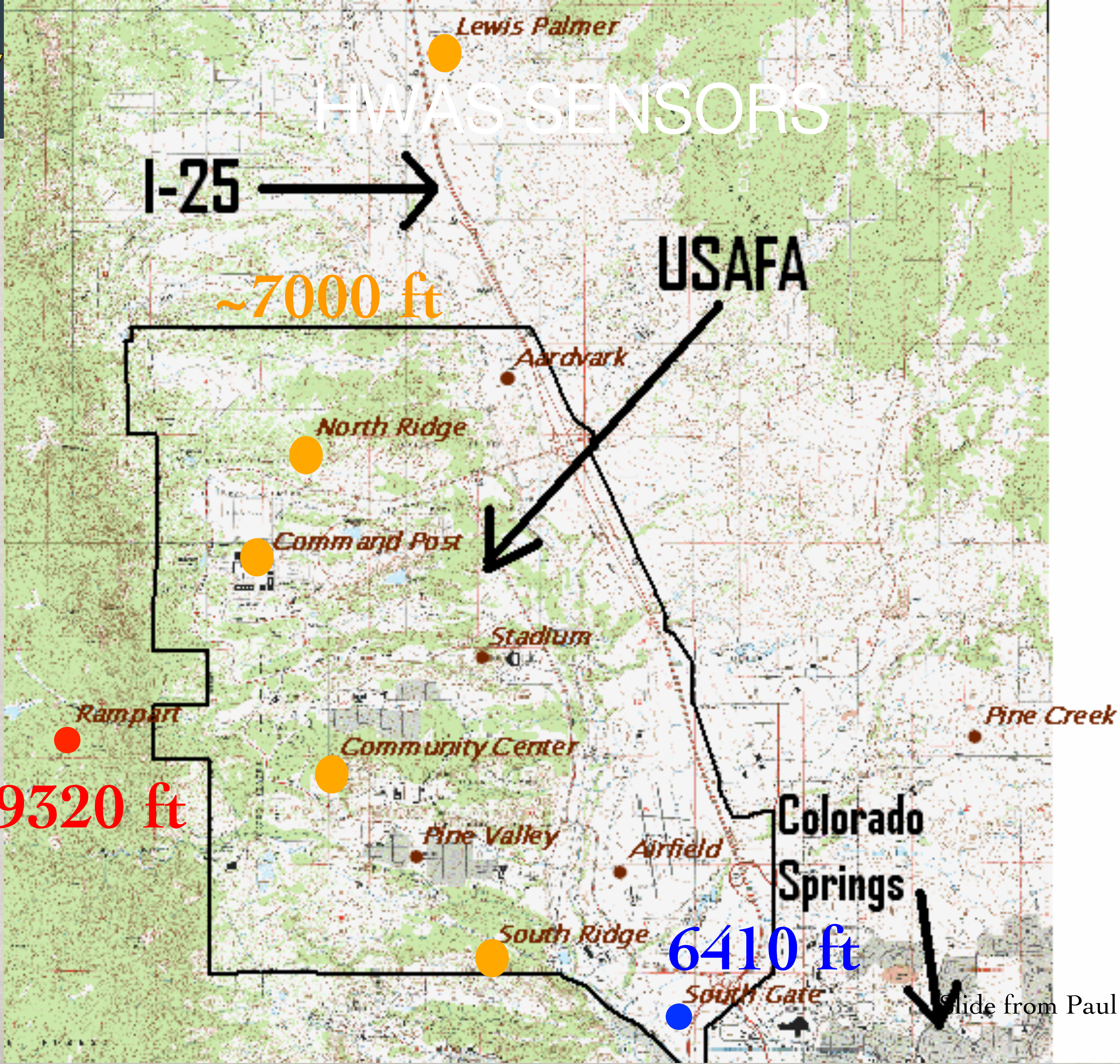
Slide from Dorita Rostkier-Edelstein

Downslope winds near CO Springs

- Dataset – High Wind Alert System (HWAS) 12
  – 2 min resolution back to 2004 at US Air Force Academy
- 110 Severe Downslope Windstorms (DWS) recorded
- Location – 3-D terrain lacks classical two-dimensional profile most studies have focused on for DWS
- Analysis Tool - Ensemble Sensitivity Analysis (ESA)
  - Case Study: 30 Dec 2008 Severe Downslope Storm
    max gust 79.1kts (91 mph) (>50kts at all 12 sensors)
  - "Perfect" model (OSSE) experiment w/ 90 members
  - Is ESA is a useful analysis tool to determine sensitivity of DWS to IC's and Observations?
  - Determine joint sensitivity between synoptic forcing and local preconditioning

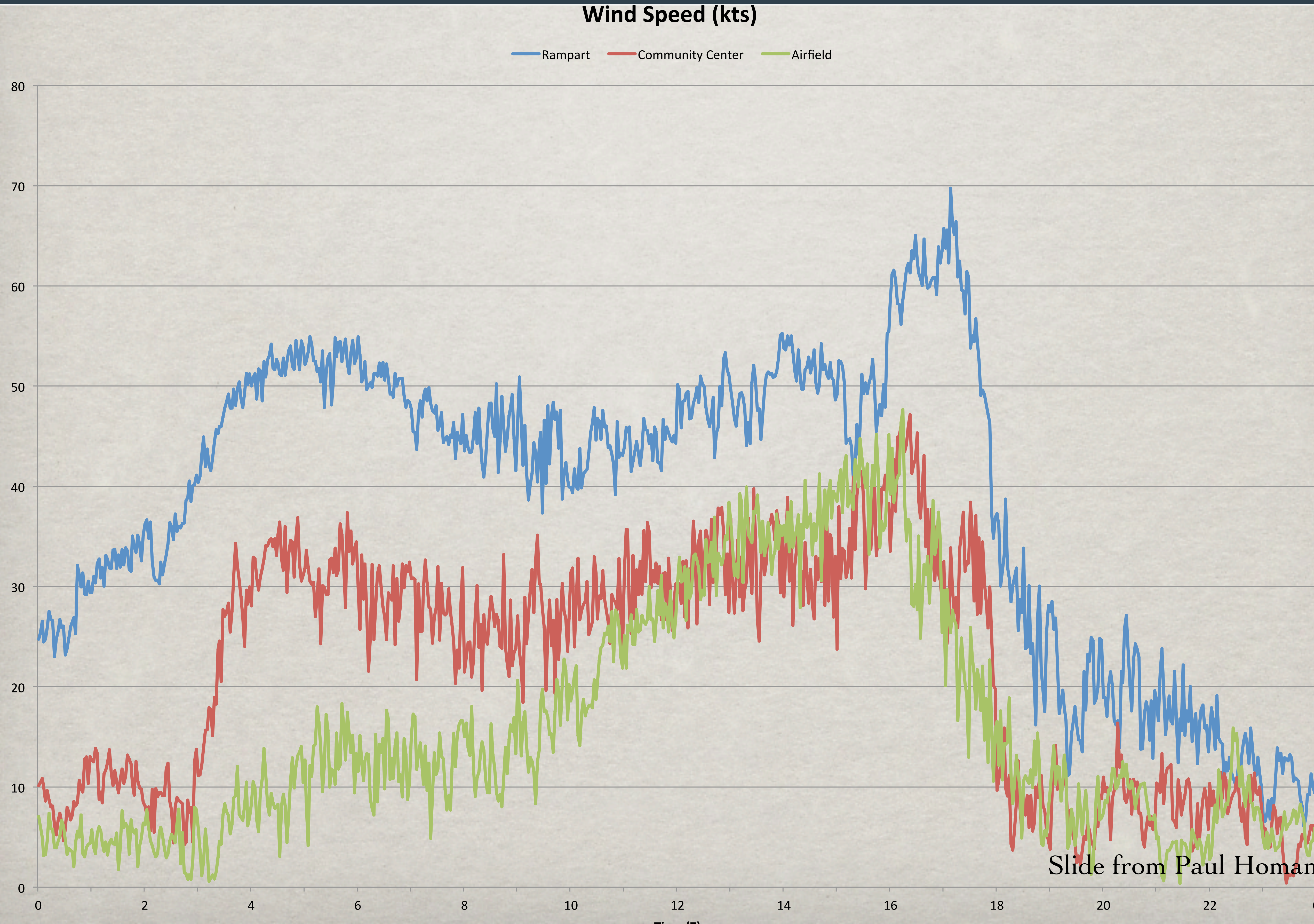
Slide from Paul Homan

HWAS SENSORS



Slide from Paul Homan

30 Dec 2008 – Observed Winds



Forecast Metric – Possible J's

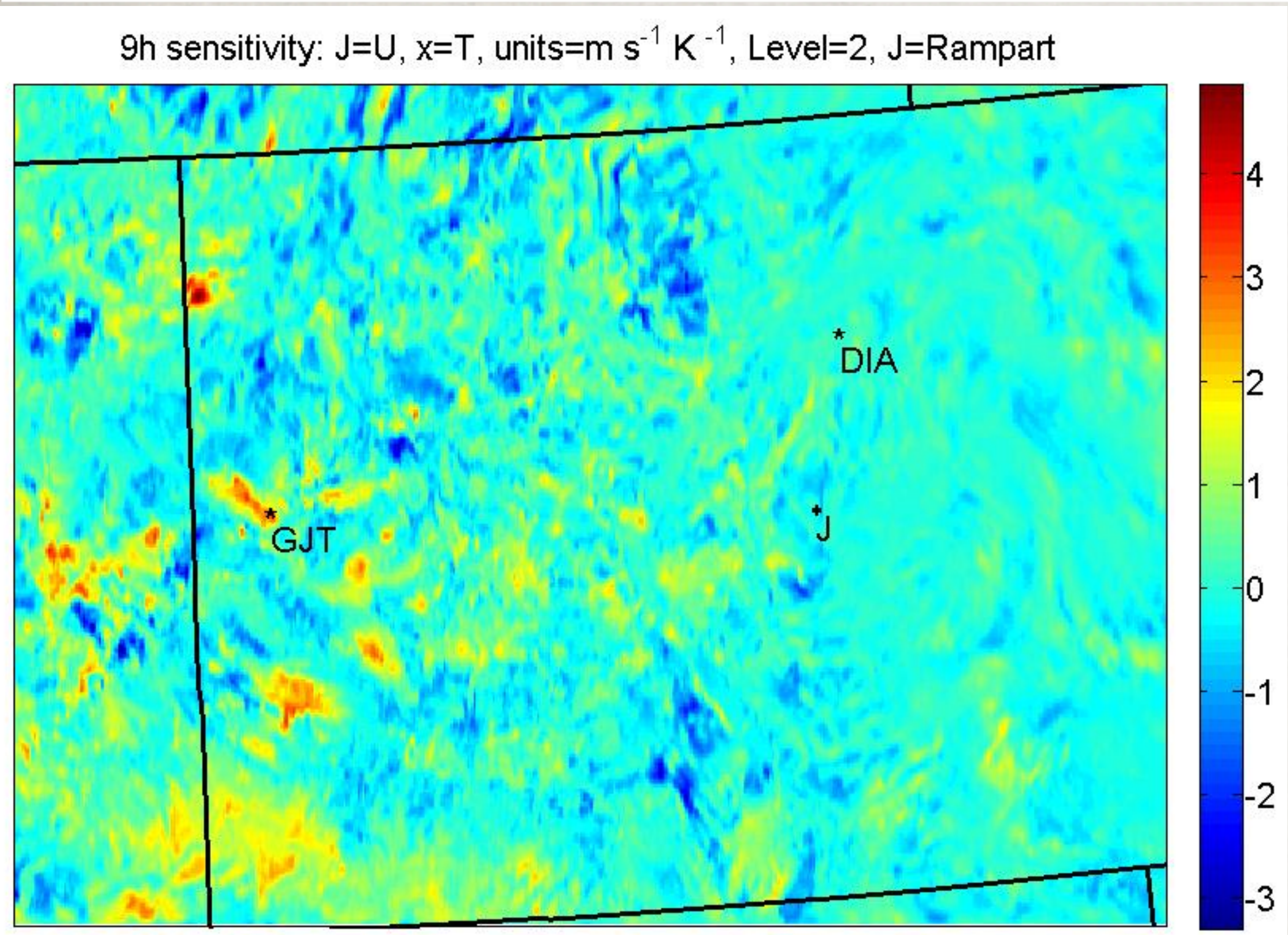
- What are we trying to forecast?

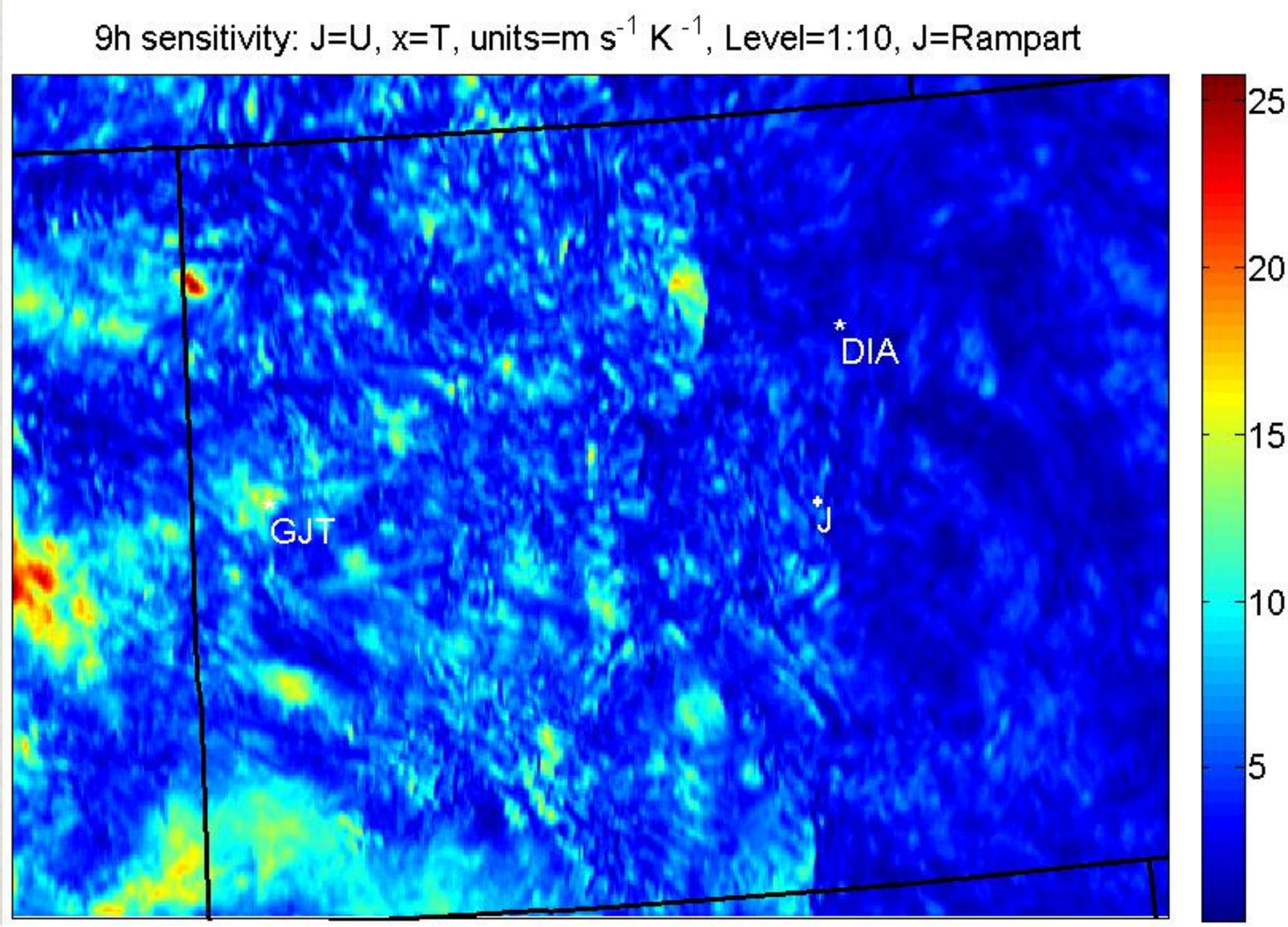
- Wind Speed - During event mostly captured by u.

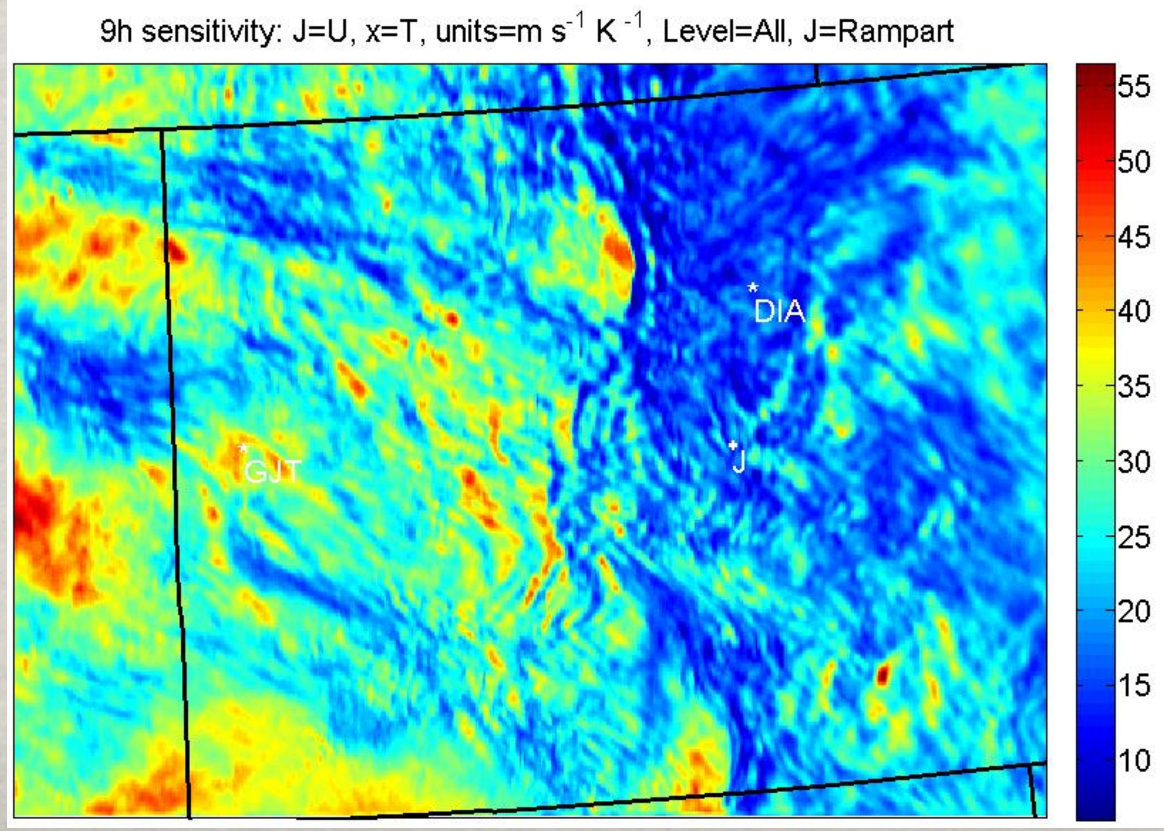
- SFC Pressure – Significant Drop during DWS (> 5 mb in 2 hrs)

Slide from Paul Homan

Synoptic Sensitivity $J = U, x = T$







Slide from Paul Homan