Lab 4: Buckling

AME 21241 – Mechanics of Solids Lab

Introduction

In many engineering applications relatively thin or slender members are subjected to compressive loading. In these cases, it is important to consider the possibility of buckling. The simplest structural element that suffers buckling is the column. A column is basically a rod that is loaded in axial compression. We may expect that the deformation of the column would be axial shortening only. Axial compression, however, tends to also cause lateral bending of the column and, should it become excessive, result in collapse.

The following sections describe a couple of buckling examples. The first is a very simplified model of a column. It is useful to understand what happens after buckling and what the effect of yielding is. It is called a “discrete system” because it has only one degree of freedom. The second is a continuous beam model of the column and yields the value for the load at which buckling deflections can become significant. It can also give some information regarding the post-buckling behavior, but that is beyond the scope of this laboratory.

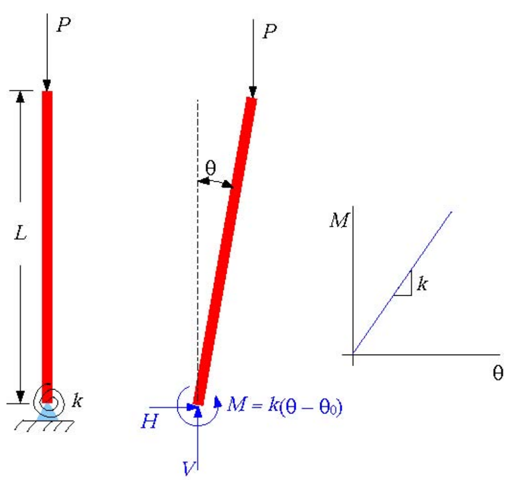

Figure 1: Discrete, perfect system

A Discrete System

Many of the main aspects regarding buckling of columns can be illustrated by considering *discrete systems* such as the one shown in Figure 1. The system consists of a rigid bar of length *L* which is attached to a pin connection and a torsional spring at its base. A vertical load *P* acts at the top of the bar.

Perfect, linear spring system

We will consider the perfect system shown in Figure 1. By ‘perfect’ we mean that the bar is initially vertical or the angle θ is initially zero. By ‘linear spring’ we mean that the relationship between moment and rotation of the torsional spring is linear as shown in Figure 1. Intuitively, we can imagine that if the load *P* is small, the bar will remain perfectly vertical but above a certain value the system will buckle, and the angle θ will be non-zero. Our first task will be to find what is the maximum load we can apply and still maintain θ = 0. This is the buckling load.

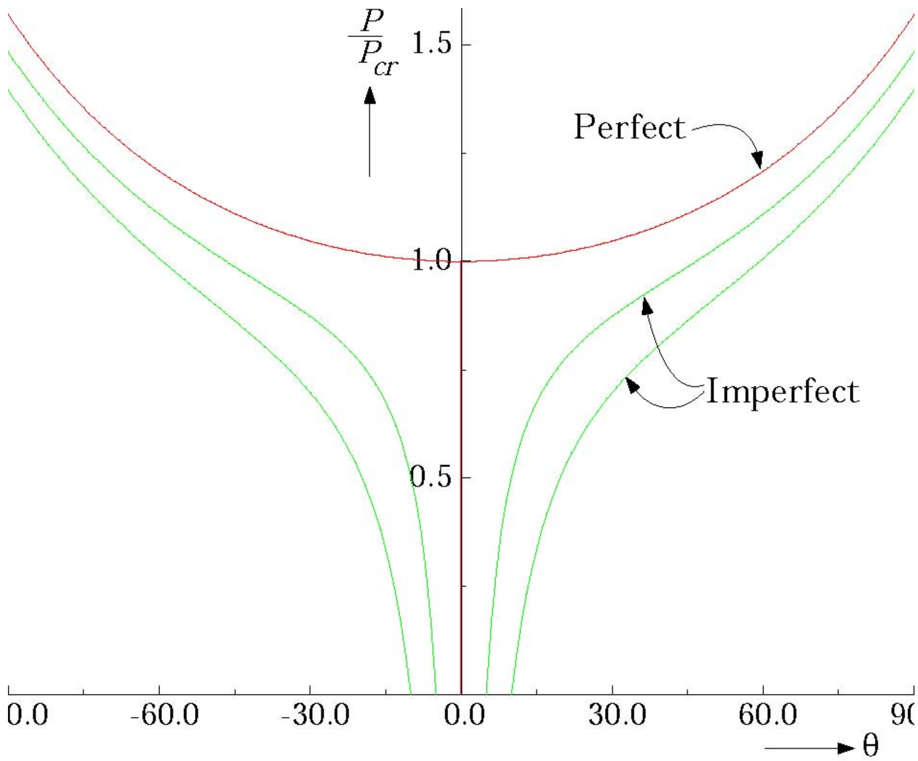

Figure 2: Response of perfect and imperfect systems

Considering the free-body diagram[1] shown and applying equilibrium, we easily find that $H = 0$, $V = P$ and $M = P\,L \sin\,\theta$. Since we are looking for any possible small deviation from the vertical configuration we can approximate $\sin\theta \approx \theta$. Substituting $M = k\theta$ into the moment equilibrium equation we obtain:

$$(k - PL)\theta_0 = 0 \tag{1}$$

This equation, (1), can be satisfied in two ways:

1. $\theta_0 = 0$, P arbitrary. This corresponds to the vertical position of the bar.
2. $\theta \neq 0$ This corresponds to the buckled configuration.

In order to satisfy (1) we need $k - PL = 0$. This yields the critical, or buckling load of the system

$$P_{cr} = \frac{k}{L} \tag{2}$$

[1] Note that the free-body diagram is drawn for the system in the buckled configuration.

So, as long as $P < P_{cr}$, the bar remains vertical. At $P = P_{cr}$ the system buckles and θ becomes non-zero. The next question is, what happens after buckling? Or, what is the relation between P and θ? In this case, we must keep the sin θ term in the equilibrium equations so we obtain:

$$k\theta - PL \sin \theta = 0$$

or

$$P = P_{cr} \frac{\theta}{\sin \theta} \tag{3}$$

The plot of P vs. θ in the range $-90^\circ \leq \theta \leq 90^\circ$ defined by Eq. (3) is labeled 'Perfect' in Figure 2. It is immediately obvious that an increasing load P is required to increase θ, so we call this a *stable response*.

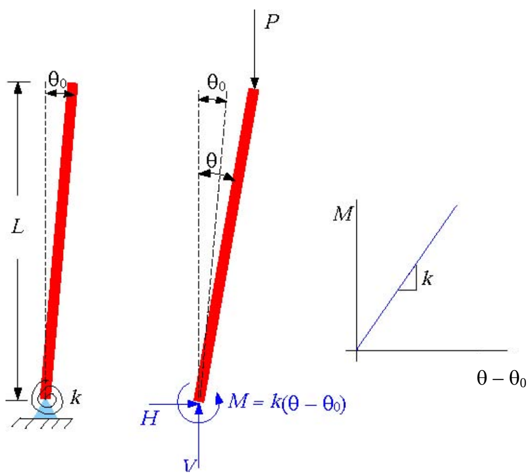

Figure 3: Imperfect system

Imperfect, linear spring system

We now consider the system shown in Figure 3. It is similar to the one we just considered but has an initial deflection θ_0 when it is unloaded. The moment equilibrium equation, obtained by considering the free-body diagram shown is still

$$M - PL \sin \theta = 0$$

but now the response of the spring is given by $M = k(\theta - \theta_0)$. Substituting,

$$k(\theta - \theta_0) - PL \sin \theta = 0$$

or

$$P = P_{cr} \frac{\theta - \theta_0}{\sin \theta} \tag{4}$$

The response predicted by Eq. (4) is also shown in Figure 2 for $\theta_0 = \pm 5^{o}$ and $\pm 10^{o}$. Note that θ starts increasing as soon as the load P starts increasing and that the predicted responses are stable with P increasing with θ. Also, Eq. (3) can be recovered by setting $\theta_0 = 0$.

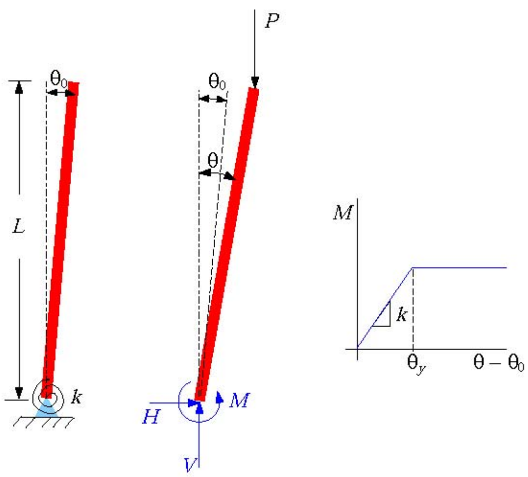

Figure 4: Imperfect, elastic-perfectly plastic system

Imperfect, elastic-plastic system

Finally, consider the response of the imperfect system shown in Figure 4. The difference between this case and the previous one is that the spring yields and becomes perfectly plastic when it is twisted by an angle θ_y as shown in the figure. The moment equilibrium equation is unaffected by the behavior of the spring and is still

$$M - PL \sin \theta = 0$$

When we substitute the spring moment rotation relations we obtain

$$k(\theta - \theta_0) - PL \sin\theta = 0 \text{ if } \theta \le \theta_0 + \theta_y$$
$$k\theta_y - PL \sin\theta = 0 \text{ if } \theta \ge \theta_0 + \theta$$

or

$$P = P_{cr} \frac{\theta - \theta_0}{\sin\theta} \; if \;\; \theta \le \theta_0 + \theta_y \tag{5}$$

$$P = P_{cr} \frac{\theta_y}{\sin\theta} \; if \;\; \theta \ge \theta_0 + \theta_y \tag{6}$$

Figure 5 shows the response of the system if $\theta_y = 20°$. Considering first the perfect case ($\theta_0 = 0$) the bar remains vertical until buckling occurs. After buckling, Eq. (5) becomes valid and the response is stable (note that Eq. (5) is the same as Eq. (4)). As soon as yielding occurs, Eq. (5) ceases to be valid, Eq. (6) governs the response and the load begins to drop as θ increases, indicating that the response is *unstable*. In other words, the system cannot support loads higher that the load at yielding. The maximum load is called the collapse load (P_{co}). In this example the collapse load is only a little bit larger than the buckling load.

The responses of imperfect systems are also shown in the Figure for $\theta_0 = \pm 5^o$ and $\pm 10^o$. The imperfect responses shown demonstrate a very important point in buckling, the *imperfection sensitivity* of the collapse load. Note that as the imperfection θ_0 increases (in absolute value), the collapse load decreases, so the larger the imperfection, the lower the collapse load of the system.

A Continuous System

For the experiment to be performed here, a continuous model of the bar is required. In this case the rigid bar in Figure 2 becomes a deformable beam as shown in Figure 6. We analyze the beam using Euler-Bernoulli beam theory as described in any introductory text on solid mechanics.

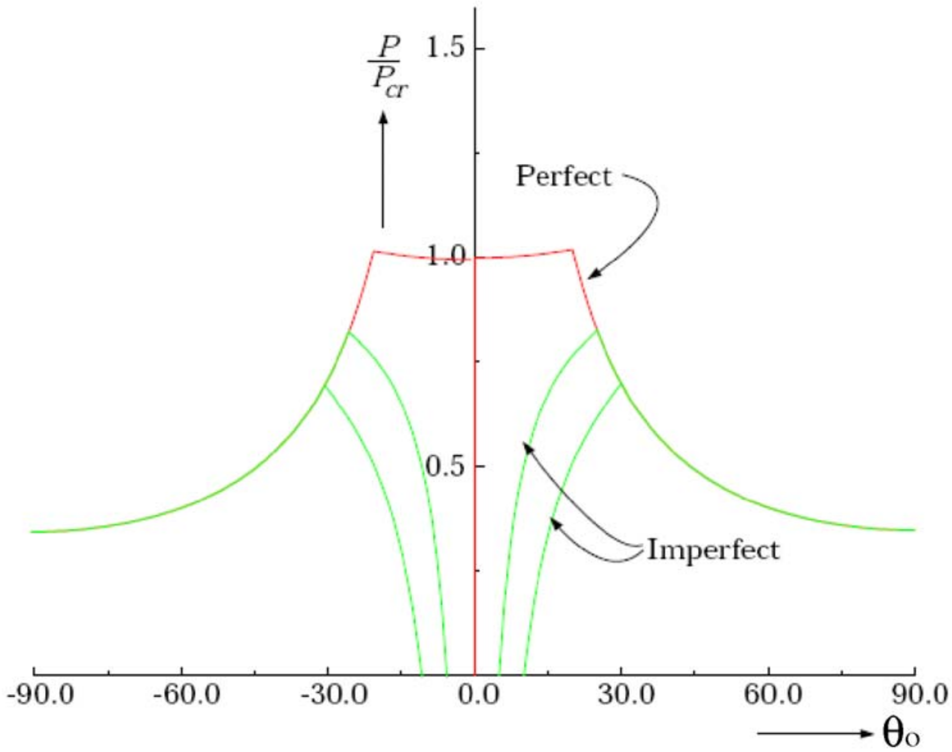

Figure 5: Response of perfect and imperfect elastic-plastic systems. $\theta_y = 20°$.

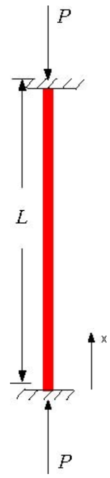

Figure 6: Continuous, perfect system

As it is customary, we adopt the “plane sections remain plane” assumption. It yields the relation between the moment and the lateral deflection as

$$EIv'' = M(x) \tag{7}$$

where E is Young’s modulus, I is the moment of inertia, the prime denotes differentiation with respect to x and M is the moment in the beam. The boundary conditions on the top and bottom of the beam can be chosen to fit the particular structure being analyzed. For this experiment, fixed-fixed end conditions are appropriate as shown in Figure 6.

Perfect system

Ideally, the boundary conditions amount to

$$v = \begin{cases} 0 \; at \; x = 0 \\ 0 \; at \; x = L \end{cases} \qquad v' = \begin{cases} 0 \; at \; x = 0 \\ 0 \; at \; x = L \end{cases}$$

where v' is the slope of the beam. In reality they may be slightly different because it is difficult to achieve perfect alignment of the specimen in the lab. At first glance we would expect the beam to compress and remain straight. If the beam is long, however, it may buckle and deflect laterally. If it is assumed to deflect, an application of static equilibrium to the sectioned beam shown in Figure 7 leads to the following equation

$$M = -Pv - M_o \tag{9}$$

Then, using Eq. (7), the equation for the deflection is:

$$v'' + \frac{P}{EI} v = -M_0 \tag{10}$$

which has the solution

$$v = A\sin\lambda x + B\cos\lambda x - \frac{M_0}{\lambda^2}$$

where $\lambda = \sqrt{P/EI}$. Applying the boundary conditions at $x = 0$ gives

$$v = \frac{M_0}{\lambda^2}(\cos\lambda x - 1) \qquad (11)$$

Application of the boundary conditions $v(L) = 0$ gives

$$\frac{M_0}{\lambda^2}(\cos\lambda L - 1) = 0 \qquad (12)$$

Setting $M_0 = 0$ gives a solution, but that is the trivial solution with no deflection. Therefore, in order to find non-zero deflections we must have

$$\lambda L = 2n\pi \qquad (13)$$

where $n = 1, 2, 3,...$. The critical load for buckling occurs at the lowest value of n or $n = 1$,

$$P_{cr} = EI\frac{4\pi^2}{L^2}$$

This is called the Euler buckling load because it was first presented by Euler when he developed the beam theory bearing his name.

Note that M_0 is not found in the analysis and the magnitude of the deflection is unknown. However, it can be expected that at the boundaries and in the center of the beam the material may yield. Consequently, it is reasonable to expect that the beam will collapse like the discrete system. See Figure 5.

Experiment Procedure

Equipment:

ATS Testing Machine	LVDT
Personal Computer	Data Acquisition Software
Calipers	Tape Measure
Magnetic Transducer	

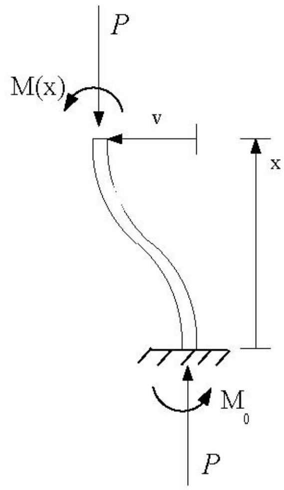

Figure 7: Sectioning of the continuous beam in the buckled condition.

Objectives:

To record the load-deflection response of a clamped-clamped column.
To identify, from the recorded response, the collapse load of the column.

Introduction:

Buckling of columns is the last topic usually covered in Mechanics of Solids. Buckling is an instability which generally occurs when “thin” structures are subjected to compressive loading. In this laboratory exercise, you will study the response of a clamped-clamped column under axial compression. During the experiment the data acquisition system will monitor the displacements of the column and the load. You will identify the main aspects of the response from the data acquired.

Experimental Set-Up:

The experimental set-up is very similar to that used in the tension test exercise. The lateral deflection of the column will be measured with a LVDT, but the extensometer will not be used. The magnetic transducer will be used to measure the axial displacement.

Procedure:

Using the calipers and the tape measure provided, measure the length, width and thickness of the specimen. Record these values.
Zero the output of the load cell.
Install the specimen in the testing machine. Your TA will show you how to do this. Make sure the top and bottom edges of the specimen contact the clamps.
Make sure the specimen is unloaded within ± 2 lb.
Set the speed of the ATS machine at 0.05 in/min.
Start the data acquisition system. Enter a file name.

Start the ATS machine. Make sure that it is compressing the column and that the data acquisition system is acquiring data properly. You should see a plot of load vs. axial deflection and transverse deflection.
Continue loading until the axial load is clearly decreasing.
Stop the data acquisition system.
Unload the column and take it out of the testing machine.

Report Guidelines:

Assuming that the Young's modulus of the material is 10 x 10^3 ksi and that the yield stress is 40 ksi (approximate nominal values for Al 6061-T6), address the following in your report.
From your measurements and the material data above: (1) Calculate the buckling load and (2) The yield load of the perfect column. Does buckling occur in the elastic or plastic range?
(3) Plot the load vs. the axial displacement. Normalize the load by the buckling load you calculated in 1.
(4) Plot the load vs. transverse deflection. Normalize the load as in the previous plot.
Describe the response of the column using the figures you generated in (3) and (4) as a guide. Comment on the initial response, what happened as the buckling load was approached and what the collapse load of the column was.
Discuss the significance of the calculation of the buckling load in this exercise.

Questions (To be answered before beginning the lab)

(1) What is buckling?

(2) What is meant by "perfect" in a discrete system?

(3) What is the difference between an imperfect linear spring system and a perfect linear spring system?

(4) What are the objectives of the lab?