University of Notre Dame
Department of Aerospace and Mechanical Engineering

AME 20241-02 Mechanics of Solids Spring 2008

EXAM #1

February 19, 2008

Name: Solutions

Question	Points	Score
1	25	
2	~~25~~ 20	
3	~~25~~ 30	
4	25	
total	100	

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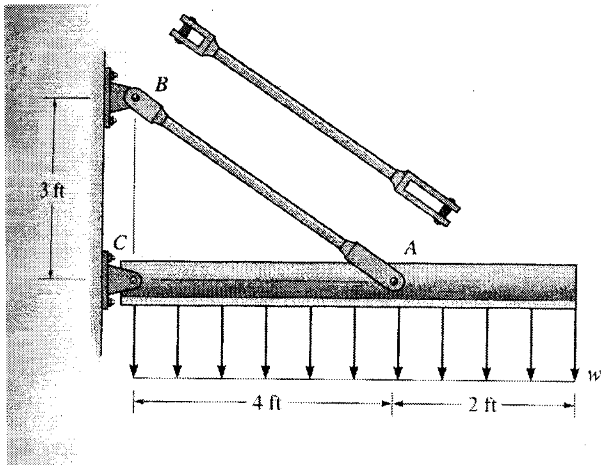
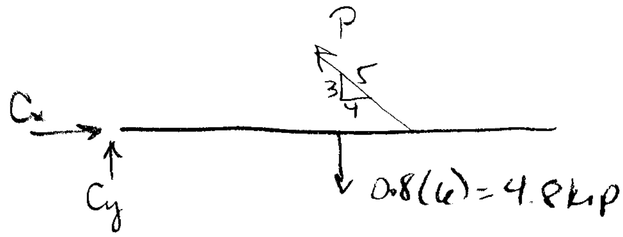
1. (25 pts.) The hanger assembly below is used to support a distributed loading of w = 0.8 kip/ft.

a) Determine the average shear stress in the 0.4 in. diameter bolt at A and the average tensile stress in rod AB, which has a diameter of 0.5 in.

b) If the yield shear stress for the bolt is $\tau_y = 25$ ksi, and the yield tensile stress for the rod is $\sigma_y = 38$ ksi, determine the factor of safety with respect to yielding in each case.

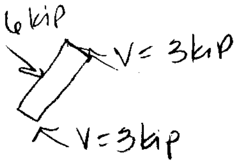




$\Sigma M_C = 0 = -4.8\text{kip}(3ft) + \frac{3}{5}P(4ft)$

$P = 6\text{kip}$ (5)

For bolt A:



(5)

$\tau = \frac{V}{A} = \frac{3}{\frac{\pi}{4}(0.4)^2} = \underline{23.9\text{ksi}}$ (5)

$FS: \frac{\tau_y}{\tau} = \frac{25}{23.9} = \underline{1.05}$

For rod AB

$\sigma = \frac{P}{A} = \frac{6}{\frac{\pi}{4}(0.5)^2} = \underline{30.6\text{ksi}}$ (5)

FS: (5)

$FS = \frac{\sigma_y}{\sigma} = \frac{38}{30.6} = \underline{1.24}$

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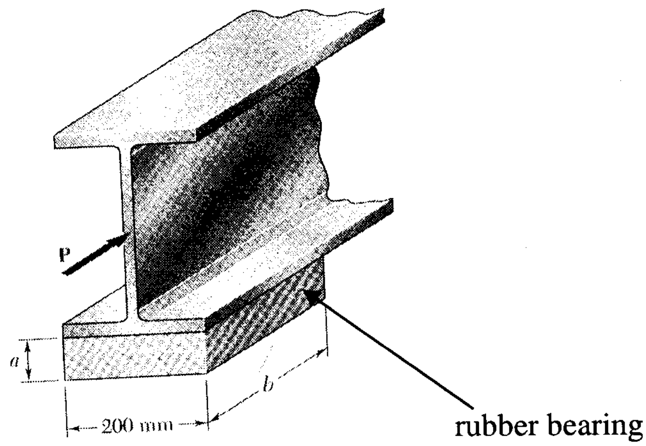
2. ~~(25 pts.)~~ 20 A rubber bearing (G = 0.9 MPa) is used to support a bridge girder as shown to provide flexibility during earthquakes. The beam must not displace more than 10 mm when a 22 kN load is applied as shown. Knowing that the maximum allowable shear stress is 4~~2~~0 kPa, determine:

a) the smallest allowable dimension b to the nearest mm

b) the largest required thickness a " " " "



(10) a) $\tau_{ave} = \dfrac{V}{b \cdot w} \Rightarrow 410 \times 10^3 \text{ Pa} = \dfrac{22 \times 10^3 \text{ N}}{b \cdot (.2 \text{ m})} \Rightarrow b = 268.3 \text{ mm}$

have to round up to keep $\tau < \tau_{allow}$ $\underline{b = 269 \text{ mm}}$

(10) b) $\tau = G\gamma \Rightarrow \gamma = \dfrac{\tau}{G} = \dfrac{410 \times 10^3}{0.9 \times 10^6} = .455$

10mm, a

$\gamma \approx \tan\gamma = \dfrac{10 \text{ mm}}{a} = .455$ (assumes small angle)

$a = 21.98$ have to round down to keep $\delta < 10 \text{ mm}$ $\underline{a = 21 \text{ mm}}$

or $\tan\gamma = \dfrac{10 \text{ mm}}{a} = .489$

$a = 20.44$ have to round down to keep $\delta < 10 \text{ mm}$

$a = \underline{20 \text{ mm}}$

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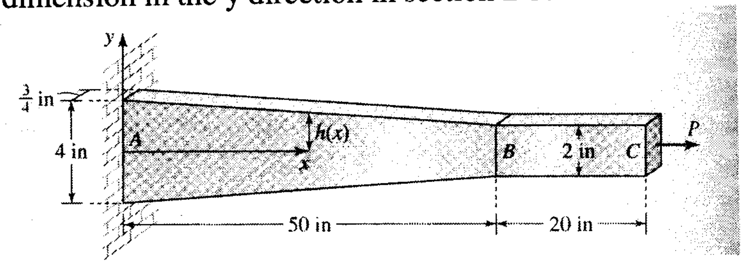
3. (~~25~~ 20 pts.) A rectangular aluminum bar (E_{al} = 10,000 ksi, υ = 0.25) of a ¾ in. thickness consists of uniform and tapered cross sections, as shown. The depth of the tapered section varies as h(x) = 2 – 0.02x. Determine:

a) The elongation of the bar under a load P = 10 kips.

b) The change in dimension in the y direction in section BC.



(20) a) $\delta = \delta_{AB} + \delta_{BC}$ (3)

$$\delta_{BC} = \frac{PL}{EA} = \frac{(10\times10^3\ lb)(20\,in)}{(10{,}000\times10^3\ lb/in^2)(3/4\,in)(2\,in)} = .0133\ in \quad (3)$$

$t = 3/4\,in$

$h(x) = 2 - 0.02x$

$$\delta_{AB} = \int_{x_A=0}^{x_B=50} \frac{P\,dx}{E\,A(x)} \quad (5) \qquad \text{where } A(x) = 2t\,h(x)$$

$$\delta_{AB} = \frac{P}{2Et}\int_{x_A=0}^{x_B=50} \frac{dx}{(2-0.02x)} = \frac{10\times10^3}{(2)(10{,}000\times10^3)(3/4\,in)(-0.02)} \ln(2-0.02x)\Big|_0^{50}$$

$$= \frac{10}{(2)(10{,}000)(3/4)(-0.02)}\left[\ln(1) - \ln(2)\right] = .0231\ in \quad (3)$$

$\delta = .0133\,in + .0231\,in = \underline{0.036\ in}$ elongation

(10) b) $\varepsilon_x = \frac{\delta_x}{L_x}$ in section BC

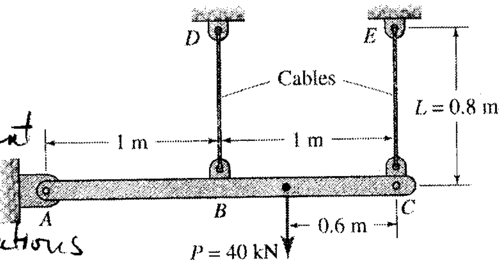
$$\varepsilon_x = \frac{.0133}{20\,in} = 655\ \mu in/in$$

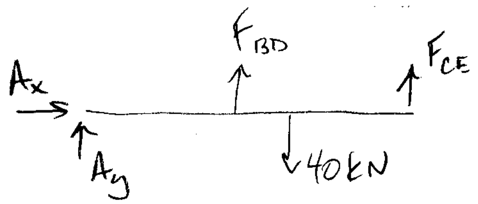
$$\varepsilon_y = -\upsilon\varepsilon_x = -(0.25)(655\ \mu in/in) = -166\ \mu in/in$$

$$\delta_y = \varepsilon_y L_y = \left(-166\ \frac{\mu in}{in}\right)(2\,in) = \underline{-333\ \mu in}$$ contraction

4. (25 pts.) A rigid horizontal bar ABC is supported by a hinge at A by by two steel cables BD and CD, which are of equal length, L = 0.8 m, and cross-sectional area A = 140 mm². Find the stress in each cable due to a vertical force of P = 40 kN.

Structure is statically indeterminant
4 unknowns
3 equilibrium equations

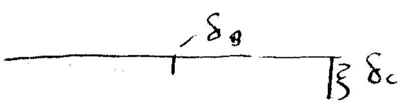




Equilibrium

$\Sigma M_A = 0: \; F_{BD} + 2F_{CE} - 40(1.4) = 0 \quad (1)$ (5)

Compatibility



$2\delta_B = \delta_C$ by similar triangles (5)

Force - Displacement

$\delta = \frac{PL}{EA}$ (5)

$2\delta_B = \delta_C \Rightarrow \frac{2F_{BD}L}{EA} = \frac{F_{CE}L}{EA} \Rightarrow 2F_{BD} = F_{CE} \quad (2)$

Solve eqns (1) + (2) to get $F_{BD} = 11.2$ kN, $F_{CE} = 22.4$ kN (5)

$\sigma_{BD} = \frac{11.2 \times 10^3}{140 \times 10^{-6}} = 80$ MPa $\quad \sigma_{CE} = 160$ MPa (5)