

Read through Chapter 3 of BS&L.

In order to preserve the parallel between mass, momentum & energy transport BS&L defined the viscous stress tensor τ_{ij} to have a sign opposite of that defined in class and used in most other textbooks. If you keep this in mind, and recall that the sign of the stress tensor is completely arbitrary (as long as you are consistent!), it should not cause you any confusion.

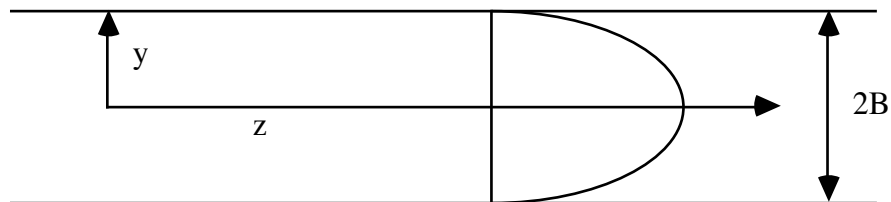
1). Consider a viscous fluid flowing in a laminar manner through a slit formed by two parallel walls a distance $2B$ apart as is depicted below.

a. Given that we have a uniform pressure gradient in the negative z direction (i.e., dp/dz equals a negative constant) and the fluid is massless with viscosity μ , calculate the flow rate per unit extension in the third direction (e.g., per unit width) using the Navier-Stokes equations.

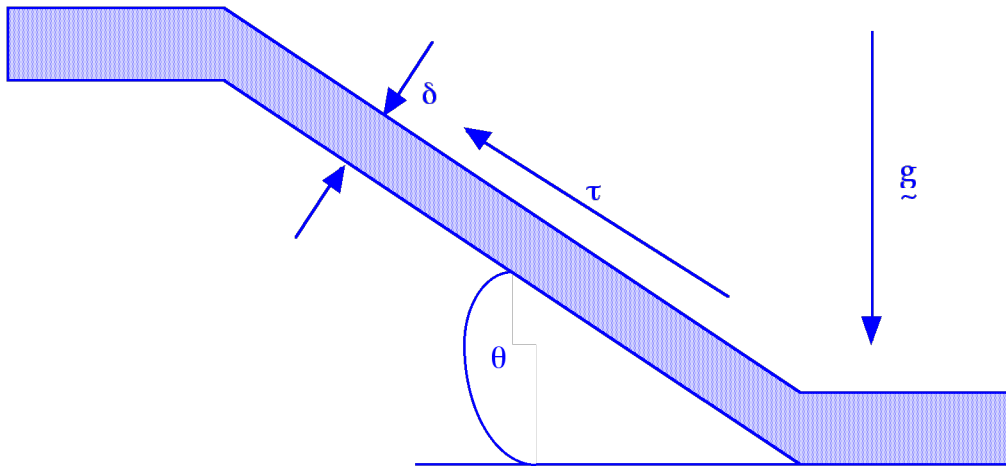
b. What is the ratio of the centerline velocity to the average velocity?

c. What is the wall shear stress (force/area exerted by the fluid on the lower wall)?

d. Now suppose we reorient the system so that gravity is in the positive z direction, the fluid has density ρ , and there is no pressure gradient (e.g., open to the atmosphere at both ends). What is the flow rate in this case?



2). Consider the velocity distribution acquired by the slow flow of a very viscous fluid down an incline, as is depicted below. In contrast to the problem considered in the lecture notes, here we have a breeze trying to blow the fluid back up the incline, in addition to the gravity trying to draw the fluid down. We model the effect of the breeze with a constant shear stress τ at the upper surface of the fluid (note: τ will be negative for the coordinate system as drawn, and is equivalent to the Marangoni stress in the tears of wine demonstration!). For some film depth δ , angle θ , density ρ and viscosity μ , how large does this shear stress have to be for the net flow rate (the integral of the velocity over the film depth) to be zero? Solve the problem assuming unidirectional flow.

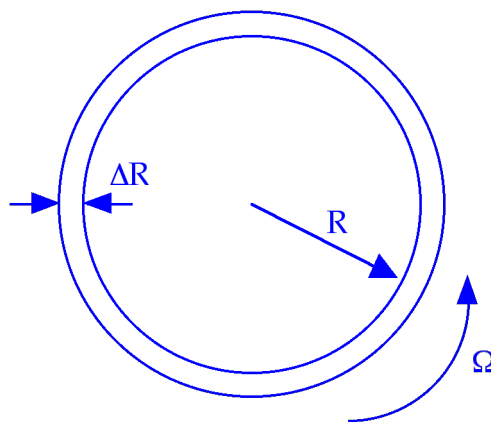


3. In class we derived a way to estimate the viscosity of a fluid using a Couette viscometer by neglecting the curvature effects (e.g., if the gap width is ΔR and the inner radius is R_1 , we required $\Delta R/R_1 \ll 1$).

a. Using cylindrical coordinates, derive the exact relationship between torque and rotation rate where the inner radius is R_0 and the outer radius is R_1 such that $\Delta R = R_1 - R_0$. Note that if you get desperate, this problem is worked out completely in BS&L...

b. The approximate formula results from ignoring curvature (e.g., $\Delta R/R_0 \ll 1$) and uses some characteristic radius R . It is interesting to see how well this approximate formula works. Plot up the ratio of the approximate formula to the exact value for four choices for the radius R in the approximation: 1) where we take R to be the inner radius R_0 , 2) where we take R to be the outer radius R_1 , 3) where we use a mixed relation where the velocity of the outer cylinder is based on R_1 and the lever arm and area for calculating the torque is based on R_0 , and 4) where we take R to be the average of the inner and outer radii. Do this for values of $[0 < \Delta R/R_0 < 0.5]$.

c. A Couette device in my lab has a bob (inner cylinder) radius of 2.375 cm and a gap width of 1.261 mm. How large is the error in the calculated viscosity if we use the four different approximate formulae for this instrument?



4. In the last homework you determined the complete *mobility tensor* for a falling body of revolution in a viscous fluid from two simple experiments: the measured velocity where the director was perpendicular to gravity, and where it was parallel to gravity. In class the time measured for a washer to fall a set distance through Karo syrup was 11.2s if flat side on and 7.5s if edge on. Using this information and the magic of index notation, we can calculate all sorts of stuff.

a. If the director p_i of the axis of revolution is given by:

$$p_i = \delta_{i1} \cos(\theta) + \delta_{i2} \sin(\theta)$$

where θ is the angle relative to vertical (e.g., gravity), derive the formula for the velocities in the 1 and 2 direction as a function of angle. Note that an angle of zero corresponds to dropping it flat side on.

b. Suppose we drop the washer at an angle and observe a fall time of 9.3s. Based on this, what would be the angle θ ?

c. The ratio of the two velocities determines the trajectory. It was about 11cm from where I let it go to the bottom. Using the information above, calculate the expected lateral displacement.

d. If the 1 sigma random error in each of the time measurements was 0.1s (pretty optimistic, I'm afraid) and the standard deviation in the height was 1cm, what is the 95% confidence interval of the predicted lateral displacement for part c? Assume that all the measurement errors are independent. You can do this any way you like, but a MC simulation (e.g., throwing the computer at it) is probably the easiest...

What you should learn from these problems:

Problem 1: There are several things you should get from this problem.

- a. Practice using the Navier-Stokes equations to solve unidirectional flows.
- b. Recognize the connection between the velocity profile, the centerline velocity, and the flow rate in channel flow, a very important geometry.
- c. Connecting the velocity profile to the shear stress.
- d. Recognizing the equivalence between gravity driven and pressure gradient driven flows (both are uniform sources of momentum/volume).

Problem 2: Some things to get from this problem:

- a. Application of the shear stress boundary condition in addition to the no-slip condition.
- b. Identifying the relationship between velocity profiles and flow rates.
- c. More practice with parabolas...

Problem 3: This problem deals with flow in cylindrical coordinates:

- a. Practice using the NS equations in cylindrical coordinates, this time using the θ component. It's not too bad if you cross out all the terms except the one that matters!
- b. Showing that the "flat earth limit" provides a useful and accurate approximation if the separation of length scales is sufficiently large.

Problem 4: Index Notation:

- a. This is an extension of the homework problem from last week, reinforcing your skills with manipulating linear problems using index notation.
- b. Showing how you can use simple measurements to get a large amount of information about a system (e.g., calculating angles of inclination and expected lateral displacements from measurements of three fall times).
- c. Refreshing your skills with doing relatively complex error propagation calculations. Note that if you write a little program that takes in the two fall times and spits out the lateral displacement, you can use a MC simulation approach to get the confidence interval...