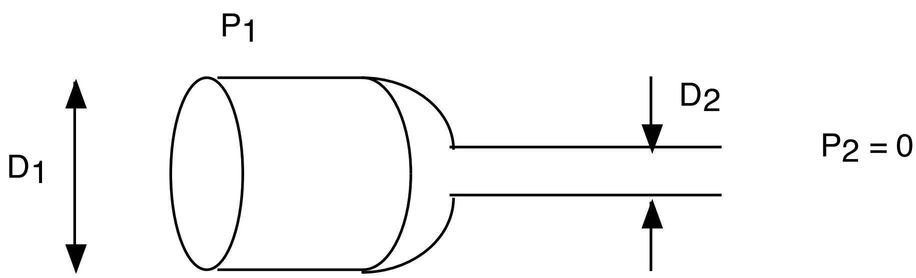
PROBLEM SET 4 CBE 30355 DUE 9/21/23

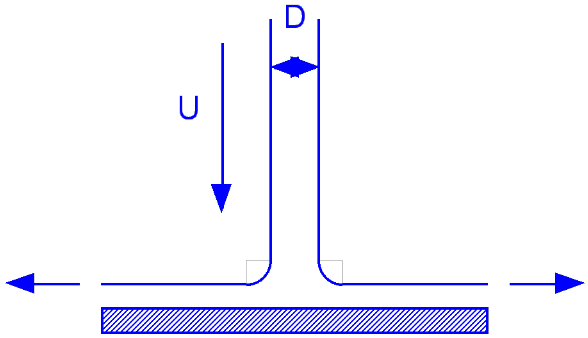
1). A firehose nozzle is basically just a contraction in a pipe. Here we want to calculate the force on the nozzle, so we can design it so it doesn't come off! We take the firehose to have an ID of 3", the corresponding nozzle diameter is 1", the upstream pressure is P_1, the downstream pressure is 0 psig (e.g., it just comes out at atmospheric pressure, which can be ignored as it pushes equally on everything), and the flow rate is 1 gal/sec of water.

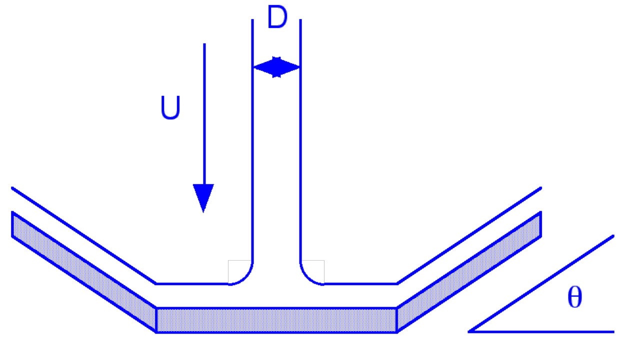
a. What's the force? Assume uniform flow (the assumptions made in class). Note that to solve this problem you have to figure out what P_1 is. If you ignore all losses, you can use Bernoulli's equation, which relates the pressure to the velocity.

b. Neglecting all losses, what is the highest fire you could use this system to put out? (Hint: Bernoulli's equation again). This height is pretty wimpy – name two ways you could increase it!



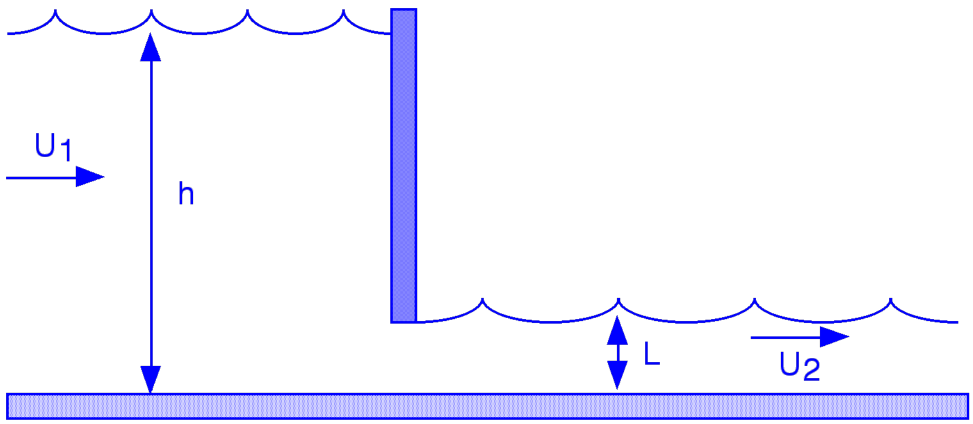
2). A jet of fluid with density ρ, velocity U, and diameter D impinges on a flat plate as depicted below. What is the force on the plate? How does this answer change if the plate is curved so the fluid sprays back out at an angle θ, and keeps the same speed U? Ignore the effects of gravity.





3). A liquid of density ρ flows through a sluice gate as shown. If the upstream and downstream flows are parallel, we can take the pressure distribution far upstream and downstream to be hydrostatic. If the upstream velocity is U_1, the upstream height is h, and the opening is L, derive an expression for the force per unit width necessary to keep the sluice gate in place. (Hint: Draw a control volume around the water upstream and downstream of the gate, and determine the velocity U_2 from conservation of mass. Then do a similar momentum balance. You can again ignore atmospheric pressure.

The fluid flowing out of the gate actually *reduces* the force on the gate from the purely hydrostatic result - the maximum possible)



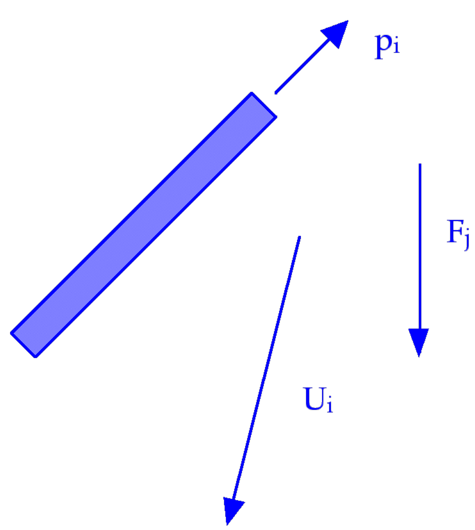
4). Index Notation: Consider a cylinder settling in a viscous fluid as depicted below. In the notes on index notation, it was shown that if an object's orientation was specified by a single director p_i, and if the relation between force and velocity was linear, then the general expression for the velocity was given by:

$$U_i = (\lambda_1 \delta_{ij} + \lambda_2 p_i p_j) F_j$$

where λ_1 and λ_2 are constants independent of orientation. To determine these two constants we measure the velocity of the cylinder when it is aligned with gravity, and when it is perpendicular to gravity. If these two *mobilities* are given by:

$$\text{Parallel: } \frac{|U|}{|F|} = A \quad ; \qquad \text{Perpendicular: } \frac{|U|}{|F|} = B$$

where $|U|$ and $|F|$ are the magnitudes of the velocities and the force, determine λ_1 and λ_2.



What you should learn from these problems:

Problem 1: There are several things you should get from this problem.

a. Application of conservation of mass (volume for incompressible fluids) to figure out velocity/flow rate relationships with change in cross-sectional area.
b. Application of Bernoulli's equation (an equation that every educated person should know…) to relate pressure to velocity along a streamline.
c. Relationship between changes in elevation and velocity, again through B's equation.

Problem 2: This one has a couple of things in it:

a. Recognition that force (and change in momentum) is a *vector*. Thus, you can apply a force without changing *speed* by changing direction.
b. Practice in drawing appropriate control volumes which simplify calculations.

Problem 3: This problem is a tricky combination of hydrostatics and conservation of momentum:

a. You need to draw the correct control volume to keep track of velocities, forces and changes in momentum.
b. You often get more momentum out of a control volume than you put in – which means there is a force being applied somewhere.

Problem 4: Index Notation:

a. This is just the first example of how you can use index notation to reveal hidden physics (we'll see more later).
b. Recognition of what it means to have a *linear relationship* and how it can dramatically simplify problems.
c. Practice with the procedure of 1) reducing the number of unknowns in a problem by constructing a "most general form" of a tensor and 2) choosing simple problems to get the remaining unknowns.
d. Figuring out what a *director* is…