PROBLEM SET 1 CBE30355 DUE 8/31/23

These first few problems should serve as a review for the vector calculus and elementary differential equations material you learned last few semesters and will use extensively this term.

1). The axial velocity u of fluid in a tube of radius a is governed by the second order differential equation:

$$\mu \frac{1}{r}\frac{d}{dr}\left(r\frac{du}{dr}\right) = \frac{\Delta P}{L}$$

with boundary conditions $u|_{r=0}$ = finite; $u|_{r=a} = 0$. The expression $\Delta P/L$ is the pressure drop per unit length of the tube (and is negative for positive flow), and is a constant. μ is the fluid viscosity and is also constant. What is the velocity distribution in the tube as a function of r? (Hint: It's directly integrable!).

2). Calculate the following quantities (Note: X denotes the cross-product and · the dot product):

a. (1,0,1)·(1,0,-1)

b. (1,1,1)·(0,1,0)

c. (1,1,1)X(2,2,2)

3). For the scalar potential function $\phi = (x + y + z)^2$ and the velocity vector field $\mathbf{u} = (z, y^2, x)$ calculate the following vector quantities:

a. $\nabla\phi$; $\nabla\cdot\mathbf{u}$

b. $\nabla^2\phi = (\nabla\cdot\nabla)\phi$; $\nabla^2\mathbf{u}$

c. $\nabla \times \mathbf{u}$

where the boldface operator $\nabla = (\partial/\partial x, \partial/\partial y, \partial/\partial z)$

4. Prove that for an arbitrary vector $\mathbf{u}$:

$$\nabla\cdot(\nabla \times \mathbf{u}) = 0$$

(In fluid mechanics, where $\mathbf{u}$ is the velocity vector, this is equivalent to saying that the vorticity [the curl of the velocity] is a solenoidal vector field [divergence free]. It is very useful in manipulating the equations of motion, particularly at high Reynolds numbers)

5. Two plates are separated by a distance of 0.5 mm. A tangential force is applied to the upper plate (in the same manner as was described in class) and it begins to move, eventually reaching a steady velocity. Answer the following:

a. If the steady-state is achieved in about 1 second, estimate the kinematic viscosity of the fluid.

b. If the force is doubled, how does the time to steady-state change? (trick question…)

c. If the gap between the plates is doubled, how does the time to steady-state change?

6. This is <u>completely</u> optional (and not for credit - solve only if you like puzzles): Prove the following vector identity for the arbitrary vector $\mathbb{u}$:

$$\nabla \times (\nabla \times \mathbb{u}) = \nabla (\nabla \cdot \mathbb{u}) - \nabla^2 \mathbb{u}$$

Hint: I usually solve this problem using index notation which is very useful for describing advanced transport problems. Detailed notes on index notation are available through the class website. We'll go over index notation in our first few in-class sessions.

In this problem set all vectors are in outlined boldface type while scalars are in regular type.

What you should learn from these problems:

Problem 1: This problem is asked to get you to do several things.

a. To remember how to sequentially integrate an expression (including the tricky bits associated with the cylindrical geometry).
b. To apply boundary conditions to determine the unknown constants of integration.
c. To get you to open your Calc II text if you've forgotten how to do this stuff…

Problem 2: Again, several things should be learned:

a. Practice with inner products and cross products.
b. Recognizing what happens when vectors are perpendicular or parallel.

Problem 3: This problem gives you practice with operators we will use extensively in this course:

a. To discriminate between gradient, divergence, laplacian, and curl.
b. To think about how all this works with vectors and tensors.

Problem 4: This problem deals with combinations of operators:

a. Practice with divergence & curl of a vector field.
b. Ultimately, this is most easily approached using index notation and the concepts of symmetric and anti-symmetric tensors...

Problem 5: This problem deals with transients:

a. Figure out how to estimate viscosities from the time to approach steady-state.
b. Determine how the characteristic time depends on the geometry and boundary conditions.
c. Start building your intuition on fluid mechanics problems.

Problem 6: A problem you really only want to do if you read ahead in the notes on index notation…