CBE 30355 TRANSPORT PHENOMENA I

Mid-Term Exam
10/14/21

This test is closed books and closed notes

Problem 1 (20 points). Unidirectional Flows: A tube of radius a is submerged to a depth H in a viscous fluid. Initially you have your finger over the top so it remains filled with air. At time t = 0 you take your finger off and it fills with liquid. We are interested in this filling process, and in particular how the height of liquid in the tube h changes with time.

a. When you first take your finger off, what is the initial driving force for the flow in the tube?

b. If the fluid is sufficiently viscous we can ignore inertial terms (we are also ignoring capillary effects, moving contact lines, entrance flow effects: just take it to be unidirectional flow!). If, at some time t the fill height is h, develop an equation for the area average velocity in the tube.

c. Use the results of part b to get a differential equation for the fill height h as a function of time. Render this dimensionless to determine the characteristic fill time. Note that it would take forever to completely fill the tube, but if you solved the equation (not too hard) you would find that it is about 84% filled in a dimensionless time of one…

d. In part b you ignored inertial effects. By scaling the equations, determine the approximate minimum viscosity for which this assumption is likely to be valid.

The following equations *may* be helpful:

$$\frac{1}{r}\frac{\partial(r\,u_r)}{\partial r}+\frac{1}{r}\frac{\partial u_\theta}{\partial \theta}+\frac{\partial u_z}{\partial z}=0$$

$$\rho\left(\frac{\partial u_z}{\partial t}+u_r\frac{\partial u_z}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_z}{\partial \theta}+u_z\frac{\partial u_z}{\partial z}\right)=-\frac{\partial p}{\partial z}+\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 u_z}{\partial \theta^2}+\frac{\partial^2 u_z}{\partial z^2}\right]+\rho g_z$$

Problem 2 (10 pts). Hydrostatics/Archimedes Law: You are given the task of determining the gold content of a "red gold" chain (actually a gold/copper alloy) that a friend has brought back from a trip to the Middle East, and thus how much it should be insured for. Using a scale, you tare out a beaker of water (e.g., get it so that it reads zero). You dangle the chain via a fine thread in the water so that it is completely immersed (but suspended above the bottom), and find that the scale reads 13.5g. You then let it settle to the bottom and find that the scale reads 171.4g. The density of gold is 19.3 g/cm^3 and costs \$57.90/gram and the density of copper is 8.6 g/cm^3 and costs \$0.0094/gram. Given this, what karat is the chain (24 karat = pure gold) and how much is the metal in the chain worth?

Problem 3 (15 points). Dimensional analysis: As demonstrated in class, when a drop of volume V, density ρ_d and viscosity μ_d is released into a fluid of viscosity μ_f and density $\rho_f < \rho_d$ it will expand into a torus that eventually breaks up after falling a height H. Here we use dimensional analysis to examine this problem.

a. Using the Buckingham Π theorem, determine the number of dimensionless groups that the problem depends on and construct an independent set.

b. Recognizing that ρ_f and ρ_d are very close, strengthen your result by only including ρ_d in the buoyancy term (e.g., $\Delta\rho g$ rather than ρ_d and g separately).

c. Empirically it is observed that the break up height H is nearly independent of the drop volume and viscosity. Under these circumstances, how should H vary with the fluid viscosity μ_f?

Problem 4 (15 points). Conservation of Momentum: Consider the tank of radius R depicted below. It is filled with water to a height H, and the top is open to the atmosphere. Someone pokes a hole in the side half-way up (e.g., at $z = H/2$). If the hole is of radius $a \ll R$ and we ignore all losses, calculate the following:

a. What is the velocity of fluid out the hole?

b. What is the initial rate of change of the fill height of the tank?

c. How far to the side does the stream project before it hits the ground?