CBE 30355 TRANSPORT PHENOMENA I

Mid-Term Exam
10/13/22

This test is closed books and closed notes

Problem 1 (15 points). Unidirectional Flows: As was discussed in class, Tanner's group from the University of Sydney used an inclined half-filled semi-circular trough to study the rheology of suspensions. In this problem we look at the operating conditions of this system.

a. If the trough is of radius R and the angle of inclination to the horizontal is θ what is the flow rate Q required to fill the trough to the brim (e.g., the edge of the semicircle)? Assume unidirectional Newtonian flow for simplicity!

b. The shear stress is $\tau_{zr} = \tau_{rz} = \mu\left(\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r}\right)$. What is the stress distribution in the trough? If the viscosity of the fluid is doubled (still just filled to the brim!), what happens to the shear stress? (Be careful!)

c. (2 points extra credit). When a suspension (rather than a Newtonian fluid) flowed down the trough, something weird happened. What was it and why did it occur?

The following equations *may* be helpful:

$$\frac{1}{r}\frac{\partial(r\,u_r)}{\partial r} + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{\partial u_z}{\partial z} = 0$$

$$\rho\left(\frac{\partial u_z}{\partial t} + u_r\frac{\partial u_z}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_z}{\partial \theta} + u_z\frac{\partial u_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 u_z}{\partial \theta^2} + \frac{\partial^2 u_z}{\partial z^2}\right] + \rho g_z$$

Problem 2 (15 points). Conservation of Momentum/Bernoulli's Equation: You are trying to put out a fire in a building at a height H. You have a pump which is capable of supplying water with a useful mechanical energy/time (e.g., effective pumping power) of PW. For this problem, neglect all losses (e.g., use Bernoulli's Equation!).

a. You want to put the fire out quickly, so you want a large flow rate Q. For the parameters of the problem, what diameter nozzle D will achieve the maximum flow rate?

b. There are always losses, and you still need to put out the fire! Should you use a nozzle larger or smaller than the one calculated in part a? *Briefly* justify your answer.

Problem 3 (15 points). Hydrostatics: Consider the balloon depicted below. At takeoff the balloon volume is actually fairly small, but as it rises the gas in the balloon expands to fill the envelope, eventually reaching its maximum volume V. The weight of the balloon, gas, and payload is fixed at a value M (assuming no leaks!). If we make the well-mixed atmosphere approximation made for Lawnchair Larry (good to about

20km), use the equation governing hydrostatics and the adiabatic gas law to develop an expression yielding the final altitude of the balloon.

Problem 4 (15 points). Conservation of Momentum/DART: Just a couple weeks ago NASA tested a concept for potentially saving the Earth: using a kinetic impactor to "nudge" an asteroid on a collision course with the planet. The particular asteroid (Dimorphos) is no danger to us, but it provides a very clever way to test the concept. This isn't fluid mechanics (although there certainly was some granular flow!) but it -is- conservation of momentum. In this problem we will examine the Double Asteroid Redirection Test.

The kinetic impactor had a mass of m_d (500 kg) and the target had a mass M ($5x10^9$ kg) so the ratio is 10^7. The relative velocity of the impactor was U_d (6600 m/s), so the energy of impact E was pretty big ($1.1x10^{10}$ J, or about 3 tons of high explosive). Using this, calculate the change in velocity ΔU of the asteroid target under the following scenarios:

a. The impactor is simply swallowed up and absorbed by Dimorphos.

b. The impactor bounces off (backwards) as if it were a perfectly elastic collision (note: there's no way this would happen at 6.6 km/s – 15,000 mph!).

c. The huge energy of the collision causes a crater to form which ejects a mass m_e 1000 times greater than the mass of the impactor. While this calculation is really complex (and is why they spent $350M to do the test!) for this problem assume that the impact energy E is converted to the kinetic energy of the ejected mass, that all the ejected material has the same velocity, and that it is all traveling back in the direction the impactor came from with a velocity much greater than the escape velocity of the asteroid (which is very small – it's not that big a rock!). Note that these are all rather optimistic assumptions, but it does illustrate the effect they are looking for.

Note that while all these velocities are *really* small, if you were to do it to an asteroid that you detected enough years in advance of impact (they're trying to map all earth crossing asteroids for this very reason!) you could actually deflect it enough to miss the earth.