Review Questions for the Third Exam Due 4/30/01

1. Derivation of global error

Consider the differential equation:

$$\frac{dy}{dx} = f(x,y)$$

If we use the Backward Euler method approximation to this equation:

$$y_{k+1} = y_k + h\, f(x_{k+1}, y_{k+1})$$

a. Derive the local error and the propagation error of this method and

b. What is the restriction on the step size h for stability?

2. Numerical solutions to non-linear differential equations:

Consider the differential equation and initial condition given by:

$$\frac{dy}{dx} = -\, y^2 \qquad y(1) = 1$$

a. Using the **trapezoidal rule** and a step size $h = 1/2$, solve this differential equation on the interval [1,2] (e.g., just three points including the initial value).

b. Recalculate the solution at $x=2$ using the same step size and the **Euler method**. Compare this result to that obtained in part a and to the exact solution $y(x) = 1/x$.

3. Statistics:

a. A series of measurements of the decay rate of a sample obtaining carbon-14 (the radioactive isotope of carbon used in radiocarbon dating) were taken as shown below. Using this information, calculate the mean, **population** variance (the variance for each measurement), and 95% confidence interval for the **average** decay rate assuming only random error. Show all of your work. The values given below are in counts per minute (cpm).

32.2
32.9
31.5
32.1
32.4
32.3
33.3
31.8
31.2
32.0

b. Given that radioactive decay is governed by the Poisson distribution, in which the standard deviation of the number of decays observed in a given time is equal to the square root of the number of decays, estimate how long each of the above measurements took.

4. Regression analysis:

The matrix of covariance of an array of variables **x** is defined as:

$$\underset{\approx}{\Sigma}_x^2 \equiv E\left\{\left(\underset{\sim}{x} - \underset{\sim}{\mu}_x\right)\left(\underset{\sim}{x} - \underset{\sim}{\mu}_x\right)^T\right\}$$

where **x** is a column vector. If we define the array **z** = **f** (**x**), derive an approximate formula for the matrix of covariance of **z**. Under what conditions will this approximation be valid?