Problem 31 Due 5/2/01

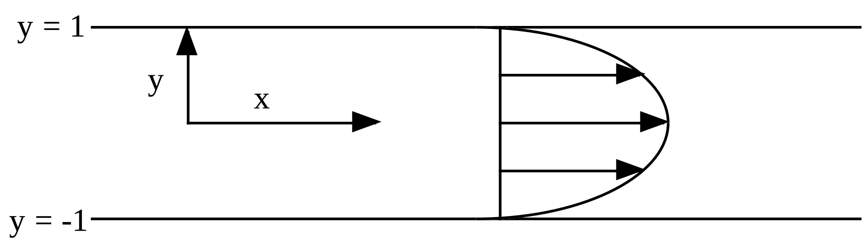
Next semester you will be studying fluid mechanics and transport phenomena. A classic problem in fluids is start-up flow in a pipe or a channel. Consider a channel of width 2 such as is depicted below. Initially, the fluid in the channel is at rest. At time t=0 a pressure gradient is applied along the channel and the fluid begins to accelerate due to this force. Because of the no-slip condition on the walls of the channel, however, the velocity at the walls remains zero. Through the action of viscosity, the walls "hold back" the fluid, leading to a parabolic velocity distribution. In dimensionless form, the velocity distribution is governed by the partial differential equation:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + 1$$

$$\left.\frac{\partial u}{\partial y}\right|_{y=0} = 0 \quad ; \quad u\Big|_{y=1} = 0$$

The first boundary condition is the symmetry condition at the centerline y = 0, while the second is the no-slip condition at the upper wall. The initial condition is just u = 0 everywhere.

Using finite difference approximations, solve this problem as a system of ODE's. I want you to give me two plots. First, I want to have a plot of the centerline velocity as a function of time. Second, I want a series plot of velocity profiles (e.g., velocity as a function of y) for different times. The times for each of the plotted profiles should be identified using the "legend" command.



Don't make this problem too hard - it is really quite easy to program, particularly if you adapt the example program 31a!