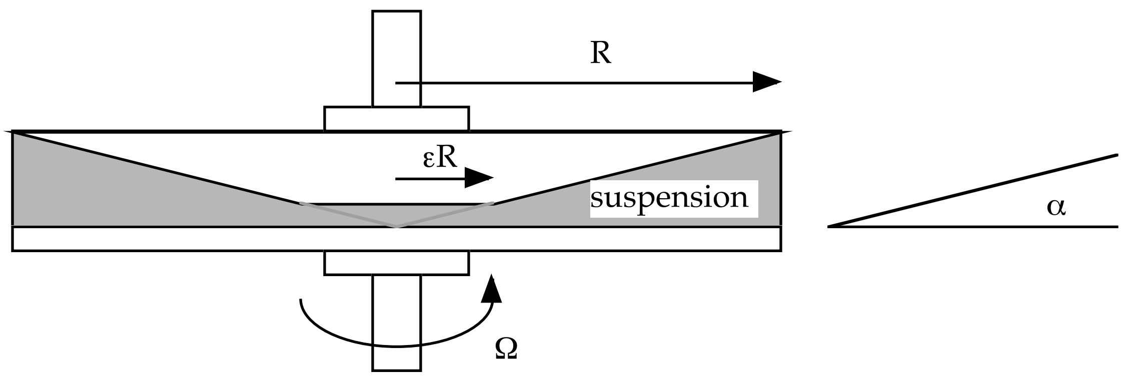
Cheg 258 Final Project **Due in my office at 5PM, 5/2/97**
Remember: You may not discuss this with others!

Radial Migration in a Truncated Cone and Plate Geometry

In experiments currently being performed at Lockheed-Martin in Palo Alto, CA, researchers are investigating the radial migration of particles in a suspension undergoing shear in a variety of geometries. The purpose of this research is to improve our understanding of migration phenomena which can lead to inhomogeneities (and hence lack of reproducibility) in solid rocket propellant motors such as those used on the space shuttle. One geometry under investigation is the truncated cone and plate geometry depicted below. In the cone and plate geometry a cone with vertex angle ($\pi - 2\alpha$) is placed in contact with a plate (actually a circular disk). The angle α is very small, so the cone is really quite flat. In the case of the truncated cone and plate geometry, the cone has its tip cut off, but the separation distance of the rest of it is otherwise the same.



A suspension of particles with volume average concentration ϕ_0 is placed between the truncated cone and plate and sheared by rotation of the lower plate at an angular velocity Ω. It turns out that particles will tend to migrate radially due to the curvature of the flow field. The details of the theory are not important here, but they can be found in the paper "Shear induced radial segregation in bidisperse suspensions," Krishnan, Beimfohr, and Leighton, J. Fluid Mech., 321, 371-393, (1996). From other experiments and theoretical considerations, the radial particle flux is expected to be:

$$N_r = - \widehat{D}_\perp \dot{\gamma} a^2 \frac{\partial \phi}{\partial r} \qquad 0 < r < \varepsilon R$$

in the parallel-plate region, and:

$$N_r = K_r \phi^2 \frac{\dot{\gamma} a^2}{r} - \widehat{D}_\perp \dot{\gamma} a^2 \frac{\partial \phi}{\partial r} \qquad \varepsilon R < r < R$$

in the cone and plate region. The parameter ε is the ratio of the radius of the truncated part of the cone to the radius R of the plate, and has a value of $\varepsilon = 0.26$ for the experimental device used by Lockheed. In these equations ϕ is the local concentration, $\hat{D}_\perp$ is the dimensionless shear-induced effective diffusivity normal to the plane of shear (e.g., in the r direction in this geometry), $\dot{\gamma}$ is the shear rate, and a is the radius of the particles. The quantity K_r is a constant (at least we think that is a reasonable description of it) which characterizes the magnitude of the tendency of the particles to migrate in the radial direction. At steady-state the flux of particles is identically zero everywhere, leading to a first order differential equation for the concentration distribution.

The shear rate in the gap between cone and plate is a function of the gap width, radial position, and rotation rate of the lower plate. For this geometry the gap width is given by:

$$h(r) = \begin{cases} \varepsilon\, R \tan(\alpha) & 0 < r < \varepsilon R \\ r \tan(\alpha) & \varepsilon R < r < R \end{cases}$$

The shear rate is just the relative velocity of the two plates divided by this width:

$$\dot{\gamma} = \frac{\Omega\, r}{h}$$

And the dimensionless diffusivity is approximately given by the correlation:

$$\hat{D}_\perp = \frac{1}{3}\phi^2\left(1 + \frac{1}{2}\exp(8.8\,\phi)\right)$$

We can calculate the concentration profile by integrating the above first order equations. Unfortunately, however, we don't know the concentration at the center $r = 0$ or outer edge $r = R$ necessary to do the integration (we would only need one or the other of these). Instead we just know that the volume average concentration is equal to the initial concentration ϕ_0:

$$\int_0^R \phi\, h\, 2\pi\, r\, dr = \int_0^R \phi_0\, h\, 2\pi\, r\, dr$$

It is very difficult to measure the concentration profile, although Lockheed is trying to do just that using Magnetic Resonance Imaging. A much simpler experiment is to measure the change in the torque exerted on the cone due to the rotation of the lower plate as the particles migrate radially outward. We wish to explore this option. The torque due to a given concentration profile is determined from the integral:

$$T = \int_0^R \dot{\gamma}\, \mu_0\, \mu_r\, 2\pi\, r^2\, dr$$

where μ_0 is the fluid viscosity and μ_r is the suspension relative viscosity. This latter quantity is a function of concentration, and is given by the approximate correlation:

$$\mu_r = \left[1 + \frac{1.5\,\phi}{1 - \phi/0.58}\right]^2$$

Note that because we are interested in the ratio of initial (e.g., that corresponding to a uniform concentration ϕ_0) and steady-state torques, the fluid viscosity μ_0, the angular velocity of the lower plate Ω, the cone angle α, and the radius of the plate R do not affect the problem. The absolute value of the torque will be functions of these values, however. These values, plus the particle radius a, will affect the time necessary to reach steady-state once shearing has begun, but they will not affect the steady-state distribution itself.

We anticipate that the value of K_r is about unity. Given this value, and given an average concentration (initial concentration before migration) $\phi_0 = 0.45$, do the following:

1). Produce a graph of the expected steady-state radial concentration profile $\phi(r)$ for $K_r = 1$ and compare it to the initial uniform concentration profile.

2). Calculate the ratio T/T_0 of the steady-state torque arising from this profile to that arising from a uniform concentration ϕ_0 at the same plate rotation speed.

3). If we wish to estimate K_r from this torque ratio to an accuracy of 10%, to what precision (relative accuracy) is it necessary to measure the initial and steady-state torques?

Remember, do not discuss this with classmates until after 5PM on Friday! You may talk to me or Jason, however. Good Luck!

Final Project Evaluation Criteria

The final project will be evaluated according to the following criteria:

50 points: Did you get the correct answer, or how close did you get?

10 points: Clarity of presentation of graphical results (e.g., labelled axes & graphs, etc.)

The remaining 40 points will be determined based on the quality of the program you used to generate the answer. Particular issues are:

10 points: Appropriateness of integration algorithm. While the Euler method is useful for figuring out what is going on, a well-implemented higher order algorithm is necessary to receive full credit.

10 points: Dealing with the discontinuity at the boundary between the flat truncated part of the cone and the rest of the cone. There are a number of ways this can be dealt with, including dealing with the inner region analytically. The key is to prevent relatively large errors from creeping into your answers.

10 points: Automatic determination of the concentration at the center. While manual shooting methods are fine for getting an idea of what the concentration at r = 0 is, some well-implemented automatic method is required for full credit.

10 points: Clarity of programming. Did you use the global command in a restrained manner (remember that the global command is a meat axe - while it is extremely useful, it should also only be used when it significantly simplifies the program)? Is your program easy for us to follow? Is it well commented? Often you will have to pass your programs on to someone continuing your project in the future. If it is not clear and well commented, a program is useless to others and loses much of its value.

I hope that these criteria will make it easier for you to prepare the final versions of you program.