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Cheg 258 Second Hour Exam **3/30/94**
Closed Book and Closed Notes

Please solve the exam on the sheets provided. Use the blue books as scratch paper only!

Each problem counts equally. Attempt all of the problems. You probably should do the easy ones first!

Problem 1. Index notation:

A. (10 points) Each of the following expressions are one of the following:

a. a scalar
b. a vector (first order tensor)
c. a second order tensor
d. a mistake

Write the appropriate letter in the blank after each of these expressions:

1. x_i _____________ 2. $x_i y_j$ ____________ 3. $x_i y_i$ ____________

4. $x_i y_j z_j$ ______________ 5. $x_i y_i z_i$ _____________

B. (10 points) The Navier-Stokes equations describe the convection and diffusion of momentum. These equations are given in vector notation by:

$$\frac{\partial \underset{\sim}{u}}{\partial t} + \left(\underset{\sim}{u} \cdot \underset{\sim}{\nabla}\right) \underset{\sim}{u} = - \underset{\sim}{\nabla}\, p + \mu \left(\underset{\sim}{\nabla} \cdot \underset{\sim}{\nabla}\right) \underset{\sim}{u} + \rho\, \underset{\sim}{g}$$

Recognizing that the divergence operator $\underset{\sim}{\nabla}$ is a vector and is given in index notation by:

$$\underset{\sim}{\nabla} \equiv \frac{\partial}{\partial x_j}$$

Rewrite the Navier Stokes equations in index notation in the space provided below.

C. (5 points) The trace of a matrix $\underset{\approx}{A}$ is defined as the sum of its diagonal elements. How would I write this using index notation? Put your answer below. (There are a couple of acceptable ways of doing this - just pick one)

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Problem 2. QR factorization (25 points):

Recall that the Householder transformation which reduces a vector a_i is given by:

$$P_{ij} = \delta_{ij} - 2\,\frac{v_i\ v_j}{v^2}$$

where:

$$v_i = a_i \pm \alpha\ \delta_{i1}$$

Realizing this, perform a QR factorization on the matrix:

$$\underset{\approx}{A} = \begin{bmatrix} 7 & 2 & 2 \\ 0 & 3 & 2 \\ 0 & 4 & 1 \end{bmatrix}$$

Write down both Q and R in the space below.

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Problem 3. Quadrature:

Suppose that we have two Simpson's rule estimates of the integral of some function f(x) over the interval [a,b]. One of the estimates used n panels, and the other used 2n panels. These estimates are given by I_n and I_{2n}, respectively. We wish to combine these two results to obtain a new estimate which is more accurate than either.

A. (10 points) What combination of I_n and I_{2n} will achieve this?

B. (10 points) What is the polynomial degree of the resulting combination (the new quadrature rule)?

C. (5 points) Gaussian quadrature weights w_i^* and nodes x_i^* are defined on the interval [-1,1]. What are the corresponding weights and nodes on the interval [a,b]?

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Problem 4. Statistics:

An automatic food bagging and boxing machine is loading boxes with crackers. The boxes are each supposed to hold 12oz of crackers, but analysis shows that the average weight of crackers in each box is actually 12.36oz with a population standard deviation of ±0.18oz. You may assume that they are normally distributed.

A. (10 points) What is the probability that any given box has less than 12oz of crackers in it? Use the table for a normal distribution given below.

B. (10 points) The boxes of crackers are packed 16 to a case. If all of the cracker boxes can be considered independent, what is the probability that the average of the weight of the boxes in a case is less than 12oz?

c. (5 points for real red-hot statisticians) What is the probability that none of the boxes in a case have a weight less than 12oz?

You may find the following table helpful:

x	P(x)
0	0.5
1	0.84134
2	0.97725
3	0.99865
4	$1 - 3.17 \times 10^{-5}$
5	$1 - 2.87 \times 10^{-7}$
6	$1 - 9.87 \times 10^{-10}$
7	$1 - 1.28 \times 10^{-12}$
8	$1 - 6.22 \times 10^{-16}$
9	$1 - 1.13 \times 10^{-19}$
10	$1 - 7.62 \times 10^{-24}$

where $P(x) = \int_{-\infty}^{x} Z(t)\,dt$

and $Z(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$

P(x) is known as the cumulative probability distribution. Z(x) is the Gaussian distribution.