Cheg 258 First Hour Exam **Date : 2/28/01**
Closed Book and Closed Notes **Name :**

Please solve the exam on the sheets provided. Use the blue books as scratch paper only !

Each problem counts equally. Attempt all the problems. You probably should do the easy ones first.

Problem 1. : **Linear Regression**

A stagnation flow towards a plate (the flow you get if you squirt a jet of water at a flat surface) causes the particles in the water to follow the trajectory:

$$y = \frac{y_0}{1 + \beta y_0 t}$$

where y_0 is the initial position of the particles (distance from the plate) and β is a measure of the strength of the flow. We want to calculate β using linear regression from the data:

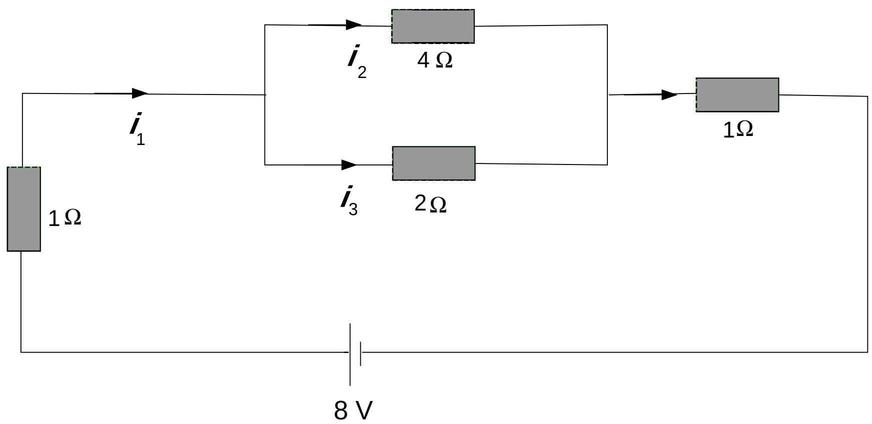
$$\mathbf{t}\ (sec) = [\ \mathbf{1,\ 2,\ 3,\ 4}\ ]$$
$$\mathbf{y}\ (\mu m) = [\ \mathbf{1.5,\ 0.8,\ 0.6,\ 0.4}\ ]$$

Set the problem up in the matrix form used in the class, explicitly identifying $\mathbf{A}$ and $\mathbf{b}$, and providing definitions for the elements of $\mathbf{x}$.

Hint: You will have to rework the model so that it is linear in the modeling parameters.

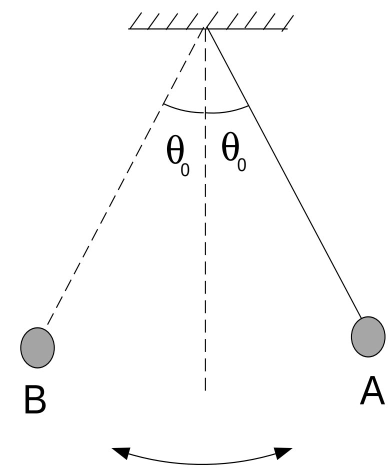
Problem 2. : **System of Linear Equations**

Given below is a simple electrical circuit for which the currents in the various segments are to be determined. Set up the problem as a system of linear algebraic equations of the form $\mathbf{Ax} = \mathbf{b}$ and solve for the unknowns $\mathbf{i_1}$, $\mathbf{i_2}$ and $\mathbf{i_3}$ using **Gaussian elimination**. Remember that the voltage drop along any segment is simply given by the product of the current and the resistance in the segment. Also, a conservation of electrical charges require that the current entering any junction must be equal to the current leaving the junction.



Problem 3. : **Error Analysis**

From elementary physics it can be shown that the angular position of a simple pendulum (see figure below) oscillating between points **A** and **B** can be described using the expression $\theta = \theta_0 \cos{(\omega t)}$. Here ω is the natural angular frequency of the oscillations given by $\omega = \sqrt{g/L}$ where L is the length of the pendulum and g is the accleration due to gravity. If the pendulum is released from position **A** at $\mathbf{t = 0}$, what is the error (standard deviation) in estimating the angular position of the pendulum at time $t = \pi/(2\omega)$ given that $\theta_0 = \pi/12\, rad$ and the length of the pendulum is only known to an accuracy $\mathbf{L = 3cm \pm 0.1cm}$ (1σ). Assume $g = 980\ cm/s^2$.



Given the above conditions how would you estimate the time for which there is a 97.5% probablity that the pendulum would have reversed its direction of motion. Remember that the pendulum reverses its direction of motion when $\theta = -\ \theta_0$.

Problem 4. : **Norms and Matrices**

A. Consider the vector $\mathbf{x}$ = (-4, -4, 2). Calculate the norms:

1. 1-norm
2. 2-norm
3. ∞-norm

B. Prove that the 2-norm condition number of an orthogonal matrix is unity.

C. Using the concept of singular value decomposition and the properties of orthogonal and diagonal matrices, prove that the 2-norm of the product $\mathbf{A^T A}$ is the square of the 2-norm of $\mathbf{A}$. Don't make this too hard, because it's not ...