Review 2 (2nd Exam Study Questions) Due 3/29/00

Problem 1. Non-linear minimization:

A. Derive Newton's method for finding the local extremum (minimum or maximum) for a function f(x) starting with some initial guess x_0

B. By using Taylor series analysis, prove that this method converges quadratically when close to the actual extremum x^*, provided that the second derivative at this point is non-zero.

C. Apply this method to find the minimum of the function $f(x) = x^3 - x$ with the initial guess $x_0 = 1/2$. Do three iterations.

Problem 2. Root Finding:

A.

1. Under what conditions is the bisection method guaranteed to converge? (be brief but complete)

2. What is the **rate** of convergence of each of the following methods?

a. Bisection ____________

b. Newton's Method _______________

c. Secant Method _______________

3. If a function locally behaves as $f(x) \sim (x - c)^3$ which of the above techniques will converge the fastest?

B. Derive the secant rule for finding the root to some function f(x). Don't try to compute the rate of convergence, however -- simply show where the rule comes from.

Problem 3. Regression Error Propagation:

Suppose you are asked to fit a set of data $b_i(t_i)$ to a model of the form:

$$b(t_i) = x_1 \phi_1(t_i) + x_2 \phi_2(t_i) + x_3 \phi_3(t_i)$$

where the $\phi_j(t_i)$ are the modelling functions and the x_j are the modelling parameters. You are to determine the best fit values of the parameters x_j (set up the regression problem in matrix form) and then to compute the function $y = f(\underset{\sim}{x})$. If we have n data points, show how we may compute the random error in y. I want equations in matrix form here: be as specific as possible, and state all of your assumptions.

Problem 4. Statistics:

An automatic food bagging and boxing machine is loading boxes with crackers. The boxes are each supposed to hold 12oz of crackers, but analysis shows that the average weight of crackers in each box is actually 12.36oz with a population standard deviation of ±0.18oz. You may assume that they are normally distributed.

A. What is the probability that any given box has less than 12oz of crackers in it? Use the table for a normal distribution given below.

B. The boxes of crackers are packed 16 to a case. If all of the cracker boxes can be considered independent, what is the probability that the average of the weight of the boxes in a case is less than 12oz?

C. What is the probability that none of the boxes in a case have a weight less than 12oz?

You may find the following table helpful:

x	P(x)
0	0.5
1	0.84134
2	0.97725
3	0.99865
4	$1 - 3.17 \times 10^{-5}$
5	$1 - 2.87 \times 10^{-7}$
6	$1 - 9.87 \times 10^{-10}$
7	$1 - 1.28 \times 10^{-12}$
8	$1 - 6.22 \times 10^{-16}$
9	$1 - 1.13 \times 10^{-19}$
10	$1 - 7.62 \times 10^{-24}$

where $P(x) = \int_{-\infty}^{x} Z(t)\,dt$

and $Z(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$

P(x) is known as the cumulative probability distribution. Z(x) is the Gaussian distribution.