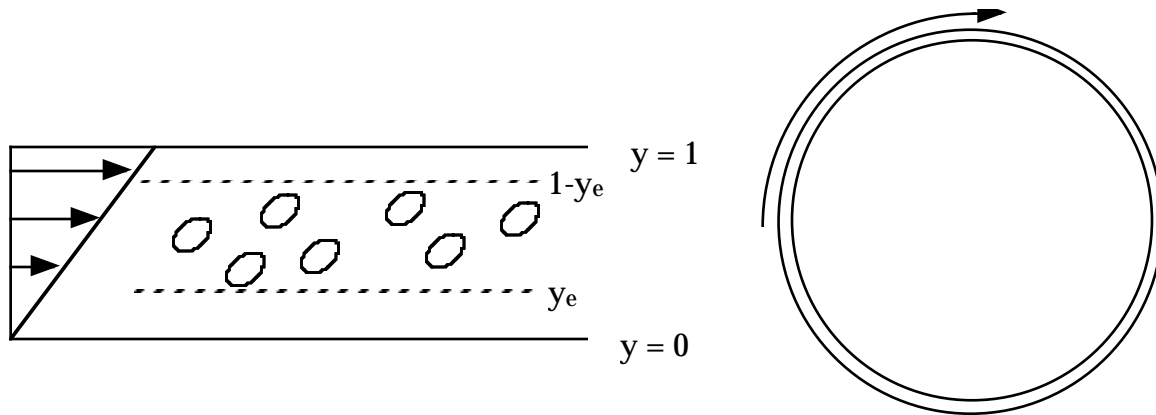


Measurement of the Gradient Diffusivity of a Dilute Emulsion

There are many examples of suspensions of liquid drops or other deformable particles in biological and chemical systems. Perhaps the most vital example is blood - which consists of about 30% by volume deformable red blood cells. The chief example in the chemical process industry is the water-oil emulsion which often forms when oil is first pumped from the ground - removing the water from the oil is a difficult and expensive process. A key problem which is as yet poorly understood is the distribution of drops in a bounded shear flow such as flow through a tube, a channel, or an artery. It has long been recognized that isolated drops will tend to migrate away from bounding walls due to the hydrodynamic interaction between the deformed drop (stretched out and oriented by the shear flow) and the wall. In a simple shear flow such as that depicted below, the drop will tend to migrate into the center of the sheared region between the plates.

If there is an emulsion - a collection of drops - the interaction between the drops will produce a dispersion which tends to resist the accumulation of drops at the centerline of the flow. At steady-state, this dispersion will exactly balance the inward migration velocity, and a steady concentration distribution will result. While the inward drift velocity is well known, the dispersivity (or gradient diffusivity) arising from both shear and droplet interaction has never been measured. It is this quantity that we wish to examine in this final project.

Consider the simple shear flow (plane Couette flow) depicted below. The shear flow is produced in the narrow gap between two concentric cylinders through the rotation of the outer cylinder. In this experiment the space between the cylinders is filled with a viscous fluid to which we have introduced a small quantity (1% by volume) of a second, less viscous phase. The mixture is sheared at a high shear rate to break up the second phase into small drops about $600\mu\text{m}$ in diameter. We also add a very small amount of the second phase which has been dyed to act as tracers. By videotaping the emulsion under shear we can measure the velocity of the dyed drops and, from the known fluid velocity profile in the gap between the cylinders, determine their radial position. The result of this experiment is thus a list of radial drop locations for the tracers. This list is given in the file linked into today's notes. You shall use this list to determine the droplet concentration distribution and the corresponding dimensionless diffusion coefficient.



The equations governing droplet drift and diffusion are complex, however the steady-state concentration distribution for a dilute emulsion is governed by the simple first order differential equation:

$$\frac{d\phi}{dy} = K \left(\frac{1}{y^2} - \frac{1}{(1-y)^2} + 2(1-2y) \right) ; \phi > 0$$

where the parameter K is a dimensionless ratio of the drift velocity to the diffusion coefficient, and also involves parameters such as the average concentration of droplets in the gap, the shear rate, the capillary number (a dimensionless parameter which determines the degree of drop deformation), the gap width, the viscosity ratio of the two fluids, etc. Basically, if you know K and if you also know all of your experimental conditions, you can calculate the diffusivity. It is your job to determine K from the distribution data.

An intriguing feature of droplet migration in a bounded shear flow is that the drops are completely eliminated from zones next to each wall. The concentration distribution is thus given by:

$$\phi = \begin{cases} 0 & 0 < y < y_e \\ \phi(y) & y_e < y < 1 - y_e \\ 0 & 1 - y_e < y < 1 \end{cases}$$

where only the concentration in the middle region $[y_e, 1-y_e]$ is governed by the differential equation. Unfortunately, the edge of the emulsion layer is an unknown function of K . We may develop an implicit relationship for y_e , however, by conservation of the total number of droplets:

$$\int_{y_e}^{1-y_e} \phi \, dy = 1$$

The theoretical problem thus reduces to the solution to two simultaneous integrations, which can be performed either numerically or (due to the simplicity of the governing differential equation) even analytically. The value of y_e for a given K , however, must be determined using some sort of non-linear root finding method.

The experimental concentration distribution can be produced in a couple of ways. One approach is to divide up the region $[0,1]$ into a large number of bins and determine the number of droplets observed in each bin. Normalizing this number by the total number of drops observed and the number of bins yields the concentration profile, which can be compared to the predicted profile for a particular value of K . An alternative approach is to plot up the cumulative probability distribution of finding a drop in the region $[0,y]$ such as was done in example 13 earlier this semester. This can be compared to the integrated concentration profile for a particular value of K . In either case, you can use the deviation between the measurements and the model to determine the best fitted value of K . The uncertainty in K can be determined using the bootstrap method demonstrated in example 18a, ideally suited to this problem because of the use of random tracers as a measure of the concentration distribution.

What to do:

- 1). Write a routine which determines both the concentration distribution and the integrated concentration distribution from the model as a function of K . This will involve use of both an integrator (if you do the integrations numerically) and a root finding routine for determination of the shooting parameter. Canned routines work fine, although you can write your own if you wish.
- 2). Take the data and generate both the experimental concentration distribution and the integrated concentration distribution.
- 3). Using a least squares (non-linear regression) process, determine the optimum value of K for the data. If you use a numerical approach to determining the model concentration distribution you will have to do some interpolation of the model results to make the comparison between model and data.
- 4). Using the bootstrap method, determine the uncertainty in the fitted value of K .

What to turn in:

- 1). Your program, which should be well commented and readable.
- 2). Your output, including the optimum value of K and an estimate of the error in K .
- 3). Two graphs, one comparing the experimental and best fit cumulative probability distribution, and one comparing the experimental and best fit concentration distribution. The second graph should have error bars for the experimental concentration estimate for each bin. You will find the matlab plotting routine "errorbar" to be useful here. Remember that because we are using tracers, the number of drops in each bin is governed by the Poisson distribution.

Final Project Evaluation Criteria:

The final project will be evaluated according to the following criteria:

30 points: Did you get the correct answer for K, or how close did you get?

20 points: Were you able to correctly estimate the error in K?

20 points: Quality of your algorithms. Did you integrate the concentration and cumulative density as a vector rather than sequentially? Did you interpolate for each point in the comparison between model and data efficiently? Did you use an appropriate number of points in the bootstrap method? Did you calculate the error in the experimental concentration distribution correctly? This is not an exclusive list, just some of the ideas to consider in writing a high quality program.

20 points: Clarity of programming. Did you use the global command in a restrained manner (remember that the global command is a meat axe - while it is extremely useful, it should also only be used when it significantly simplifies the program)? Is your program easy for us to follow? Is it well commented? Often you will have to pass your programs on to someone continuing your project in the future. If it is not clear and well commented, a program is useless to others and loses much of its value.

10 points: Clarity of presentation of graphical results (e.g., labelled axes & graphs, legend, title, etc.)

I hope that these criteria will make it easier for you to prepare the final versions of your program. Remember, do not discuss this with classmates until after 6:00PM on Thursday! You may talk to me or the TA's, however. Good Luck!