Name: _____

Cheg 258 Third Hour Exam **4/27/94**
Closed Book and Closed Notes

Please solve the exam on the sheets provided. Use the blue books as scratch paper only!

Each problem counts equally.

Problem 1. Statistics: Boxes of crackers again. You are a quality control engineer for a plant that boxes up crackers. Each box is supposed to hold 12oz of crackers. As the quality control engineer, you are supposed to insure that there is no more than a 2.3% probability that the average weight of the boxes in a case of 16 boxes is less than 12oz. Past experience shows that the packaging machine has a one standard deviation error of 0.24oz. Using this information, answer the following questions:

A. (10 points) If the weight of each of the boxes in a case is independent, determine what the minimum target weight of the packaging machine should be (how much more than 12oz).

B. (10 points) If all the boxes in a case have a covariance of one (perfect covariance), what would be the minimum target weight?

C. (5 points) Which scenario is more reasonable? Justify your answer in one sentence.

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Problem 2. Root Finding:

A. (10 points):

1. Under what conditions is the bisection method guaranteed to converge? (be brief but complete)

2. What is the **rate** of convergence of each of the following methods?

a. Bisection _____

b. Newton's Method _____

c. Secant Method _____

3. If a function locally behaves as $f(x) \sim (x - c)^3$ which of the above techniques will converge the fastest?

B. (15 points): Derive the secant rule for finding the root to some function f(x). Don't try to compute the rate of convergence, however -- simply show where the rule comes from.

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Problem 3. Integration:

A. (10 points) For each of the following rules, write down whether they are implicit or explicit and the **local** order of the error in step size.

Integration Rule	Implicit? (Y/N)	**Local** Error (order in step size h)
Euler Method	_____	_____
Backward Euler	_____	_____
Trapezoidal Rule	_____	_____
Two-Stage Runge Kutta	_____	_____
Four-Stage Runge Kutta	_____	_____

B. (15 points) **Derive** the error propagation formula for the Backward Euler rule (use the back of the sheet if necessary).

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Problem 4. Systems of Equations:

A. (10 points) The velocity profile due to a heated wire (this is one of the experiments in senior lab) is governed by the pair of coupled non-linear ODE's given below. Write down the equivalent set of first order coupled ODE's.

$$f''' = -g + \frac{1}{Pr}\left[\frac{1}{5}(f')^2 - \frac{3}{5} f f''\right]$$

and

$$g' = \frac{-3}{5} f g$$

For those of you who are interested, f ' is the dimensionless vertical velocity (f is the streamfunction) and g is the dimensionless temperature. Pr is the Prandtl number, a constant determined by the fluid used in the experiment.

B. (10 points) Write out the Jacobian matrix for this set of equations.

c. (5 points) Briefly explain how you would determine the stability of this set of differential equations.