CBE 30356 Transport II
Problem Set 5
Due via Gradescope, 11:55 PM 2/23/23

1). As has been repeatedly emphasized in class, unidirectional flow of an incompressible fluid is the same as heat transport in solids: both the equations and the solutions are the same! In this problem consider a long vertical tube of radius a filled with a viscous liquid of density ρ and viscosity μ, initially at rest. At time $t = 0$ you "let it go" (e.g., think of a filled straw when you take your finger off the end!) and it starts to move. Solve the following:

a. Write down the governing equations and render them dimensionless.

b. Determine the asymptotic solution at long times, and calculate the shear stress at the wall $r = a$.

c. Set up the problem for the decaying solution and get the lead eigenvalue, which determines how fast the asymptotic solution is reached. Note: do this numerically unless you particularly like to work with Bessel functions…

d. Plot up the wall shear stress as a function of time, including the long time asymptote.

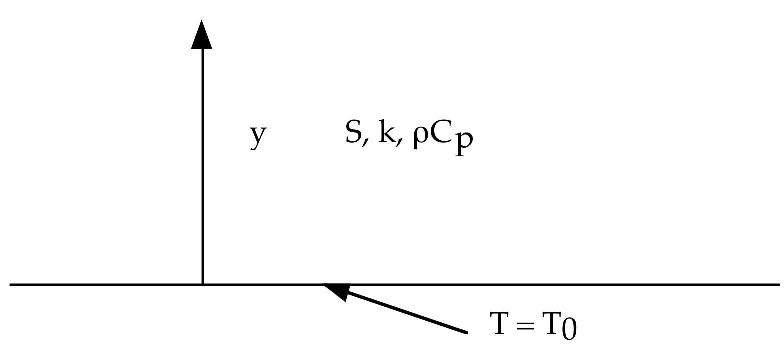
Hint: Think about the relationship between this problem and the one you solved for homework last week…

2). For very short times it is actually more appropriate to look at this sort of problem as a transient boundary layer. Consider the semi-infinite space depicted below. The initial temperature is T_0, and the wall is maintained at a temperature T_0 at all times. There's a heat source S in the material, however, and that means that it is getting hotter in time (but not at $y = 0$). Far from the lower wall the temperature gradient should vanish (even though it is getting warmer in time).

a. Write down the equations and boundary conditions governing this problem and render them dimensionless.

b. Using affine stretching, show that the problem admits a self-similar solution, and obtain the similarity rule and variable in canonical form.

c. Determine the heat flux at the wall to within an unknown O(1) constant.



3). For the problem above, get the transformed differential equation and, using the shooting method, get the constant. Plot up the dimensionless scaled temperature profile (e.g., $f(\eta)$).

4). Consider the problem discussed in class where there was a periodic heating of a semi-infinite medium. Suppose instead of there being a periodic temperature at the bottom we impose a periodic heat flux $q_y = q_0 \sin(\omega t)$. Solve for the amplitude of the temperature at the wall as a function of frequency and the properties of the material.