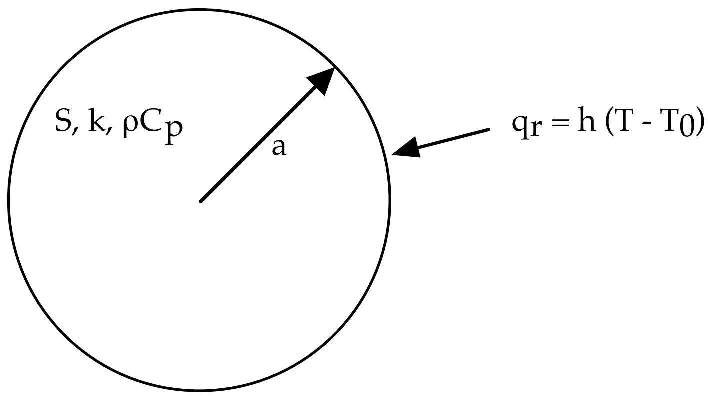
CBE 30356 Transport II
Problem Set 4
Due via Gradescope, 11:55 PM 2/16/23

1). The first three problems deal with the heated rod depicted below. Consider a rod of radius a, thermal conductivity k, and volumetric heat capacity ρC_p. Initially the rod is at equilibrium with its surroundings at a temperature T_0. At time t = 0 we have a source of energy per unit volume S which warms things up. Heat is lost to the surroundings governed by a heat transfer coefficient h. So:



a. Write down the differential equation and boundary conditions governing this problem.

b. Render them dimensionless and show that the only parameter which appears in the dimensionless equations is the Biot number.

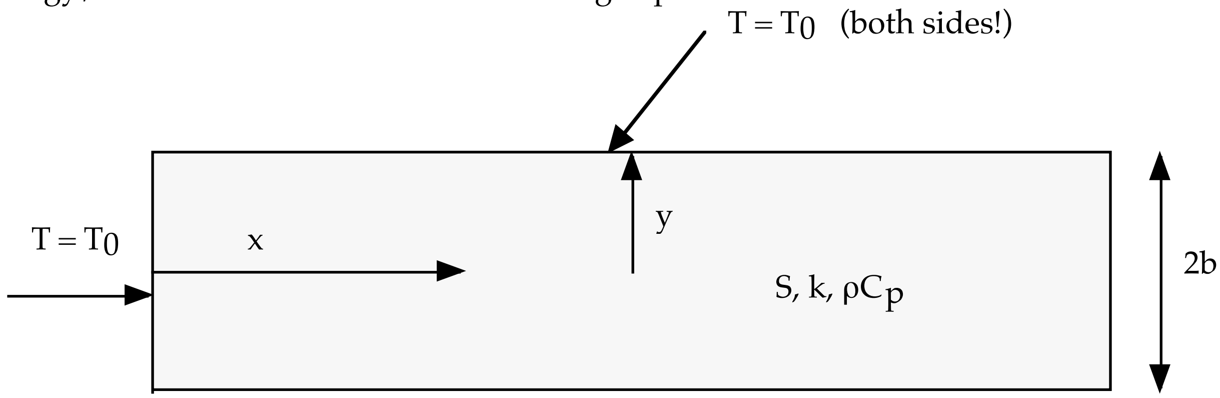
c. Solve for the asymptotic solution at long times.

d. Plot up the temperature profile for Bi = 0.1, 2, and ∞ on the same graph. Note that the Bi = ∞ parabola is the same as Poiseuille flow through a tube!

2). OK, now for the transient! For the problem above, solve the transient problem numerically and plot up the area average temperature of the rod as a function of time. Use a Biot number of 2. Note that this will require using orthogonality, integration, etc., for getting the coefficients, but the example programs will be a lot of help! This problem actually admits an analytic solution (in terms of Bessel functions) but it is pretty messy and a whole lot easier to get numerically!

3). Solve problem 2 again, only this time do it using the finite difference marching solution described in the notes. There are a couple of issues here: First, because you are in the cylindrical coordinate system your (second derivative) conductive term looks a little different. The notes on how the matrix was set up in slsolve.m will show you what to do there. Second, your boundary condition at $r^* = 1$ now involves both the function and the derivative. Again, examining the code for slsolve.m will show you how to deal with that condition. Solve the problem for a Biot number of 2, calculate the average temperature of the rod, and graphically compare it to your answer for question 2.

4). Consider the rectangular slab (semi-infinite in the x-direction) depicted below. The surface at x = 0 and y = ±b are maintained at a temperature T0. There is a source of energy/volume S in the slab that heats things up.



a. Write down the differential equation and boundary conditions that govern this problem and render them dimensionless.

b. We are only interested in the asymptotic solution at long times here, so you can throw out the transient term! Solve for this steady state solution when you are far from the end x = 0 (e.g., it will just be a function of y, and should be *very* familiar!).

c. Now for the harder part. Subtracting off the solution you obtained in part b, solve for the temperature distribution as a function of x and y using the same sort of separation of variables solution you did in the last homework (there will be different constants, though, which you will need to obtain – Wolfram alpha works well there!). Don't forget to add the large x solution back in!

d. Plot up the depth (y-direction) average temperature as a function of x.

Note: while this problem in heat transfer isn't all that important, the mathematically identical problem for unidirectional flow in a rectangular duct very much is! It turns out that the slowdown of fluid velocity near the side walls (corresponding to the lower depth averaged temperature you are plotting in this problem) is -the- dominant source of dispersion in a typical microfluidic "Lab on a Chip" system and has large implications to chip design.