CBE 30356 Transport II
Problem Set 12
Due via Gradescope, 11:55 PM 5/2/23

1). Concentration polarization in simple shear. It was demonstrated in class that dead-end filtration resulted in a continually increasing concentration polarization layer at a membrane surface. In practice this is mitigated by cross-flow filtration, resulting in a *steady, position dependent* concentration polarization layer. Consider as a model of such a system a tangential flow given by a simple shear flow $u_z = \dot{\gamma}\, y$ where y is the distance normal to the membrane surface at y = 0. The membrane filters out a species with diffusivity D at a constant permeate flux velocity $u_y|_{y=0} = -u_0$. The upstream concentration (and that far from the membrane in the y direction) is a constant value c_0, and the boundary condition at the membrane itself is the no-flux condition, with diffusion balancing convection.

a. Write down the convective diffusion equation and boundary conditions for this problem, and render the equations dimensionless.

b. Show by scaling that diffusion in the flow (z) direction is negligible.

c. Set up the equation for the asymptotic concentration distribution as a function of z and y. Your dimensionless concentration distribution will look *a lot* like the transient dead-end filtration problem (in fact, if the velocity profile were flat rather than the more realistic simple shear it would be mathematically identical!).

d. Solve the equation. Note that (just as in the Nusselt-Graetz constant heat flux problem) you get the last unknown coefficient by applying a mass balance condition at z = 0 (the entering flow average concentration has to be c_0).

2. A glass surface is negatively charged with surface potential -100mV. It is in a 1 mM aqueous NaCl solution at 298°K. Answer the following:

a. Calculate the Debye layer thickness.

b. Calculate the surface charge density.

c. Using the relationship between the potential and ion charge concentration provided by the Boltzmann distribution, plot up the distribution of Na^+ and Cl^- counterions in the diffuse layer and the net free charge (e.g., the difference between the two!).

d. By integrating the free charge distribution, show that the entire system (charged membrane plus diffuse layer) satisfies the electro-neutrality condition (e.g., no net charge).

e. A tangential electric field (e.g., *along* the surface) of 100 V/cm is applied. This results in a force/volume on the net free charge in the double layer, and is resisted by viscous drag. Balancing these two mechanisms, calculate the *electroosmotic velocity* resulting from the field.