Cheg 258 Third Hour Exam **5/8/00**
Closed Book and Closed Notes

Problem 1). Adaptive Step Size Control:

a. The Euler method is a one stage Runge Kutta rule specified by:

$$k_1 = h\, f(t_n, y_n)$$

$$(y_{n+1})^{1s} = y_n + k_1$$

This can be extended to a two stage rule by adding the second stage:

$$k_2 = h\, f(t_n+h, y_n+k_1)$$

$$(y_{n+1})^{2s} = y_n + (k_1 + k_2)/2$$

Describe how you can use these two rules to determine the optimum step size to achieve a tolerance of τ in an adaptive integration scheme. Be as specific as possible (e.g., an equation is preferred). Remember that the error estimate will be dominated by the one stage rule!

b. Apply this method to determine the step size required for the first step in integrating the equation $y' = 1 - 2y$; $y(0) = 0$ to a tolerance of 0.01.

Problem 2). Systems of Equations:

a. The velocity profile due to a heated wire (this is one of the experiments in senior lab) is governed by the pair of coupled non-linear ODE's given below. Write down the equivalent set of first order coupled ODE's.

$$f''' = - g + \frac{1}{Pr}\left[\frac{1}{5}(f')^2 - \frac{3}{5} f f''\right]$$

and

$$g' = \frac{-3}{5} f g$$

For those of you who are interested, f' is the dimensionless vertical velocity (f is the streamfunction) and g is the dimensionless temperature. Pr is the Prandtl number, a constant determined by the fluid used in the experiment.

b. Write out the Jacobian matrix for this set of equations.

c. Briefly explain how you would determine the stability of this set of differential equations.

Problem 3). Error Propagation / Optimization:

A chocolatier can make either chocolate covered cherries, chocolate covered nuts, or a mixture of the two. She makes a greater profit (on average) for the cherries than for the nuts, but the market is a bit more volatile. In this problem we wish to determine the optimum fraction of cherries and nuts he should make. The average figures are as follows:

$$\mu_c = \$3.00 \text{ / box} \qquad \sigma_c^2 = 1.25 \text{ (\$/ box)}^2$$
$$\mu_n = \$2.50 \text{ / box} \qquad \sigma_n^2 = 0.25 \text{ (\$/ box)}^2$$
$$\sigma_{cn}^2 = 0.25 \text{ (\$/ box)}^2$$

Note that there is some covariance between the profit on cherries and nuts. On the following I want equations and numbers, but words will yield partial credit!

a. Given that she makes a fraction x of chocolate covered cherries and (1-x) of nuts, calculate her expected profit per box and the standard deviation of that profit.

b. If she wishes to maximize her profit under adverse economic assumptions (e.g., she wishes to maximize her profit at the 85% (1σ) confidence level), what should be her strategy? You don't actually need a calculator to solve this, although it might help.

Problem 4). Quadrature:

Accurately estimate the integral:

$$I = \int_0^{\pi} \cos(x)\ln(x)\,dx$$

using three-point Gaussian quadrature. Show all of your work.