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Cheg 258 Second Hour Exam **4/15/98**
Closed Book and Closed Notes

Please solve the exam on the sheets provided. Use the blue books as scratch paper only!

Each problem counts equally. Attempt all of the problems. You probably should do the easy ones first!

Problem 1. Root Finding.

A. Derive the rate of convergence of Newton's method for solving for the root of a function f(x).

B. Apply Newton's method to find a root of the polynomial $f(x) = x^3 - 2x$ starting from the initial guess $x_0 = 1$. Do two iterations (e.g., get x_2).

C. Use the secant method for the same function, starting with the two initial guesses $x_0 = 1$ and $x_1 = 2$. Do one iteration.

NAME ________________________________

Problem 2. Quadrature and Error:

A laboratory experiment requires measurement of the average temperature of a reactor over its width, e.g. the integral:

$$\overline{T} = \frac{1}{H}\int_0^H T(x)\,dx$$

Unfortunately, temperature measurements could only be made at $x_1 = H/2$ and $x_2 = H$. You are responsible for estimating $\overline{T}$ from these measurements (e.g., $T_1 = T(x_1)$, $T_2 = T(x_2)$).

A. If the measurements are highly accurate (in this case, if $\sigma_T << H\frac{dT}{dx}$), determine the most accurate way of estimating $\overline{T}$. I want a derivation here - justify your answer.

B. If the error in the measurements is very large such that $\sigma_T >> H\frac{dT}{dx}$, how does your answer change?

NAME ______________________________

Problem 3. Optimization and Error:

A farmer friend of yours is trying to figure out how to reduce the uncertainty in his farming income using futures contracts and has come to you for advice. A futures contract is an agreement to sell some quantity of crops at a preset price, with the crops and money actually being exchanged at harvest. This has the advantage of "locking in" current (usually average) prices, but has the disadvantage that if he doesn't actually produce as much as he has contracted to sell, he then has to purchase the difference on the open market at harvest time. Now for the problem:

The farmer produces x_1 bushels of corn, a random variable with a standard deviation of σ_{x_1}. The price he will get per bushel for this corn at harvest time is another random variable x_2 with a standard deviation of σ_{x_2}. Because prices tend to be high when crop yields are low, there is a negative covariance between these variables:

$$\sigma^2_{x_1 x_2} = -\lambda\, \sigma_{x_1}\, \sigma_{x_2}$$

where λ is a positive number between zero and one. The price he will get per bushel via the futures contract is a fixed value p, and the number of bushels sold in this contract is z, the variable you wish to determine. For the purposes of this problem we shall take the expectation value of the price at harvest time (e.g., $\mu_{x_2} = E(x_2)$) to be the same as the futures contract price p. You may use the following values:

$$E(x_2) = p = \$2.50 \ / \ \text{bushel} \qquad \sigma_{x_2} = \$1.00 \ / \ \text{bushel}$$

$$E(x_1) = 10{,}000 \ \text{bushels} \qquad \sigma_{x_1} = 2{,}000 \ \text{bushels}$$

$$\lambda = 0.5 \ \ (\text{e.g., } \sigma^2_{x_1 x_2} = -\$1{,}000)$$

A. Determine the farmer's expected revenue and standard deviation as a function of z. It is actually easier to do this with the variables first rather than substituting in the numbers right off.

NAME ______________________________

B. Determine the value of z which minimizes the uncertainty in the farmer's income. By how much is the uncertainty reduced over the case $z = 0$ (no futures trading)?

C. How might your recommendations change if the futures price p is less than the farmer expects to get at harvest (e.g., $p < E(x_2)$)? Please be brief.

D. (one point extra credit) There are 55 students in the class, and questions of the day are asked at random. How many students will be called on before the probability that a student gets called on twice exceeds 50%?