Cheg 258 Second Hour Exam **3/31/00**
Closed Book and Closed Notes

Please solve the exam on the sheets provided. Use the blue books as scratch paper only!

Each problem counts equally. Attempt all of the problems. You probably should do the easy ones (e.g., the last one) first!

Problem 1. Non-linear root finding:

A. Derive the secant method for finding roots to functions. Under what conditions does it fail to converge?

B. Apply this method to the function $f(x) = x^3 - 8$ choosing as initial points $x = 0$ and 1. Do two iterations.

Problem 2. Weighted Linear Regression:

In the laboratory you are attempting to fit a power law model of the form $y = \alpha x^{\beta}$ where α and β are the parameters you are trying to estimate. The data consists of measurements at just three values of x, however error estimates are provided as given in the table below.

A. Using weighted linear regression, show how you would obtain optimal estimates of α and β. Note that I am not interested in the numbers here, but rather if you know how to set the problem up. Define all matrices and vectors clearly. The measured data points are independent (no covariance).

B. Using the error estimates for the measured values rather than any scatter in the data, show how you can calculate the error in the fitted coefficient α.

x	y	σ_y
1	0.30	0.10
2	0.65	0.15
3	1.20	0.20

Problem 3). Optimization:

We examine the planning of an orchard of cherry trees. Suppose we have $20,000 to invest in planting cherry trees. We can plant a mix of sour (dessert) cherries and sweet cherries (the ones you like to eat). The sour cherry trees cost $20 each, and the sweet cherry trees cost $10 each. The sweet cherries, however, can be sold for $100 per tree per year when mature if fully pollinated, while the sour cherries can be sold for only $25 per tree when mature. Finally, the yield per tree of the sweet cherry trees depends on their proximity to the sour cherry trees, since they are not self-pollinating. If the number of sour trees is given by x_1 and the number of sweet trees is x_2, we shall take the yield (the fraction of the fully pollinated yield) of the sweet trees to be $x_1 / (x_2 + x_1)$.

Show how you would calculate the optimum number of sweet and sour cherry trees. Be as specific as possible, defining all functions and techniques employed.

Problem 4. Error Propagation and Statistics:

It is desired to calculate the radius of a sphere by measurement of its mass and knowledge of its density. The density of the material making up the sphere is reported to be:

$$\rho = 1.32 \pm 0.02 \text{ g/cm}^3$$

where the error is one standard deviation. How accurately do we have to measure the mass to get the radius to a precision of ±2% at the 95% confidence level?

(one point extra credit) There are 52 students in the class, and questions of the day are asked at random. If we ask a total of 80 questions of the day, what fraction of the class will never be called?