Cheg 258 First Hour Exam **2/24/95**
Closed Book and Closed Notes

Please solve the exam on the sheets provided. Use the blue books as scratch paper only!

Each problem counts equally. Attempt all of the problems. You probably should do the easy ones first!

Problem 1. Gaussian Elimination Error:

The -orthogonal- matrix **A** is given by:

$$\mathbf{A} = \begin{matrix} -0.1231 & 0.9045 & 0.4082 \\ -0.4924 & 0.3015 & -0.8165 \\ -0.8616 & -0.3015 & 0.4082 \end{matrix}$$

Compute the upper bound $||\Delta\mathbf{x}|| \;/\; ||\mathbf{x}||$ of the error in the solution of $\mathbf{A}\,\mathbf{x} = \mathbf{b}$ given that we have some bounded relative error in b of $||\Delta\mathbf{b}|| \;/\; ||\mathbf{b}||$. The norms are all 1-norms.

Problem 2. Finite Differences

A high order finite difference approximation for the first derivative of a function f(x) which is defined only for $x \leq x_0$ (e.g., up to the right edge of a domain) is given by:

$$f'(x_0) \approx [1.5 * f(x_0) - 2 * f(x_0 - h) + 0.5 * f(x_0 - 2h)] / h$$

a). What is the order in h of the algorithm error of this approximation?

b). Combining this algorithm error with numerical error in the calculation of f(x) determine the approximate optimum value of h and the minimum possible error.

Problem 3. Systems of Equations:

Suppose that you have a stock of red, yellow, and blue paints in your paint store, but - since primary colors are passe' - they are not selling well. A marketing survey (those guys in the business school have to do something, after all) has shown that the hot new colors are orange, purple, and green. You can make alot more money by getting rid of the excess red, yellow, and blue paints by recombining them to make the new colors. Your goal is to determine how much of each of the new paints you can make. Assume the following information:

Excess paints to be used:
red = 3 cans
blue = 6 cans
yellow = 5 cans

Mixing rules:
green = half blue and half yellow
orange = half red and half yellow
purple = half red and half blue

Set this problem up as a matrix problem of the form **A x** = **b** and solve it using gaussian elimination. Use the back of this sheet as well if you need more space.

Problem 4. Linear Regression:

An chemical undergoes spontaneous first order decomposition, and we want to determine the rate of decrease from a series of concentration observations. The concentration is modelled by the exponential decay:

$$C = C_0 * \exp(- \lambda t)$$

where C_0 is the initial concentration and λ is the decay constant. We want to calculate λ using linear regression from the data:

t(sec)	C ($x10^{-2}M$)
0	10.0
1	3.50
2	1.50
3	0.50
4	0.17

Set the problem up in the matrix form used in class, explicitly identifying **A** and **b**, and providing definitions for the elements of **x**.