Cheg 258 First Hour Exam **3/1/00**
Closed Book and Closed Notes

Please solve the exam on the sheets provided. Use the blue books as scratch paper only!

Each problem counts equally. Attempt all of the problems. You probably should do the easy ones first!

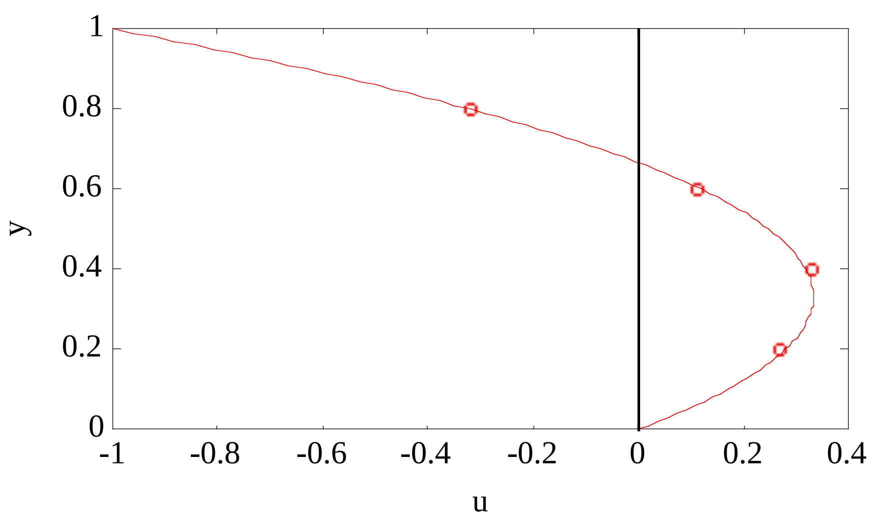
Problem 1. Linear Regression:

In an experiment done in my laboratory in undergraduate research a few years ago we looked at particle motion in Taylor-Dean flow. We won't worry about the details here, but we will examine this flow field. Suppose we have a velocity distribution which is described by the quadratic function:

$$u = c\, y\, (a - y)$$

where a and c are parameters characterizing the velocity profile. Using a probe, we measure the velocity at the following positions:

y	u
0.2	0.27
0.4	0.33
0.6	0.11
0.8	-0.32



We are interested in determining the point in the interior of the flow ($y = a$)where the velocity is zero. Show how you would calculate this using linear regression, and state any assumptions that you make. Explicitly define any matrices you use in the calculation.

Problem 2. Error Analysis:

What is the error in this value calculated in problem 1? Again, state all of your assumptions. I am not looking for a number, but rather I want to see all of the equations you use, and for you to be as specific as possible. Define all matrices as explicitly as you can.

Problem 3. Systems of Equations:

In your laboratory you are doing protein separations. You are using UV absorbance to measure the protein concentration. For dilute solutions, the protein absorbance is proportional to the protein concentration, and the total absorbance is just the sum of that resulting from each species independently. Suppose you have calibrated the absorbance at three different wavelengths for three protein species:

	Absorbance measurements (units = 1/(g/liter))		
Species \ λ	200 nm	300 nm	400 nm
BSA	.1	.2	.1
BHb	.1	.2	.3
Insulin	.1	.3	.2

If you measure the absorbance at 200nm, 300nm, and 400nm to be 0.06, 0.15, and 0.11, respectively, solve for the concentration of each of the species.

Set this problem up as a 3x3 matrix problem of the form **A x** = **b** and **solve it using gaussian elimination**. Use the back of this sheet as well if you need more space.

Problem 4. Gaussian Elimination Error:

a. The -orthogonal- matrix **A** is given by:

$$\mathbf{A} = \begin{matrix} -0.3015 & 0.8616 & 0.4082 \\ -0.9045 & -0.1231 & -0.4082 \\ -0.3015 & -0.4924 & 0.8165 \end{matrix}$$

Compute the upper bound $||\Delta\mathbf{x}|| \;/\; ||\mathbf{x}||$ of the error in the solution of $\mathbf{A}\ \mathbf{x} = \mathbf{b}$ given that we have some bounded relative error in b of $||\Delta\mathbf{b}|| \;/\; ||\mathbf{b}||$. The norms are all 1-norms.

b. What is the magnitude of the largest singular value of the matrix above? (Hint: you don't need any more space than that given below!)

c. (one point extra credit) There are 52 students in the class, and I will ask approximately 80 questions of the day over the course of the semester. What is the probability that you will be called on at least once?