CBE 20258 Computer Methods
Mid-Term Exam
March 3, 2016

You may have one page of notes for this exam

Problem 1. (20 points) Error Propagation. It's tricky to measure the diameter of a cylinder without calipers, but a student proposes to do it by using a ruler to measure the length, weighing it, and knowing what the density of it is. If we know the mass to a precision of 1% and the density to 0.5%, how accurately do we need to measure the length to get the diameter to a precision of 1% as well?

Problem 2. (20 points) Model Linearization. In the second project you used three values to determine the time at which a maximum correlation occurred. Here we try seeing what using five values an a bit of regression can do! Suppose we have five measurements of y_i at times t_i:

We are fitting this to the model $y = y_m - a\,(t - t_s)^2$, where the goal is to determine t_s.

a. Set up this problem as a linear regression problem (e.g., linearize it!), clearly identifying all variables and matrices used.

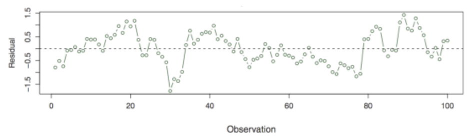
b. Show how you can calculate the matrix of covariance of the fitting parameters, and how these can be used to get the standard deviation of t_s, the quantity of interest.

c. Show how you can calculate the 95% confidence interval for t_s. (Hint: the t-table is given below)

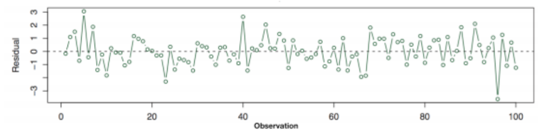
d. List three assumptions that can lead to the answer in part c being incorrect.

dof \ tcdf	0.0005	0.005	0.025	0.05	0.15	0.25
1	-636.6192	-63.6567	-12.7062	-6.3138	-1.9626	-1
2	-31.5991	-9.9248	-4.3027	-2.92	-1.3862	-0.8165
3	-12.924	-5.8409	-3.1824	-2.3534	-1.2498	-0.7649
4	-8.6103	-4.6041	-2.7764	-2.1318	-1.1896	-0.7407
5	-6.8688	-4.0321	-2.5706	-2.015	-1.1558	-0.7267
10	-4.5869	-3.1693	-2.2281	-1.8125	-1.0931	-0.6998
100	-3.3905	-2.6259	-1.984	-1.6602	-1.0418	-0.677
1000	-3.3003	-2.5808	-1.9623	-1.6464	-1.037	-0.6747

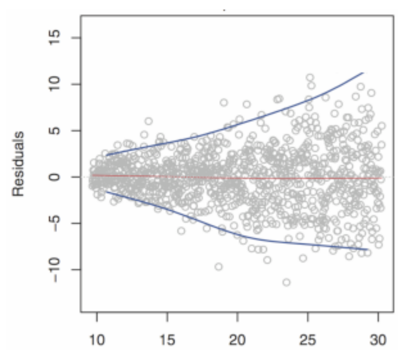
Problem 3. (20 points) Residuals. It is always very important to plot your residuals! In four separate unweighted regression problems the following residuals were observed (e.g., $r = Ax - b$). Based on the material covered in class, ***briefly*** (1) describe any problems with the linear regression model that resulted in these residuals, and ***briefly*** (2) suggest how you could use the techniques described in class to either improve the analysis and/or the calculation of the matrix of covariance of the fitted parameters.



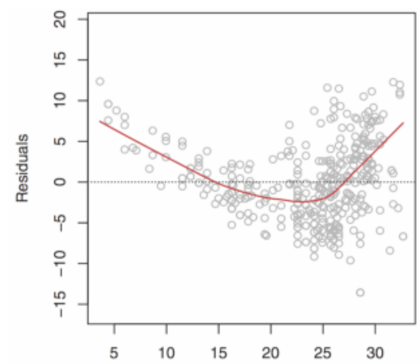
a.



b.



c.



d.

Problem 4. (20 points) Singular Value Decomposition

a. SVD is more computationally expensive than direct calculation of an inverse, but once you've got it, A^{-1} is fast to get as well ($O(n^2)$ additional operations, rather than $O(n^3)$). If $A = U \Sigma V^T$, show what A^{-1} is in terms of these matrices.

b. Why is SVD superior to QR or LU in solving an underdetermined set of equations?

c. Singular value decomposition is extremely valuable in the interpretation of the transfer problem $Ax = b$. Using SVD on matrix A, determine what pattern of output vector b requires the largest input vector x. What is this vector in terms of the 2-norm of b and the decomposition matrices U, Σ, and V?

Problem 5. (20 points) Systems of Equations. It is the end of the semester and you are about ready to pack up and go home. Looking around your kitchen you discover that you have four cups of butter, eight cups of flour, and four cups of sugar left over, as well as a pretty good selection of spices. While you are happy to take the spices home (they're pretty expensive these days!) you want to get rid of the rest of the ingredients by baking cookies for your friends. Looking through your copy of The Joy of Cooking, you find the following cookie recipes, with required ingredients:

Shortbread: 1c butter, 2c flour, 0.5c sugar
Roll cookies: 0.5c butter, 2.5 c flour, 0.5c sugar
Brandy snaps: 0.5c butter, 1c flour, 0.75c sugar

You find that you have all the extra spices these recipes call for. How many recipes of each type of cookie do you need to make to exactly use up all the butter, flour, and sugar?

Set this problem up as a 3x3 matrix problem of the form $A\,x = b$ and solve it.