CBE 20258 Computer Methods
Mid-Term Exam
March 6, 2014

You may have one page of notes for this exam

Problem 1. (20 points) Means and Variances. A student measures the capacity of a flask by filling it with fluid and weighing it. The following measurements are reported:

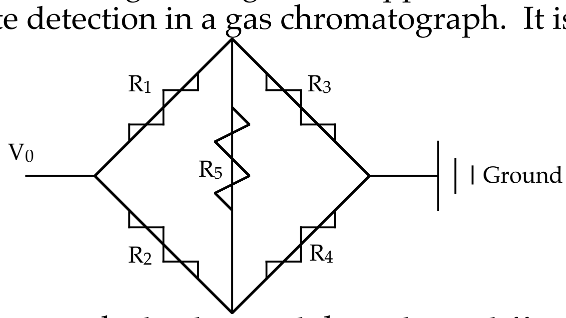
Mass = [28.7; 29.1; 29.0; 28.8]

a. What is the 90% confidence interval of the mean of these measurements? Show -all- of your work (e.g., only use calculator statistical functions to check your answers). The t-distribution table is given below.

b. The same student now repeats the measurement 10^4 times. What can you say (quantitatively and qualitatively) about the confidence interval in the mean?

dof \ tcdf	0.0005	0.005	0.025	0.05	0.15	0.25
1	-636.6192	-63.6567	-12.7062	-6.3138	-1.9626	-1
2	-31.5991	-9.9248	-4.3027	-2.92	-1.3862	-0.8165
3	-12.924	-5.8409	-3.1824	-2.3534	-1.2498	-0.7649
4	-8.6103	-4.6041	-2.7764	-2.1318	-1.1896	-0.7407
5	-6.8688	-4.0321	-2.5706	-2.015	-1.1558	-0.7267
10	-4.5869	-3.1693	-2.2281	-1.8125	-1.0931	-0.6998
100	-3.3905	-2.6259	-1.984	-1.6602	-1.0418	-0.677
1000	-3.3003	-2.5808	-1.9623	-1.6464	-1.037	-0.6747

Problem 2. (20 points) Electrical Circuits. A Wheatstone Bridge is an electrical circuit that is used in many chemical engineering sensor applications. As just one example, it forms the basis for solute detection in a gas chromatograph. It is depicted below:



A voltage V_0 is applied across the bridge, and the voltage differential between the two nodes in the middle is measured.

a. Using Ohm's Law V = I R, set up the system of five equations which the current through each of the five resistors must satisfy. Remember that current is conserved, and that voltage is a single value at any position. (e.g., voltage drop along any paths connecting the same two nodes must be identical)

b. Recast these equations in matrix form (e.g., Ax = b), clearly identifying A, b, and x.

Problem 3. (20 points) Error Propagation. A student is trying to determine the radius of a cylinder by filling it with a fluid to some height and weighing the result. We have the formula:

$$M = \pi R^2 H \rho$$

We have the measurements:

$$M = 301 \pm 1 \text{ g} ; H = 20 \pm 0.1 \text{ cm} ; \rho = 1.2 \pm 0.01 \text{ g/cm}^3$$

a. What is the value of the radius R and its standard deviation?

b. Which measurement is most important to improve to get a more precise result?

Problem 4. (20 points) Singular Value Decomposition

a. Explain how you can use SVD to compress an image. What vectors and values do you keep, and why does it work?

b. You are tasked to solve the square, non-singular problem A x = b. Is it better to use PLU decomposition or SVD decomposition? Explain your answer.

c. In an underdetermined problem there are an infinite number of solutions. What is the difference in the solution obtained by SVD and LU? Which approach does the Matlab "\" operator use in this case?

Problem 5. (20 points) Michaelis-Menten kinetics. In class we showed how you could use weighted regression to improve the Lineweaver-Burk linearization of the Michaelis-Menten rate expression. That's not the only linearization: a better-conditioned way of approaching the same problem is the Hanes-Woolf linearization:

$$\frac{[S]}{r} = \frac{1}{V_{\max}}[S] + \frac{K_m}{V_{\max}}$$

a. Suppose we have some measured rates r_i at substrate concentrations $[S]_i$. Show how you would solve for V_{max} and K_m using unweighted linear regression, identifying all matrices and vectors used in the calculation.

b. How do you determine the error in V_{max} and K_m in this case? What assumptions are you making? Be specific!

c. If the measured rates (rather than the "b" used in the linearized version) have an independent random error with the same standard deviation σ_r, it is more appropriate to use weighted linear regression. Show how you would use weighted linear regression to improve Hanes-Woolf, clearly identifying all matrices and vectors you use.