Instability of two-dimensional electron transport in ungated semiconductors

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Nomenclature

e	electron charge [C]
m	effective electron mass [kg]
n	electron concentration [1/m^3]
N_D	doping concentration [1/m^3]
L	length of semiconductor [m]
t	time [s]
T_e	electron temperature [K]
v	electron velocity [m/s]
V	potential [V]
x	coordinate [m]
ϵ_s	permittivity [F/m]
τ	momentum relaxation time [s]

Abstract

Hydrodynamic instability of a two-dimensional cavity flow of electrons in an ungated semiconductor is studied. The electron flow is driven by a voltage difference and electron-phonon interaction is neglected. A dynamical system starting from a simplified hydrodynamic model is constructed. The governing nondimensional equations are linearized about the steady-flow solution and its instability is studied through the spectrum of eigenvalues. The spectrum shows that the system is essentially unstable oscillatory, which is getting more stable as the imposed voltage drop is decreased. The frequencies of oscillation are found to be of the order of THz under specific circunstances.

1 Introduction

In the last years the study of mechanism of terahertz emissions or sources from semiconductor devices has become more attractive and important. Currently, compact, tunable and small THz sources are required to detect a wide range of process and chemical reactions to characterize and test a wide variety of chemical and biological systems.

There have been some research effort to study semiconductor devices as THz sources. Modelling of ballistic transport in a AlGaAs/InGaAs FET and its analogy with shallow water in order to explain the plasma oscillations and radiation emmisions has been reported [1]. This description is generalized and applied in high electron mobility transistors (HEMT) [2–4]. Instabilities in one-dimensional transport have been found using the hydrodynamic model [5]. Mechanism of current saturation in FET due to chocking of electron flow and plasma waves using the hydrodynamic model and its similarity with shallow water has been described [6]. Nonlinear oscillations due to ballistic transport in FET and analogous phenomena with hydraulic jumps are described using equations which coincide with those for shallow water [7]. Hydrodynamic model applied to a two-dimensional electron plasmas in FET, nonlinear dynamic response and how the boundary conditions are determinant in the nonlinear effects have been studied [8]. Current instabilities and plasma waves in an ungated two-dimensional electron layers using the hydrodynamic model and ist comparision with a gated electron layer were reported [9]. Transit-time effects in plasma instability related to the electron drift across the high field region in high elctron mobility transistors have been studied [10]. Plasma oscillations in both gated two-dimensional layers and stripes and high electron mobility transistors have been analized [11, 12].

Experimental research effort has been very important to detect terahertz radiation in recent years. Experimental technique using two-colour diode laser and its limitations have been described [13]. Sub-terahertz and terahertz radiation in silicon FETs and nanometer gate high electron mobility transistors due to plasma waves have been reported [14, 15]. New techniques and algorithms to determine the radiation spectrum of terahertz sources have been analized [16]. Theoretical and experimental studies have showed nonresonantand resonant detection of terahertz radiation in both Si MOSFETs and gated two-dimensional structures such as GaAs HEMT [17,18]. Under particular conditions, nanometer field effect transistor made of InGaAs/InAlAs can produce terahertz emission [19, 20].

Early analyses [1, 2] used a capacitance approximation and a gated piece of semiconductor. Instead, we include the Poisson's equation in an ungated two-dimensional cavity of semiconductor. As usual, we will also neglect the interaction between the electrons and the phonons, which is the cause of generation of heat.

2 Mathematical Model

Let us consider a two-dimensional piece of semiconductor as it is shown in Fig.1. The two-dimensional hydrodynamic model for electrons is [21].

$$\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}-\frac{e}{\epsilon_s}(n-N_D) = 0, \tag{1}$$

$$\frac{\partial n}{\partial t}+\frac{\partial (un)}{\partial x}+\frac{\partial (vn)}{\partial y} = 0, \tag{2}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}-\frac{e}{m}\frac{\partial V}{\partial x}+\frac{u}{\tau} = 0, \tag{3}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}-\frac{e}{m}\frac{\partial V}{\partial y}+\frac{v}{\tau} = 0. \tag{4}$$

The boundary and charge neutrality conditions are

$$V(0,y,t) = -V_0, \tag{5}$$

$$V(L,y,t) = 0, \tag{6}$$

$$n(0,y,t) = n_0, \tag{7}$$

$$v(x,0,t) = 0, \tag{8}$$

$$v(x,H,t) = 0, \tag{9}$$

$$\frac{\partial V}{\partial y}(x,0,t) = 0, \tag{10}$$

$$\frac{\partial V}{\partial y}(x,H,t) = 0, \tag{11}$$

$$\int_0^L\int_0^H n(x,y,t)\;dxdy = N_D LH. \tag{12}$$

Defining the aspect ratio as $R = H/L$, the nondimensional versions of Eqs. (1), (2), (3) and (4) are

$$\frac{\partial^2 V}{\partial x^2} + \frac{1}{R^2}\frac{\partial^2 V}{\partial y^2} - \alpha\,(n-\beta) = 0, \tag{13}$$

$$\frac{\partial n}{\partial t} + \frac{\partial (un)}{\partial x} + \frac{1}{R}\frac{\partial (vn)}{\partial y} = 0, \tag{14}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + \frac{1}{R}v\frac{\partial u}{\partial y} - \frac{\partial V}{\partial x} + \frac{\sqrt{\alpha}}{\gamma}u = 0, \tag{15}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + \frac{1}{R}v\frac{\partial v}{\partial y} - \frac{1}{R}\frac{\partial V}{\partial y} + \frac{\sqrt{\alpha}}{\gamma}v = 0, \tag{16}$$

and the boundary conditions and charge neutrality conditions are

$$V(0,y,t) = -1, \tag{17}$$

$$V(1,y,t) = 0, \tag{18}$$

$$n(0,y,t) = 1, \tag{19}$$

$$v(x,0,t) = 0, \tag{20}$$

$$v(x,1,t) = 0, \tag{21}$$

$$\frac{\partial V}{\partial y}(x,0,t) = 0, \tag{22}$$

$$\frac{\partial V}{\partial y}(x,1,t) = 0, \tag{23}$$

$$\int_0^1\int_0^1 n(x,y,t)dxdy = \beta, \tag{24}$$

where the voltage, charge density, length, velocity and time scales are V_0, n_0, L, H, $\sqrt{eV_0/m}$, and $L\sqrt{m/eV_0}$. The dimensionless groups are

$$\alpha = \frac{en_0L^2}{V_0\epsilon_s}, \tag{25}$$

$$\beta = \frac{N_D}{n_0}, \tag{26}$$

$$\gamma = \sqrt{\frac{\tau^2e^2n_0}{\epsilon_s m}}. \tag{27}$$

Substituting the Eq. (24) into the Eq. (13) by integration in the xy plane , we get

$$\int_0^1 \left(\frac{\partial V}{\partial x}(1,y,t) - \frac{\partial V}{\partial x}(0,y,t)\right)dy + \int_0^1 \left(\frac{\partial V}{\partial y}(x,1,t) - \frac{\partial V}{\partial y}(x,0,t)\right)dx = 0, \tag{28}$$

subtituting the Eqs. (22) and (23) into Eq. (28), we get

$$\int_0^1 \left(\frac{\partial V}{\partial x}(1,y,t) - \frac{\partial V}{\partial x}(0,y,t)\right)dy = 0, \tag{29}$$

3 Two-dimensional electron flow

We consider Eqs.(1), (2), (3) and (4) looking for a steady state with non-zero electric field and electron flow. The nondimensional steady state solution is

$$
\begin{aligned}
&(30) \quad \overline{V}(x,y) &=& \quad x-1, \\
&(31) \quad \overline{n}(x,y) &=& \quad \beta, \\
&(32) \quad \overline{u}(x,y) &=& \quad \gamma/\sqrt{\alpha}, \\
&(33) \quad \overline{v}(x,y) &=& \quad 0.
\end{aligned}
$$

For small perturbation with respect to a time-independent solution we have

$$
\begin{aligned}
&(34) \quad V(x,y,t) &=& \quad \overline{V}(x,y)+V'(x,y,t), \\
&(35) \quad n(x,y,t) &=& \quad \overline{n}(x,y)+n'(x,y,t), \\
&(36) \quad u(x,y,t) &=& \quad \overline{u}(x,y)+u'(x,y,t), \\
&(37) \quad v(x,y,t) &=& \quad \overline{v}(x,y)+v'(x,y,t),
\end{aligned}
$$

upon linearization, the nondimensional system of equations becomes

$$
\begin{aligned}
&(38) \quad \frac{\partial^2 V'}{\partial x^2}+\frac{1}{R^2}\frac{\partial^2 V'}{\partial y^2}-\alpha n' &=& \quad 0, \\
&(39) \quad \frac{\partial n'}{\partial t}+\frac{\gamma}{\sqrt{\alpha}}\frac{\partial n'}{\partial x}+\beta\left(\frac{\partial u'}{\partial x}+\frac{1}{R}\frac{\partial v'}{\partial y}\right) &=& \quad 0, \\
&(40) \quad \frac{\partial u'}{\partial t}+\frac{\gamma}{\sqrt{\alpha}}\frac{\partial u'}{\partial x}-\frac{\partial V'}{\partial x}+\frac{\sqrt{\alpha}}{\gamma}u' &=& \quad 0, \\
&(41) \quad \frac{\partial v'}{\partial t}+\frac{\gamma}{\sqrt{\alpha}}\frac{\partial v'}{\partial x}-\frac{1}{R}\frac{\partial V'}{\partial y}+\frac{\sqrt{\alpha}}{\gamma}v' &=& \quad 0.
\end{aligned}
$$

For this case, boundary and charge neutrality conditions are

$$
\begin{aligned}
&(42) \quad V'(0,y,t) &=& \quad 0, \\
&(43) \quad V'(1,y,t) &=& \quad 0, \\
&(44) \quad n'(0,y,t) &=& \quad 0, \\
&(45) \quad v'(x,0,t) &=& \quad 0, \\
&(46) \quad v'(x,1,t) &=& \quad 0, \\
&(47) \quad \frac{\partial V'}{\partial y}(x,0,t) &=& \quad 0, \\
&(48) \quad \frac{\partial V'}{\partial y}(x,1,t) &=& \quad 0, \\
&(49) \quad \int_0^1\left(\frac{\partial V'}{\partial x}(1,y,t)-\frac{\partial V'}{\partial x}(0,y,t)\right)dy &=& \quad 0.
\end{aligned}
$$

We are interested in finding a plasma wave-like behavior in the linearized system. Let

$$
\begin{aligned}
&(50) \quad V'(x,y,t) &=& \quad \widehat{V}(x,y)e^{\omega t}, \\
&(51) \quad n'(x,y,t) &=& \quad \widehat{n}(x,y)e^{\omega t}, \\
&(52) \quad u'(x,y,t) &=& \quad \widehat{u}(x,y)e^{\omega t}, \\
&(53) \quad v'(x,y,t) &=& \quad \widehat{v}(x,y)e^{\omega t}.
\end{aligned}
$$

We re-write Eqs. (38)-(41) as

$$\frac{\partial^2 \widehat{V}}{\partial x^2} + \frac{1}{R^2}\frac{\partial^2 \widehat{V}}{\partial y^2} - \alpha \widehat{n} = 0, \tag{54}$$

$$\omega \widehat{n} + \frac{\gamma}{\sqrt{\alpha}}\frac{\partial \widehat{n}}{\partial x} + \beta\left(\frac{\partial \widehat{u}}{\partial x} + \frac{1}{R}\frac{\partial \widehat{v}}{\partial y}\right) = 0, \tag{55}$$

$$\omega \widehat{u} + \frac{\gamma}{\sqrt{\alpha}}\frac{\partial \widehat{u}}{\partial x} - \frac{\partial \widehat{V}}{\partial x} + \frac{\sqrt{\alpha}}{\gamma}\widehat{u} = 0, \tag{56}$$

$$\omega \widehat{v} + \frac{\gamma}{\sqrt{\alpha}}\frac{\partial \widehat{v}}{\partial x} - \frac{1}{R}\frac{\partial \widehat{V}}{\partial y} + \frac{\sqrt{\alpha}}{\gamma}\widehat{v} = 0, \tag{57}$$

and Eqs. (42)-(49) as

$$\widehat{V}(0,y) = 0, \tag{58}$$

$$\widehat{V}(1,y) = 0, \tag{59}$$

$$\widehat{n}(0,y) = 0, \tag{60}$$

$$\widehat{v}(x,0) = 0, \tag{61}$$

$$\widehat{v}(x,1) = 0, \tag{62}$$

$$\frac{\partial \widehat{V}}{\partial y}(x,0) = 0, \tag{63}$$

$$\frac{\partial \widehat{V}}{\partial y}(x,1) = 0, \tag{64}$$

$$\int_0^1 \left(\frac{\partial \widehat{V}}{\partial x}(1,y) - \frac{\partial \widehat{V}}{\partial x}(0,y)\right) dy = 0, \tag{65}$$

and writing

$$\widehat{V} = \widetilde{V}(y)e^{\lambda x}, \tag{66}$$

$$\widehat{n} = \widetilde{n}(y)e^{\lambda x}, \tag{67}$$

$$\widehat{u} = \widetilde{u}(y)e^{\lambda x}, \tag{68}$$

$$\widehat{v} = \widetilde{v}(y)e^{\lambda x}, \tag{69}$$

we get

$$\frac{d^2 \widetilde{V}}{dy^2}(y) + \lambda^2 R^2 \widetilde{V}(y) - \alpha R^2 \widetilde{n}(y) = 0, \tag{70}$$

$$\frac{d\widetilde{v}}{dy}(y) + \frac{R}{\beta}\left(\omega + \lambda\frac{\gamma}{\sqrt{\alpha}}\right)\widetilde{n}(y) + \lambda R \widetilde{u}(y) = 0, \tag{71}$$

$$\lambda \widetilde{V}(y) - \left(\omega + \lambda\frac{\gamma}{\sqrt{\alpha}} + \frac{\sqrt{\alpha}}{\gamma}\right)\widetilde{u}(y) = 0, \tag{72}$$

$$\frac{d\widetilde{V}}{dy}(y) - R\left(\omega + \lambda\frac{\gamma}{\sqrt{\alpha}} + \frac{\sqrt{\alpha}}{\gamma}\right)\widetilde{v}(y) = 0, \tag{73}$$

If we take the derivative with respect to y in Eq.(73), the whole system can be reduced to the following equation

$$\frac{d\widetilde{V}^2}{dy^2}(y) + k^2 \widetilde{V}(y) = 0, \tag{74}$$

with

(75) $k^2 = \lambda^2 R^2$,

and the boundary conditions (61),(62), (63) and (64) can be written as

(76) $\frac{d\widetilde{V}}{dy}(0) = 0$,

(77) $\frac{d\widetilde{V}}{dy}(1) = 0$.

The general solution for Eq.(74) is

(78) $\widetilde{V}(y) = C_1 e^{iky} + C_2 e^{-iky}$,

by applying the boundary conditions (76) and (77), the solution for Eq.(74) has the following form

(79) $\widetilde{V}(y) = D\cos(k_n y)$,

with $k_n = \pm 2\pi n$. Therefore, the general solution can be written as

(80) $\widetilde{V}(y) = \sum_0^\infty D_n \cos(k_n y)$.

It is easy to show the proposed solution in Eqs. (66)-(69) satisfies the boundary conditions (58), (59), (60) and (65) independently of the eigenvalue k. Let us define $\sigma = \pm i k_n$. Re-writing Eqs. (66)-(69) as

(81) $\widehat{V} = V_0 e^{\lambda x + \sigma y}$,

(82) $\widehat{n} = n_0 e^{\lambda x + \sigma y}$,

(83) $\widehat{u} = u_0 e^{\lambda x + \sigma y}$,

(84) $\widehat{v} = v_0 e^{\lambda x + \sigma y}$,

the system of Eqs. (54)-(57) can be written as

$$\begin{bmatrix} -\lambda^2 - \sigma^2/R^2 & \alpha & 0 & 0 \\ 0 & \omega + \lambda\gamma/\sqrt{\alpha} & \beta\lambda & \beta\sigma/R \\ -\lambda & 0 & \omega + \lambda\gamma/\sqrt{\alpha} + \sqrt{\alpha}/\gamma & 0 \\ -\sigma/R & 0 & 0 & \omega + \lambda\gamma/\sqrt{\alpha} + \sqrt{\alpha}/\gamma \end{bmatrix} \begin{Bmatrix} V_0 \\ n_0 \\ u_0 \\ v_0 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

For a non-trivial solution, the determinant should vanish. This gives

(85) $-(\lambda^2 R^2 + \sigma^2)(\omega\sqrt{\alpha}\gamma + \alpha + \gamma^2\lambda)\left[\left(\omega + \frac{\gamma}{\sqrt{\alpha}}\lambda\right)\left(\omega + \frac{\gamma}{\sqrt{\alpha}}\lambda + \frac{\sqrt{\alpha}}{\gamma}\right) + \alpha\beta\right] = 0.$

The roots for λ are

(86) $\lambda_1 = -\frac{\omega\sqrt{\alpha}\gamma + \alpha}{\gamma^2}$,

(87) $\lambda_2 = i\frac{\sigma}{R}$,

(88) $\lambda_3 = -i\frac{\sigma}{R}$,

$\lambda_4 = \frac{\alpha}{2\gamma^2}\left[-\left(\frac{2\gamma\omega}{\sqrt{\alpha}} + 1\right) + i\sqrt{4\gamma^2\beta - 1}\right]$,

(89) $= a + ib$,

$\lambda_5 = \frac{\alpha}{2\gamma^2}\left[-\left(\frac{2\gamma\omega}{\sqrt{\alpha}} + 1\right) - i\sqrt{4\gamma^2\beta - 1}\right]$,

(90) $= a - ib$.

The root λ_1 satisfies $(\omega + \gamma\lambda/\sqrt{\alpha} + \sqrt{\alpha}/\gamma) = 0$ and it implies $\widehat{V} = 0$, $\widehat{n} = 0$ and

$$\frac{\partial \widehat{u}}{\partial x} + \frac{1}{R}\frac{\partial \widehat{v}}{\partial y} = 0.$$

This is a trivial solution, therefore we are not going to consider λ_1 as an eigenvalue. Since $\sigma = ik_n$ and $k_n = \pm 2\pi n$, we have two cases.

3.1 *Case $k_n = k_0 = 0$*

$$\begin{aligned}
\widehat{V}(x,y) &= \left(A + Bx + Ce^{(a+ib)x} + De^{(a-ib)x}\right)\widetilde{V}(y), \\
\frac{\partial \widehat{V}}{\partial x}(x,y) &= \left(B + C(a+ib)e^{(a+ib)x} + D(a-ib)e^{(a-ib)x}\right)\widetilde{V}(y), \\
\widehat{n}(x,y) &= \frac{1}{\alpha}\left[Ce^{(a+ib)x}(a+ib)^2 + De^{(a-ib)x}(a-ib)^2\right]\widetilde{V}(y).
\end{aligned}$$

Thus Eqs. (58), (59), (60) and (65) can be written as

$$\begin{aligned}
A + C + D &= 0, \\
A + B + Ce^{a+ib} + De^{a-ib} &= 0, \\
C(a+ib)^2 + D(a-ib)^2 &= 0, \\
C(a+ib)(1-e^{a+ib}) + D(a-ib)(1-e^{a-ib}) &= 0,
\end{aligned}$$

and

$$(91)\quad \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & e^{a+ib} & e^{a-ib} \\ 0 & 0 & (a+ib)^2 & (a-ib)^2 \\ 0 & 0 & (a+ib)(1-e^{a+ib}) & (a-ib)(1-e^{a-ib}) \end{bmatrix} \left\{\begin{matrix} A \\ B \\ C \\ D \end{matrix}\right\} = \left\{\begin{matrix} 0 \\ 0 \\ 0 \\ 0 \end{matrix}\right\}$$

so that

$$(92)\quad (a+ib)(1-e^{a-ib}) - (a-ib)(1-e^{a+ib}) = 0,$$

3.1.1 *For $4\gamma^2\beta > 1$*

The solution for ω in terms of the dimensionless groups is [5]

$$(93)\quad \omega = -\frac{\gamma}{\sqrt{\alpha}}\left[\frac{\alpha}{2\gamma^2} + b\cot b + W\left(-\frac{be^{-b\cot b}}{\sin b}\right)\right].$$

3.1.2 *For $4\gamma^2\beta = 1$*

In tis case, The solution for ω in terms of the dimensionless groups is [5]

$$(94)\quad \omega = -\frac{\gamma}{\sqrt{\alpha}}\left[\frac{\alpha}{2\gamma^2} + 1 + W\left(-e^{-1}\right)\right].$$

3.1.3 *For $4\gamma^2\beta < 1$*

By writing $b = ib'$ where $b' = b = (\alpha\sqrt{1-4\beta\gamma^2})/2\gamma^2$ is now real, ω becomes [5]

$$(95)\quad \omega = -\frac{\gamma}{\sqrt{\alpha}}\left[\frac{\alpha}{2\gamma^2} + b'\coth b' + W\left(-\frac{b'e^{-b'\coth b'}}{\sinh b'}\right)\right].$$

The case $k_n = k_0 = 0$ has been well described in [5].

3.2 Case $k_n \neq 0$

$$\begin{aligned}
\widehat{V}(x,y) &= \left(Ae^{\frac{k_n}{R}x} + Be^{-\frac{k_n}{R}x} + Ce^{(a+ib)x} + De^{(a-ib)x}\right)\widetilde{V}(y),\\
\frac{\partial \widehat{V}}{\partial x}(x,y) &= \left(A\frac{k_n}{R}e^{\frac{k_n}{R}x} - B\frac{k_n}{R}e^{-\frac{k_n}{R}x} + C(a+ib)e^{(a+ib)x} + D(a-ib)e^{(a-ib)x}\right)\widetilde{V}(y),\\
\widehat{n}(x,y) &= \frac{1}{\alpha}\left[Ce^{(a+ib)x}\left((a+ib)^2 - \frac{k_n^2}{R^2}\right) + De^{(a-ib)x}\left((a-ib)^2 - \frac{k_n^2}{R^2}\right)\right]\widetilde{V}(y).
\end{aligned}$$

Thus Eqs. (58), (59), (60) and (65) can be written as

$$\begin{aligned}
A + B + C + D &= 0,\\
Ae^{\frac{k_n}{R}} + Be^{-\frac{k_n}{R}} + Ce^{a+ib} + De^{a-ib} &= 0,\\
C\left((a+ib)^2 - \frac{k_n^2}{R^2}\right) + D\left((a-ib)^2 - \frac{k_n^2}{R^2}\right) &= 0,\\
A\frac{k_n}{R}(1 - e^{\frac{k_n}{R}}) - B\frac{k_n}{R}(1 - e^{-\frac{k_n}{R}}) + C(a+ib)(1 - e^{a+ib}) + D(a-ib)(1 - e^{a-ib}) &= 0,
\end{aligned}$$

and

$$(96)\quad \begin{bmatrix} 1 & 1 & 1 & 1 \\ e^{\frac{k_n}{R}} & e^{-\frac{k_n}{R}} & e^{a+ib} & e^{a-ib} \\ 0 & 0 & (a+ib)^2 - \frac{k_n^2}{R^2} & (a-ib)^2 - \frac{k_n^2}{R^2} \\ \frac{k_n}{R}(1-e^{\frac{k_n}{R}}) & -\frac{k_n}{R}(1-e^{-\frac{k_n}{R}}) & (a+ib)(1-e^{a+ib}) & (a-ib)(1-e^{a-ib}) \end{bmatrix} \begin{Bmatrix} A \\ B \\ C \\ D \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

so that

$$\begin{aligned}
&\left(e^{-\frac{k_n}{R}}\frac{k_n}{R}(1 - e^{\frac{k_n}{R}}) + e^{\frac{k_n}{R}}\frac{k_n}{R}(1 - e^{-\frac{k_n}{R}})\right)\left((a-ib)^2 - (a+ib)^2\right)\\
&+\left(e^{a-ib}\left((a+ib)^2 - \frac{k_n^2}{R^2}\right) - e^{a+ib}\left((a-ib)^2 - \frac{k_n^2}{R^2}\right)\right)\left(\frac{k_n}{R}(1 - e^{\frac{k_n}{R}}) + \frac{k_n}{R}(1 - e^{-\frac{k_n}{R}})\right)
\end{aligned}$$

$$(97)\quad +\left(\left((a-ib)^2 - \frac{k_n^2}{R^2}\right)(a+ib)(1 - e^{a+ib}) - \left((a+ib)^2 - \frac{k_n^2}{R^2}\right)(a-ib)(1 - e^{a-ib})\right)\left(e^{\frac{k_n}{R}} - e^{-\frac{k_n}{R}}\right) = 0.$$

The solutions for Eq. (97) are $(a+ib)^2 = k_n^2/R^2$ and $(a-ib)^2 = k_n^2/R^2$.

3.2.1 For $4\gamma^2\beta > 1$

The solution for ω in terms of the dimensionless groups is

$$(98)\quad \omega_n = \frac{\sqrt{\alpha}}{2\gamma}\left[\pm\frac{4\gamma^2\pi n}{\alpha R} - 1 \pm i\sqrt{4\gamma^2\beta - 1}\right],$$

3.2.2 For $4\gamma^2\beta = 1$

In this case, the solution for ω in terms of the dimensionless groups is

$$(99)\quad \omega_n = \frac{\sqrt{\alpha}}{2\gamma}\left[\pm\frac{4\gamma^2\pi n}{\alpha R} - 1\right],$$

3.2.3 For $4\gamma^2\beta < 1$

In the last case, the solution for ω in terms of the dimensionless groups is

$$\omega_n = \frac{\sqrt{\alpha}}{2\gamma}\left[\pm\frac{4\gamma^2\pi n}{\alpha R} - 1 \pm \sqrt{1-4\gamma^2\beta}\right], \tag{100}$$

The most interesting case is when ω has imaginary components, which corresponds to oscillatory modes in time. As an example, we can take typical values that corresponding to a GaAs semi-conductor: effective mass of electron = 6.6% of its actual mass, $\epsilon_s = 113.28 \times 10^{-12}$ C^2/m^2N, $L = 100$ nm, $n_0 = 5 \times 10^{17}$ cm^{-3}, $N_D = 5 \times 10^{17}$ cm^{-3}, $\tau = 0.4 \times 10^{-12}$ s, $V_0 = 1$ V. This gives $\alpha = 7.072$, $\beta = 1$, and $\gamma = 17.365$. The corresponding plasma frequency is then $\omega_p = 43.41 \times 10^{12}$ s^{-1}. This value was used in the Figs. (2), (3), (4) and (5). Considering the case $4\gamma^2\beta > 1$, the spectrum for a typical case with GaAs is shown in Fig. (2). For other cases, Figs. (3), (4) and (5) are showing the electron density and electric field for the first unstable mode at different aspect ratios R. It is shown just the first square of the electric field for aspect ratios bigger than $R = 1$, since the pattern is the same through all the cavity.

4 Conclusions

The simplified hydrodynamic model gives a simple way to study the dynamics in an two-dimensional cavity of semiconductor under a voltage difference. The pattern of the spectrum of eigenvalues tell us that there is a predominant unstable region, which depends strongly in the applied voltage to the piece of semiconductor. As the applied voltage decreases, the spectrum is getting more stable. The spectrum is also determined by the aspect ratio of the cavity. As the aspect ratio decreases, the unstable eigenvalues are getting more unstable. The unstable region, which means eigenvalues with positive real component, has oscillatory components which describe terahertz frequencies under specific parameter values. According to this, we have a source of terahert emission in the form of plasma waves from a ungated two-dimensional piece of semiconductor.

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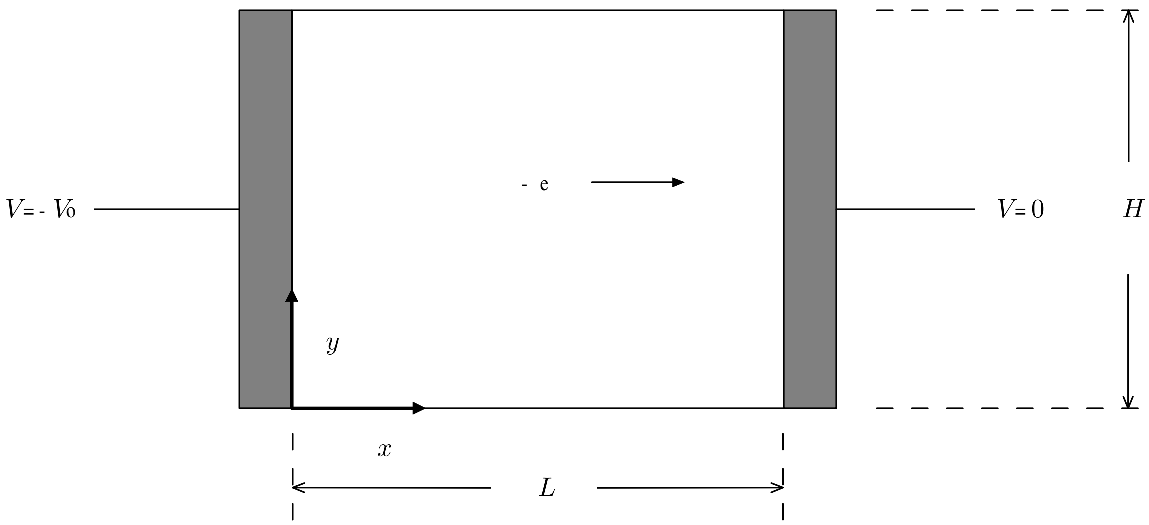

Figure 1: Schematic of semiconductor material.

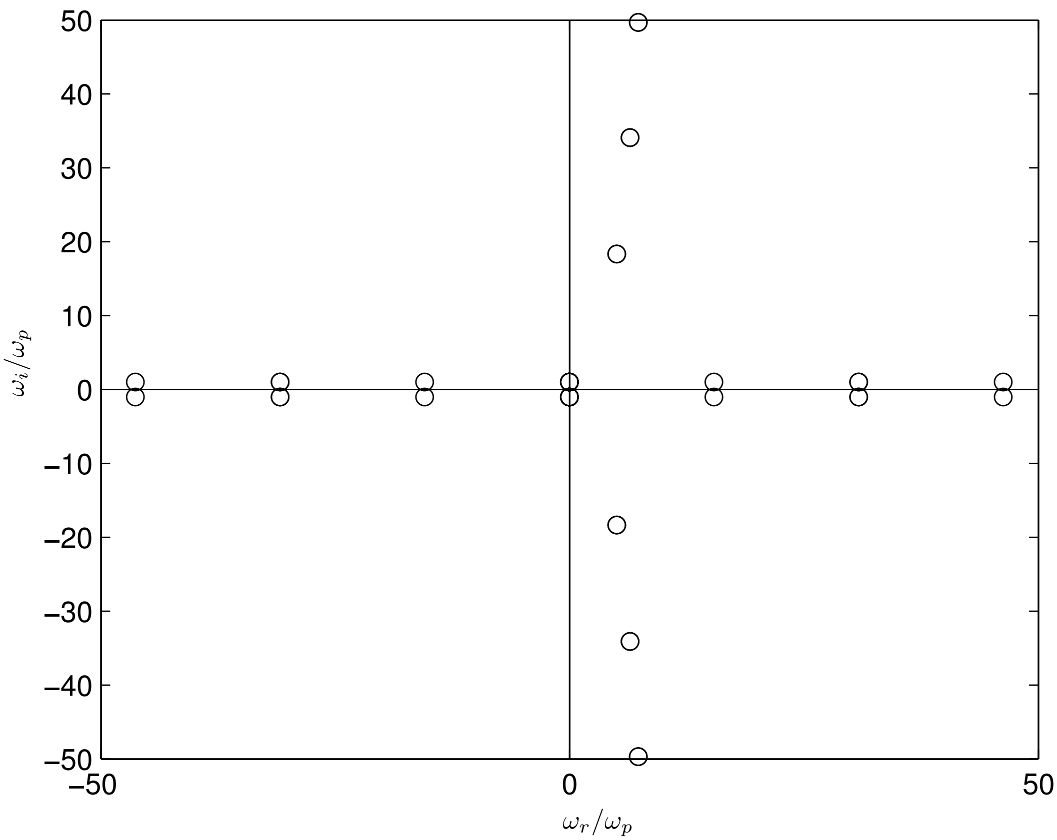

Figure 2: Spectrum of eigenvalues for $\alpha = 7.072$, $\beta = 1$ and $\gamma = 17.365$ with $R = 1$

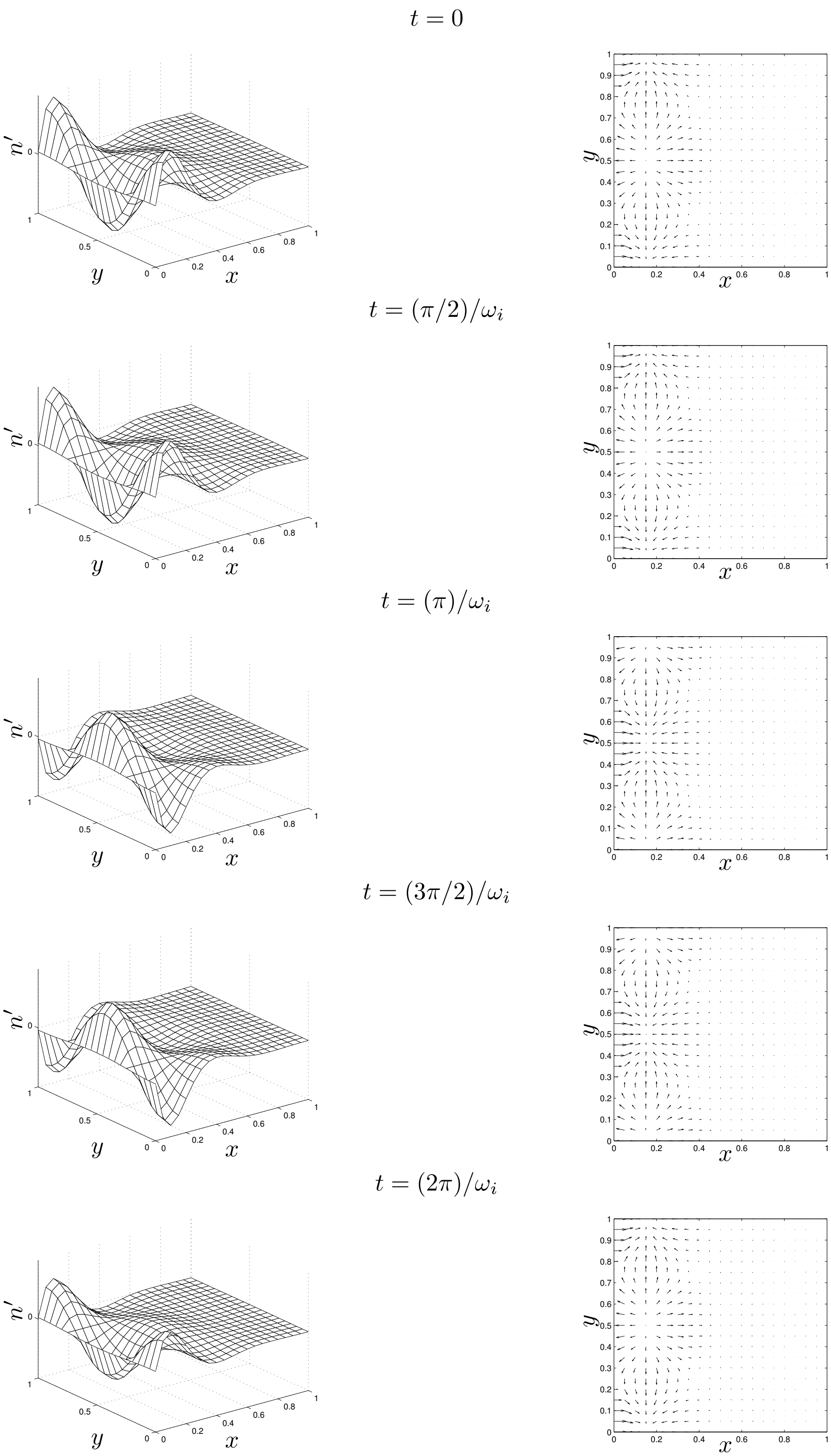

Figure 3: Electron density eigenfunction (left) and electric field (right) for $\alpha = 10$, $\beta = 1$, $\gamma = 2$ and $R = 1$ at $\omega_i \sim 1.15\omega_p$ (arbitrary scale).

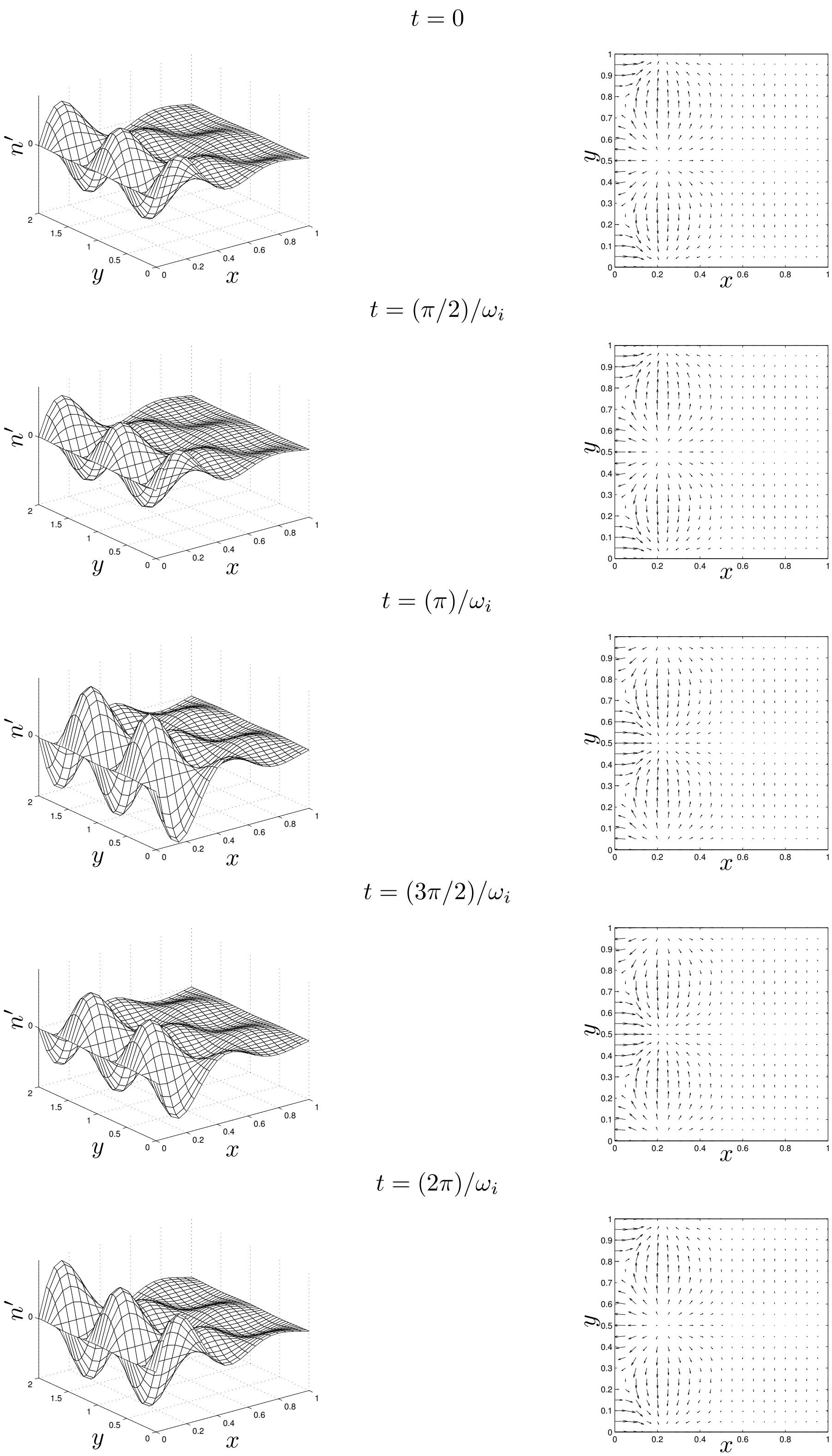

Figure 4: Electron density eigenfunction (left) and electric field (right) for $\alpha = 10$, $\beta = 1$, $\gamma = 2$ and $R = 2$ at $\omega_i \sim 1.15\omega_p$ (arbitrary scale).

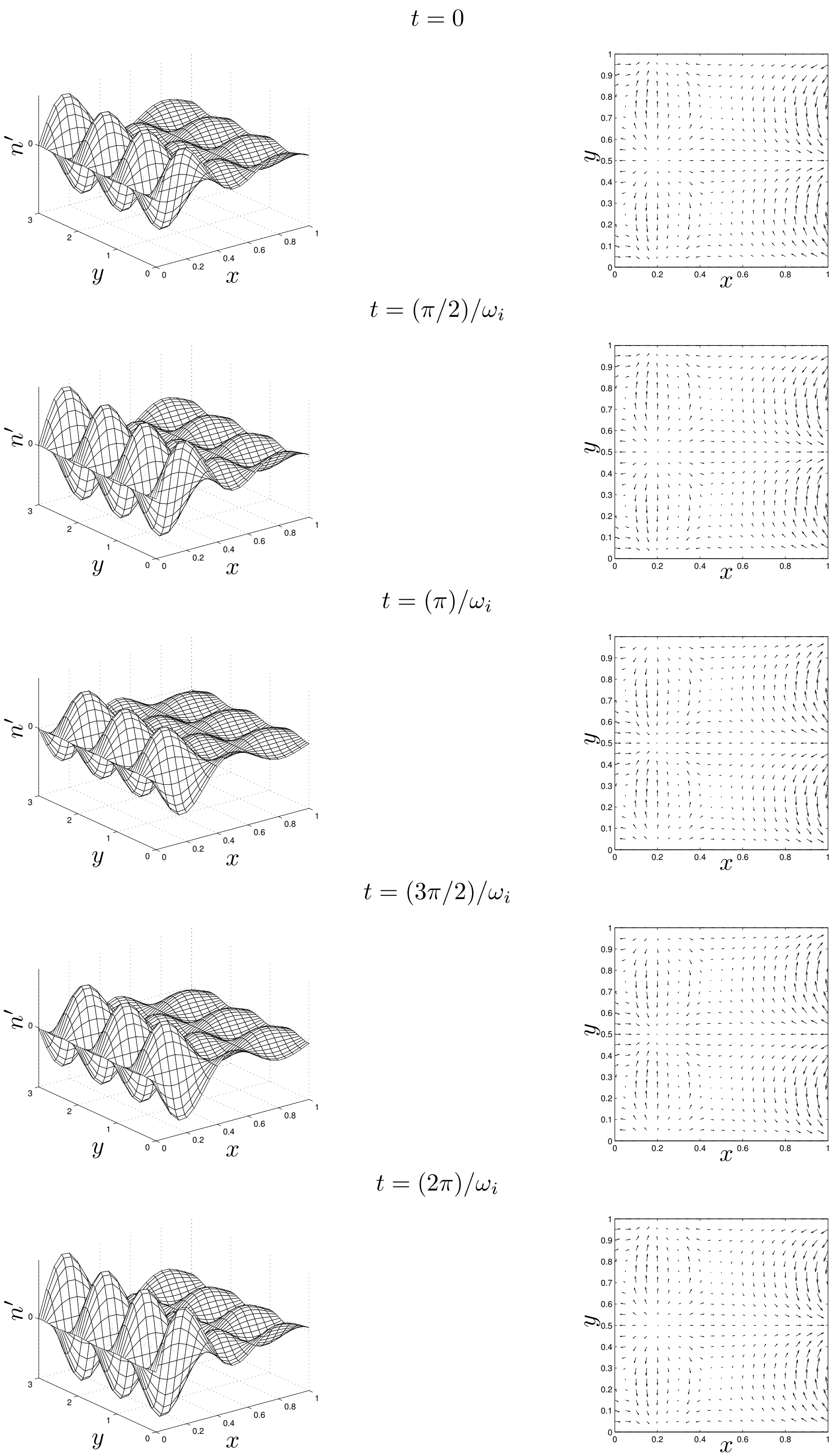

Figure 5: Electron density eigenfunction (left) and electric field (right) for $\alpha = 10$, $\beta = 1$, $\gamma = 2$ and $R = 3$ at $\omega_i \sim 1.15\omega_p$ (arbitrary scale).