Phonon-limited Electron Mobility In a Semiconductor Nanowire

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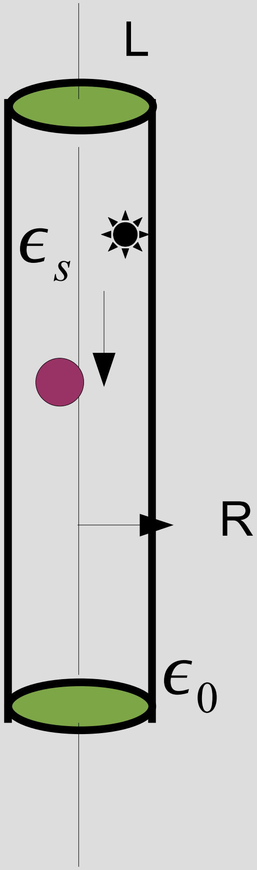
Topics We Will Discuss

1. Introduction to the problem
2. Motivation of the work
3. Results
4. Future work to be done

Introduction to the problem



- We have a nanowire of radius R

- Modeling the potential due to acoustic phonon deformation potential.

- Calculate the quantum mechanical electron scattering matrix element

- Effect on electron mobility

Motivation of The Work

1. For 3D semiconductor , for sufficient impurities , we can neglect phonon-scattering.

2. For 2D semiconductor even at low temperature, for best samples (cleaned) scattering due to phonon has been observed.

3. So the relative importance of phonon-scattering compare to impurity scattering even at low temperature can not be neglected .

B.J.F. Lin et.all, APL,49 ,695 (1984).
K. Hirakawa and H.sakaki, PRB 33, 8291, (1986).

Approach &Results

1.The simplest coupling of acoustic phonon with electron is through deformation potential D.

2. The perturbing potential can be modeled as

$$V(\vec{r},t)=i\sqrt{\frac{\hbar q}{2c\rho}}D[e^{i(\vec{q}.\vec{r}-\omega_q t)}-e^{i(\vec{q}.\vec{r}+\omega_q t)}]$$

3.Where $\omega_q=cq$, and number of phonons in a particular mode is given by BE distribution

$$N_q=[\exp(\frac{\hbar w_q}{kT})-1]^{-1}.$$

John Davis, The Physics of Low−dimensional Semiconductor.

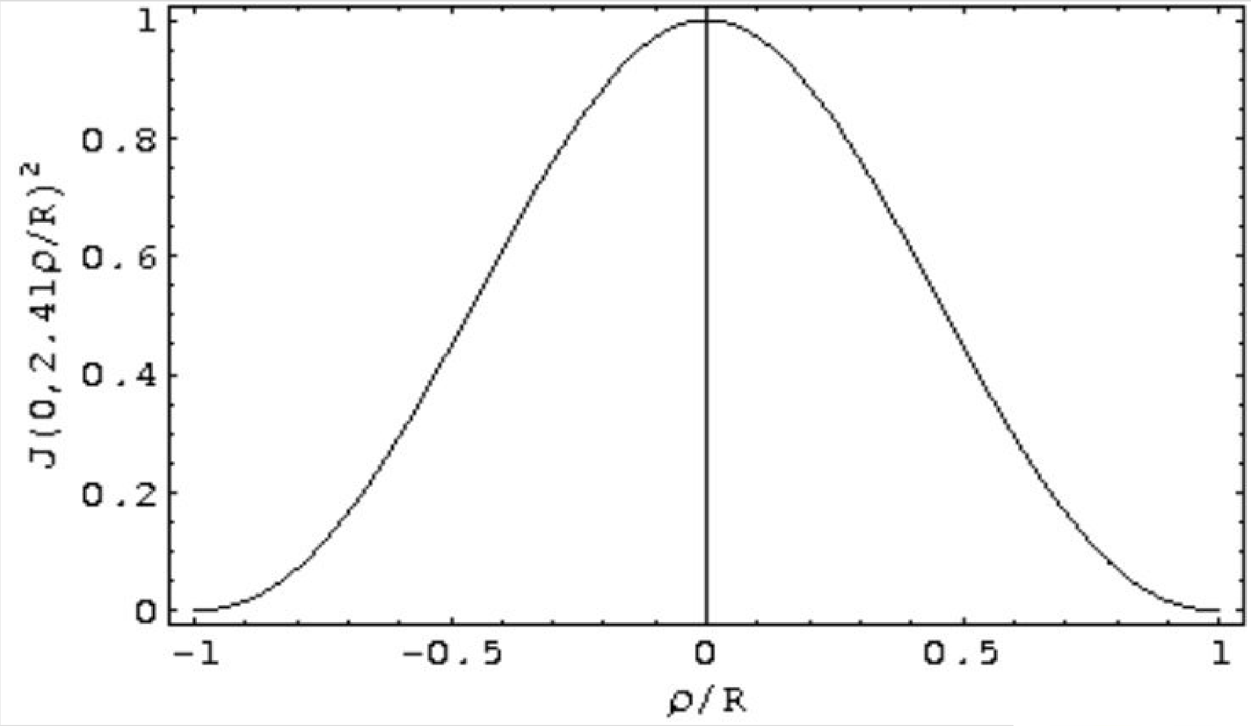
Quantum Electron Wave-function In The Quantum Nano Wire

1. Solve the Schroedinger equation in cylindrical co−ordinates−−−−textbook problem .

2. $$\Psi_{m,n}(\rho,\theta,z)=\frac{1}{(\pi R^2 L)^{\frac{1}{2}}} J_m\left(z_{m,n}\frac{\rho}{R}\right)e^{in\theta}e^{ikz}$$

3. $$E_{m,n}=\frac{\hbar^2 k^2}{2m_z}+\frac{\hbar^2 z^2_{m.n}}{2m\perp R}$$

4. Quantum wave−function goes to zero at the ends of the wire.



Electron Scattering Rate From The Electrostatic Potential Inside The Quantum Wire

1. Assume Fermi energy lies in the 1^{st} sub band. So m=0 and n=1.

2. Carriers will scatter from the calculated perturbing potential inside the wire..

$$w_{i\to f}(k_z, q, R) = N_q \frac{2\pi}{\hbar} M_{if}^2 \delta(E_f - (E_i \pm \hbar\omega_q)).$$

Fermi−golden rule

$$M_{if}(q, R) = A\int_0^{2\pi} d\theta \int_0^R \rho\, d\rho\, J_0^2(z_{m,n}\frac{\rho}{R}) e^{i\vec{Q}.\vec{\rho}}$$

$$.\times\int dz e^{i(k_f-(k_i\mp\hbar q_z))z}$$

$$A = \sqrt{\frac{\hbar q}{2c\rho}} \frac{1}{\pi R^2 L}$$

Scattering Rate

1. Use quasi-elastic approximation, $E \gg \hbar\omega_q$

2. For the degenerate electron gas $E_F \gg k_B T$, $N_q = N_q + 1 = \frac{k_B T}{\hbar\omega_q}$
So the scattering time for emission and absorption are equal.

3. We have to include the angle dependence of the scattering rate for $(1-\cos\theta)$ 1D system.

Electron Mobility Calculation In The Nanowire.

$$\frac{1}{\tau_j(E)} = \frac{2L}{\hbar}\int_{-\infty}^{\infty} dk_{z'} \sum M_{if}^2(Q)\,\delta\left(\frac{\hbar^2 k_{z'}^2}{2m} - \frac{\hbar^2 k_z^2}{2m}\right)(1-\cos\theta)$$

2.Now the only possibility is $k_{z'} = -k_z \quad (1-\cos\theta)=2$

3. $$\frac{1}{\tau_j(k_z)} = \frac{2NmK_B T D^2}{\pi^3 \hbar^3 c^2 R^4}\,\frac{1}{k_z}\sum \int_0^{\rho} \rho\, d\rho\, J_0\left(2.41\frac{\rho}{R}\right)^2 J_0(Q\rho)$$

4. All scattering are random and have no correlation among them, so we can do a statistical average to get mean scattering rate.

5. $$\tau_{avg}(T) = \frac{\int_0^{\infty} dE\, g(E)\left[\frac{-\partial F_0}{\partial E}\right]\tau_j}{\int_0^{\infty} dE\, g(E)\left[\frac{-\partial F_0}{\partial E}\right]}$$

$$g(E) = \sqrt{\left(\frac{m\hbar^2}{2(E-E_0)}\right)}$$

$$\mu(T) = \frac{e\,\tau_{avg}(T)}{m}$$

Do we have Confinement Effect on Phonon Band structure?

1. In polar semiconductor scattering is mainly dominated by polar optical phonon.

2. As dimensional confinement of the electrons modifies its band structure, phonon also should have the confinement effect.

3. $V(\rho, z) = \int_{-\infty}^{\infty} dq\, e^{iqz}\, V(\rho, q)$

4. $\left(-q^2 + \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial}{\partial \rho}\right)\right) V(\rho, q) = 0$

Solution

1. The differential equation is very known-one . But the question is what will be the boundary conditions ?

2. Boundary conditions will reflect the dielectric mismatch in the solution.

3. Now the potential due to phonon incorporate the dielectric mismatch which ultimately effects the mobility of the electrons.

E.A. Muljarov et. al. Phys.Rev.B, vol 62, 2001

Classical Electrodynamics, J.D. Jackson, p 119−122

K.W .Kim et.all. JAP, 70(1),1991.

Future Work

1. Calculate the POP-limited electron mobility using calculated POP-potential field.

2. To investigate the effect of the dielectric mismatch on electron-phonon coupling hence the mobility.

3. Calculate a numerical value of acoustic phonon-limited electron mobility.