EE566 Solid State Devices

Spring 2009
Dept of Electrical Engineering
University of Notre Dame
Instructor: Debdeep Jena (djena@nd.edu, x8835)

Assignment 2

SOLUTIONS

Some commentary on the solutions: (The detailed solutions are in the following pages – by Khalifa). In this page, I point out the common errors committed, and clarify some points.

Problem 1: For 2DEGs, the carrier density approaches a constant value as T approaches 0K: $n_{2d} \to \frac{m^*}{\pi\hbar^2}(E_F - E_C)$ as $T \to 0K$. In fact, for carrier densities exceeding $10^{12}/\text{cm}^2$, the low-temperature limit works well at room temperature as well. For numerical evaluation, one can always use the exact form.

Though carrier transport in BJTs and diodes is 1-dimensional, the carriers are free to move in all 3 dimensions. Therefore, 3D statistics has to be used for such structures. However, in nanotubes, nanowires, and in general quantum wires, one needs to use 1-D statistics for carriers. The solution attached was provided some time ago by K. Hosseini as part of the assignments of EE 566.

a)

$$n_{2D} = \int_{E_c}^{E_{top}} g_{2D}(E) f(E) dE$$

but $f(E) = \frac{1}{1+e^{\frac{E-E_F}{kT}}}$ and $g_{2D}(E) = \frac{m^*}{\pi\hbar^2}\theta(E-E_C)$

$$n_{2D} = \int_{E_c}^{E_{top}} \frac{m^*}{\pi\hbar^2} \frac{1}{1+e^{\frac{E-E_F}{kT}}} \theta(E-E_C) dE$$

little error is introduced by letting $E_{top} \to \infty$

$$n_{2D} = \int_{E_c}^{\infty} \frac{m^*}{\pi\hbar^2} \frac{1}{1+e^{\frac{E-E_F}{kT}}} dE = \int_{E_c}^{\infty} \frac{m^*}{\pi\hbar^2} \frac{1}{1+e^{\frac{E-E_c}{kT}} e^{-\frac{E_F-E_c}{kT}}} dE$$

let $\zeta = \frac{E-E_c}{kT} \Rightarrow d\zeta = \frac{dE}{kT}$ OR $dE = kT d\zeta$

let $\eta = \frac{E_F - E_c}{kT}$

$\zeta \to 0$ as $E \to 0$ and $\zeta \to \infty$ as $E \to \infty$

$$n_{2D} = \int_0^{\infty} \frac{m^*}{\pi\hbar^2} \frac{kT}{1+e^{\zeta-\eta}} d\zeta = \frac{m^* kT}{\pi\hbar^2} \int_0^{\infty} \frac{1}{1+e^{\zeta-\eta}} d\zeta$$

$$n_{2D} = \underbrace{\frac{m^* kT}{\pi\hbar^2}}_{N_c^{2D}} \underbrace{\frac{1}{\Gamma(0+1)} \int_0^{\infty} \frac{\zeta^0}{1+e^{\zeta-\eta}} d\zeta}_{\Im_0(\eta)} = \underbrace{\frac{m^* kT}{\pi\hbar^2}}_{N_c^{2D}} \ln(1+e^{\eta})$$

$$\therefore n_{2D} = N_c^{2D} \Im_0(\eta) = N_c^{2D} \ln(1+e^{\eta})$$

We can make a plot of $E_F - E_c$ as a function of the 2DEG density by noting:

$$n_{2D} = N_c^{2D} \Im_0(\eta) = N_c^{2D} \ln(1+e^{\eta})$$

$$\frac{n_{2D}}{N_c^{2D}} = \ln(1+e^{\eta}) \Rightarrow 1+e^{\eta} = e^{\frac{n_{2D}}{N_c^{2D}}} \Rightarrow e^{\eta} = e^{\frac{n_{2D}}{N_c^{2D}}} - 1$$

$$\therefore \eta = \ln\left(e^{\frac{n_{2D}}{N_c^{2D}}} - 1\right) \quad \text{OR} \quad E_F - E_c = kT \ln\left(e^{\frac{n_{2D}}{N_c^{2D}}} - 1\right)$$

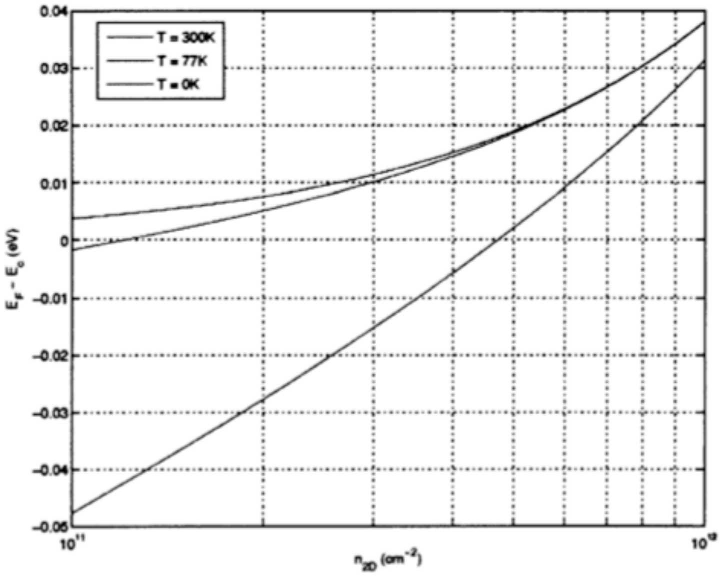
at lwo Temperatures, $N_c^{2D} = \frac{m^* kT}{\pi\hbar^2}$ becomes very small

since n_{2D} is given to be in the range $10^{11} - 10^{12} cm^{-2}$, $e^{\frac{n_{2D}}{N_c^{2D}}}$ becomes $\gg 1$

$$\therefore E_F - E_c \approx kT \ln\left(e^{\frac{n_{2D}}{N_c^{2D}}}\right) = kT \frac{n_{2D}}{N_c^{2D}} = \frac{kT n_{2D}}{\frac{m^* kT}{\pi\hbar^2}} = \frac{n_{2D}\pi\hbar^2}{m^*} \quad \text{OR} \quad n_{2D} \approx \frac{m^*(E_F - E_c)}{\pi\hbar^2}$$

which shows that at low temperatures, n_{2D} becomes independent of temperature

The following figure shows how $E_F - E_c$ varies as a function of the 2DEG density



Examples of devices where electron motion is 2-dimentional:
Devices which utilizes quantum well such us quantum well lasers, quantum well modulators, and (quantum well injection transit
time) QWITT diode oscillator

b)

$$n_{1D} = \int_{E_c}^{E_{top}} g_{1D}(E) f(E) dE$$

but $f(E) = \dfrac{1}{1+e^{\frac{E-E_F}{kT}}}$ and $g_{1D}(E) = \sqrt{\dfrac{2m^*}{\pi^2\hbar^2(E-E_C)}}$

$$n_{2D} = \int_{E_c}^{E_{top}} \sqrt{\frac{2m^*}{\pi^2\hbar^2(E-E_C)}} \frac{1}{1+e^{\frac{E-E_F}{kT}}} dE$$ and noting that little error is introduced by letting $E_{top} \to \infty$

$$n_{1D} = \int_{E_c}^{\infty} \sqrt{\frac{2m^*}{\pi^2\hbar^2(E-E_C)}} \frac{1}{1+e^{\frac{E-E_F}{kT}}} dE = \sqrt{\frac{2m^*}{\pi^2\hbar^2}} \int_{E_c}^{\infty} \frac{(E-E_C)^{-1/2}}{1+e^{\frac{E-E_c}{kT}} e^{-\frac{E_F-E_c}{kT}}} dE$$

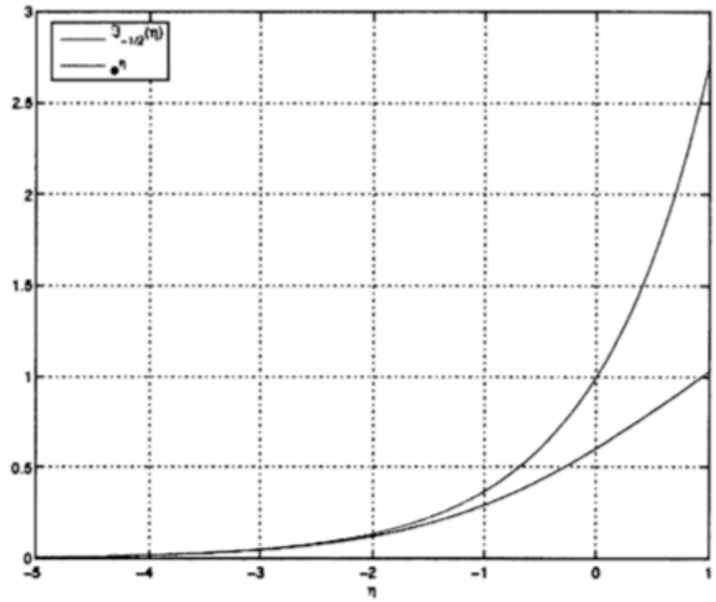
let $\zeta = \dfrac{E-E_c}{kT} \Rightarrow d\zeta = \dfrac{dE}{kT}$ OR $dE = kTd\zeta$

let $\eta = \dfrac{E_F - E_c}{kT}$

$\zeta \to 0$ as $E \to 0$ and $\zeta \to \infty$ as $E \to \infty$

$$n_{1D} = \sqrt{\frac{2m^*}{\pi^2\hbar^2}} \int_{E_c}^{\infty} \frac{(kT\,\zeta)^{-1/2}}{1+e^{\zeta-\eta}} dE = \sqrt{\frac{2m^*kT}{\pi\hbar^2}} \frac{1}{\sqrt{\pi}} \int_{E_c}^{\infty} \frac{\zeta^{-1/2}}{1+e^{\zeta-\eta}} dE = \underbrace{\sqrt{\frac{2m^*kT}{\pi\hbar^2}}}_{N_c^{1D}} \underbrace{\frac{1}{\Gamma\left(-\frac{1}{2}+1\right)} \int_{E_c}^{\infty} \frac{\zeta^{-1/2}}{1+e^{\zeta-\eta}} dE}_{\Im_{-1/2}(\eta)} = N_c^{1D} \Im_{-1/2}(\eta)$$

As is evident from the figure below, $\Im_{-1/2}(\eta)$ is closely approximated by e^{η} under the non-degeneracy condition $\eta \le -2 \Rightarrow n_{1D} = N_c^{1D}\Im_{-1/2}(\eta) \approx N_c^{1D}e^{\eta} = N_c^{1D}e^{\frac{E_F - E_c}{kT}}$



Examples of devices where electron motion is 1-dimentional:
Carbon nano-tubes and Carbon nano-wire

Problem 2: This problem is designed to illustrate the powerful concept of "quasi-neutrality". As many of you have realized, on the surface, by assuming $n(x) \sim N_D(x)$, the problem is rather simple to solve by balancing the drift and diffusion currents. However, it is clear that the electric field obtained has to originate from a charge imbalance in the doped region; Poisson's equation tells us that $F(x) = \frac{q}{\varepsilon_S}\int_{-\infty}^{x} dx[N_D(x) - n(x)]$ is zero! The exact solution has to be <u>numerical</u> - you can attempt it, but be assured that the answer you will be get will be VERY close to what is got by assuming "quasi-neutrality". You know that the semiconductor is not charge neutral at every point, but since the electric field that develops due to the diffusion of carriers is very small, it is close to neutral, i.e., quasi-neutral. We will discuss quasi-neutrality in quantitative details when we discuss p-n junctions soon.

a) Under equilibrium, the total carrier currents inside a semiconductor must be identical to zero:

$$J_n = J_{n|drift} + J_{n|drift} = 0$$

$$q\mu_n nE(x,T) + qD_n\frac{dn}{dx} = 0 \Rightarrow E(x,T) = -\frac{D_n}{\mu_n}\frac{1}{n(x)}\frac{dn}{dx}$$

$$\frac{D_n}{\mu_n} = \frac{kT}{q}\frac{n}{dn/d\eta_c} \quad \text{where } \eta_c = \frac{E - E_F}{kT}$$

$$\text{Assuming non-degeneracy: } n \to N_c e^{\eta_c}, \frac{n}{dn/d\eta_c} \to 1$$

$$\therefore \frac{D_n}{\mu_n} = \frac{kT}{q} \quad \text{and} \quad E(x,T) = -\frac{kT}{q}\frac{1}{n(x)}\frac{dn}{dx}$$

Assuming Total ionization for the dopant: $n(x) = N_d(x)$

$$\therefore E(x,T) = -\frac{kT}{q}\frac{1}{N_D(x)}\frac{dN_D(x)}{dx}$$

b) The magnitude of the electric field is directly proportional to the temperature (e.g. as temperature increase, the magnitude of the electric field increase , but the direction of the electric field remains the same)

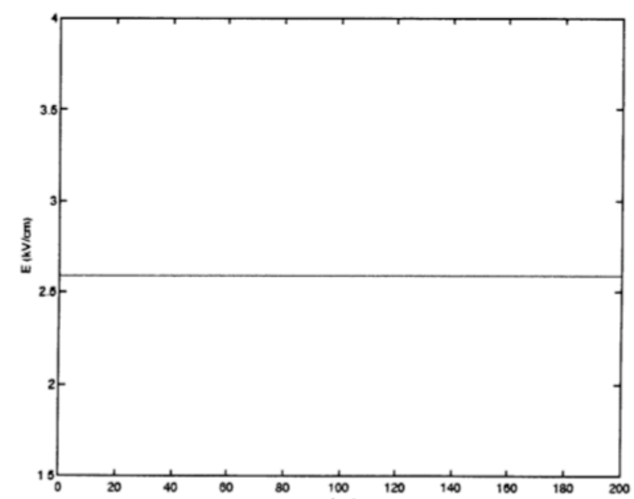
c) For a constant doping profile $N_d(x) = \text{constant} = N_d$: $\frac{dN_D(x)}{dx} = 0$ and thus $E(x,T) = 0$

d)

$$N_d(x) = N_0 e^{-x/\lambda} \Rightarrow \frac{d}{dx}N_d(x) = -\frac{N_0}{\lambda}e^{-x/\lambda}$$

$$E(x,T) = -\frac{kT}{q}\frac{1}{N_D(x)}\frac{dN_D(x)}{dx} = -\frac{kT}{q}\frac{1}{N_0 e^{-x/\lambda}}\frac{-N_0}{\lambda}e^{-x/\lambda}$$

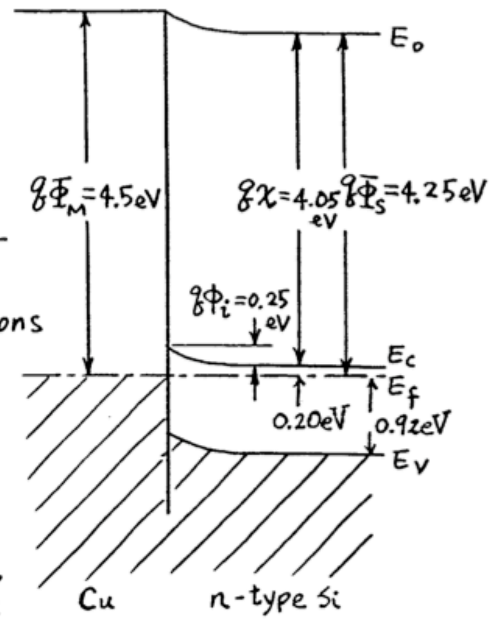
$$E(x,T) = \frac{kT}{q}\frac{1}{\lambda} = 2.5875\times10^5\frac{V}{m} = 2.5875\frac{kV}{cm}$$



Problem 3: (Problem 3.3 from MKC)

3.3

(a) $\phi_i = 0.25$ V (silicon to copper)

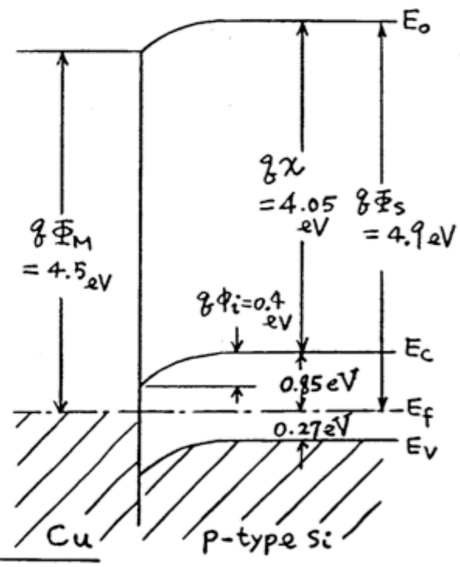


(b)

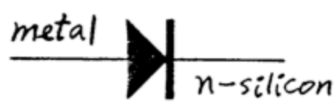
(i) The built-in electric field is directed from the silicon toward the copper. If hole-electron pairs are generated within the space-charge-region, the holes move toward the copper and the electrons move toward the silicon bulk. This represents a current in the diode from silicon toward the copper.

(ii) If the diode is an open circuit, the net current generated by the light must be reduced to zero. This occurs when the built-in field is reduced to zero. The collected holes and electrons raise the potential of the metal with respect to the silicon, thus forward biasing the junction. The bias at the junction can be measured externally and its maximum value is the built-in voltage $\phi_i = 0.25$ V.

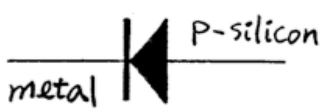
(c) $q\Phi_S = 4.9$ eV, then:



(d) Part (a) refers to a Schottky barrier to n-type silicon.



Part (c) refers to a Schottky barrier to p-type silicon.



Problem 4 (Problem 3.7 from MKC)

3.7

(a) From Eq. (3.3.18), we have $\phi(x) = \frac{qN_d x_d x}{\epsilon_s} - \frac{qN_d x^2}{2\epsilon_s}$

Using the condition $\phi(l) = \phi_B$, we get

$\phi_B + \frac{qN_d l^2}{2\epsilon_s} = \frac{qN_d x_d l}{\epsilon_s}$ and since $x_d = \sqrt{\frac{2\epsilon_s(\phi_i - V_a)}{qN_d}}$,

$$\phi_B + \frac{qN_d l^2}{2\epsilon_s} = l\sqrt{\frac{2q(\phi_i - V_a)}{\epsilon_s}}\sqrt{N_d}$$

$$0.65 + 7.726\times10^{-20} N_d = 1.243\times10^{-9}\sqrt{N_d}$$

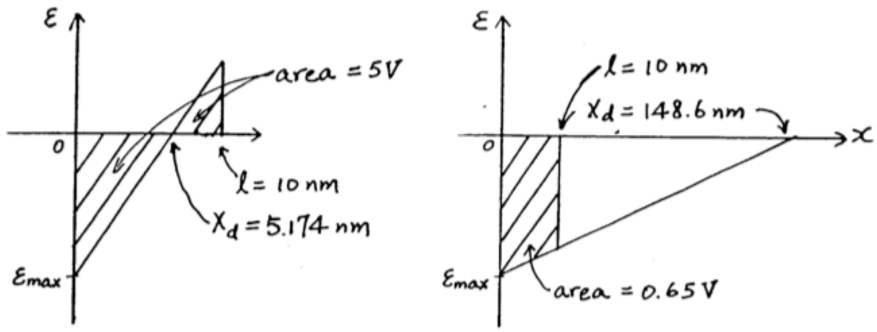
$$\therefore\ 5.969\times10^{-39} N_d^2 - 1.445\times10^{-18} N_d + 0.4225 = 0$$

Solving this quadratic equation for N_d, we obtain

$$N_d = 2.929\times10^{17}\ cm^{-3} \quad \text{or} \quad 2.417\times10^{20}\ cm^{-3}$$

The solution $N_d = 2.417\times10^{20}\ cm^{-3}$ is spurious because it gives $X_d = \sqrt{\frac{2\epsilon_s(\phi_i - V_a)}{qN_d}} = 5.174\ nm$

and corresponds to the situation below left



The correct answer is $N_d = 2.929\times10^{17} cm^{-3}$, $X_d = 148.6\ nm$, which corresponds to the situation above right. Thus $N_d \le 2.929\times10^{17}\ cm^{-3}$ in order to avoid the onset of efficient tunneling. We could have also obtained the approximate value of N_d by

$$|\mathcal{E}_{max}|\, l \simeq \phi_i,$$

$$|\mathcal{E}_{max}| \simeq \frac{0.65V}{10nm} = 6.5\times10^5\ V/cm$$

$$= \frac{qN_d X_d}{\epsilon_s} = \sqrt{\frac{2qN_d(\phi_i - V_a)}{\epsilon_s}}$$

$$N_d \simeq 2.73\times10^{17}\ cm^{-3}.$$

(b) The condition $N_d \le 2.929\times10^{17}\ cm^{-3}$, in turn, requires $\rho \ge 0.04\ \Omega\text{-cm}$ (from Fig. 1.15) for the epitaxial layer resistivity in order that Schottky diodes can be made. In practice, resistivity values of roughly 1 Ω-cm are used.

(c)