

EE566 Solid State Devices

Spring 2007

Dept of Electrical Engineering

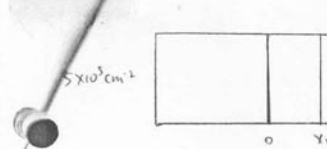
University of Notre Dame

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Assignment 8 SOLUTIONS

PROBLEM 1:

EE566, Spring '05



$N_A = 5 \times 10^{16} \text{ cm}^{-3}$
 $D_n = 20 \text{ cm}^2 \text{ s}^{-1}$
 $x_B = 5 \mu\text{m}$

$\beta_F \equiv I_C / I_B$
ASSGN ~~8~~

Here, we ignore the recombination current in base, so $\alpha \approx 1$

Solu by Albert Wang
alternative method (similar) by Neeraj attached.

$$\beta = \frac{\gamma \alpha_T}{1 - \gamma \alpha_T} \approx \frac{\gamma}{1 - \gamma} = \frac{\frac{1 + |I_{PE}/I_{EE}|}{|I_{PE}/I_{EE}|}}{\frac{|I_{PE}/I_{EE}|}{1 + |I_{PE}/I_{EE}|}} = \frac{|I_{EE}|}{|I_{PE}|}$$

Since the base current is dominant by the reverse injection into emitter,
 $I_{PE} \propto e^{qV_{BE}/V_T}$

$$\beta / \beta_{F0} = \frac{|I_{EE}| / |I_{PE}|}{|I_{EE0}| / |I_{PE0}|} = \frac{|I_{EE}|}{|I_{EE0}|} \frac{e^{qV_{BE0}/V_T}}{e^{qV_{BE}/V_T}}$$

(4/4)

$I_{EE0}, I_{PE0}, V_{BE0}$ are the values in the middle bias range.

$I_{EE} \propto \frac{dn}{dx}$, if we ignore the recombination in base, the minority distribution is linear in base.

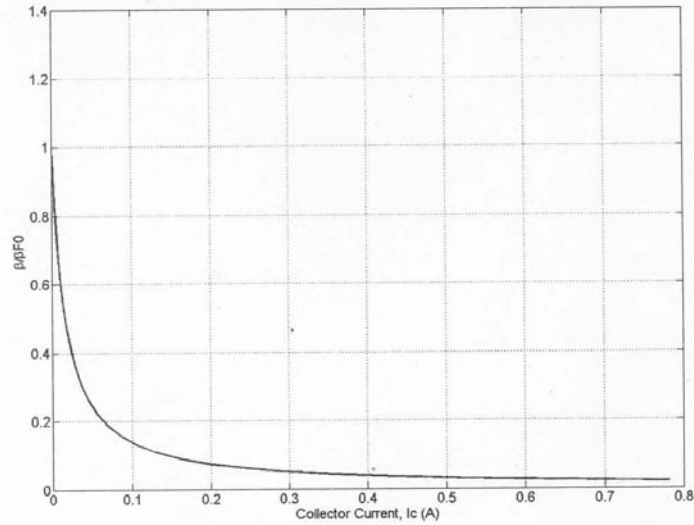
$I_{EE} \propto \frac{n_p(x_B) - n_{p0}}{x_B}$, at Base-collector junction,
 $n = 0$

$\therefore I_{EE} \propto \frac{-n_{p0}}{x_B}$

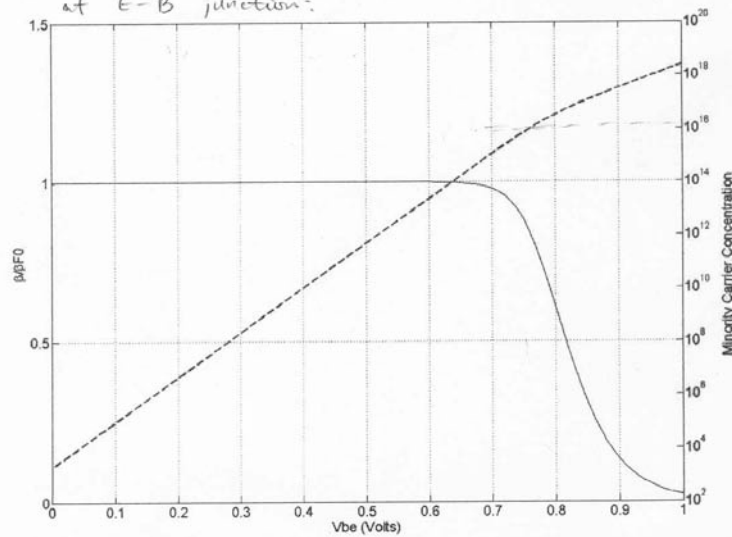
$$\therefore \frac{|I_{EE}|}{|I_{EE0}|} = \frac{n_{p0}}{n_{p00}} = \frac{\frac{N_{A0}}{2} \left[\left(1 + \frac{4n_i^2 \exp\left(\frac{qV_{BE}}{KT}\right)}{N_{A0}^2} \right)^{1/2} - 1 \right]}{\frac{N_{A0}}{2} \exp\left(\frac{qV_{BE0}}{KT}\right)}$$

$$\therefore \beta / \beta_{F0} = \frac{\frac{N_{A0}}{2} \left[\left(1 + \frac{4n_i^2 \exp\left(\frac{qV_{BE}}{KT}\right)}{N_{A0}^2} \right)^{1/2} - 1 \right]}{\frac{n_i^2}{N_{A0}} e^{qV_{BE}/V_T}}$$

Plot in Matlab, we can see as the I_c increase the gain decreased.



if we plot the β and n_{p0} injected minority carrier density at E-B junction:



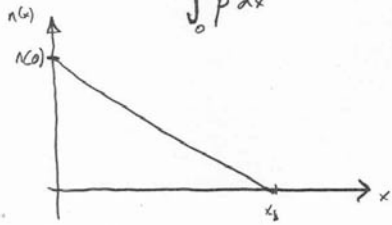
at around 0.7-0.8V, the injected minority density is comparable with Doping in base. So we found the gain β decreases.

Solved by Jak Neopov

$$7.9 \quad \frac{\beta_F}{\beta_{FO}} = \frac{I_{C1}}{I_{C0}}$$

$$I_{C0} = A q D_n n_i^2 \exp(V_{BE}/V_T) / W_B N_A$$

$$I_{C1} = A q D_n n_i^2 \exp(V_{BE}/V_T) \int_0^{x_B} p dx$$



$$\int_0^{x_B} p dx = N_A x_B + \int_0^{x_B} n(x) dx$$

$$= N_A x_B + \frac{n(0) x_B}{2}$$

$$I_F = \frac{q D_n n_i^2 \exp(V_{BE}/V_T)}{N_A W_B}$$

$$I_K = \frac{q D_n N_A(0) A}{W_B}$$

↑
current for high injection region

← Minority Carrier Profile Across the Base.

$$n(0) = \frac{N_A(0)}{2} \left[\left(1 + \frac{4 n_i^2 \exp(V_{BE}/V_T)}{N_A^2(0)} \right)^{1/2} - 1 \right]$$

$$\frac{\beta_F}{\beta_{FO}} = \frac{I_{C1}}{I_{C0}} = \frac{N_A(0) x_B}{(N_A(0) x_B + \frac{x_B N_A(0)}{2} \left[\left(1 + \frac{4 n_i^2 \exp(V_{BE}/V_T)}{N_A^2(0)} \right)^{1/2} - 1 \right])}$$

$$= \frac{1}{1 + \frac{1}{4} \left(1 + \frac{4 n_i^2 \exp(V_{BE}/V_T)}{N_A^2(0)} \right)^{1/2} - 1/4}$$

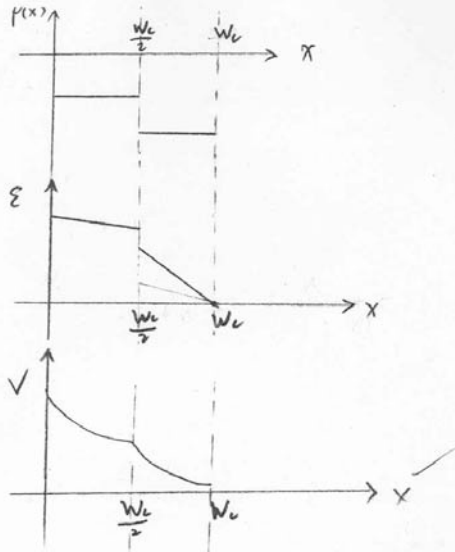
$$= \frac{1}{\frac{1}{4} \left(1 + \frac{4 n_i^2 \exp(V_{BE}/V_T)}{N_A^2(0)} \right)^{1/2} + 3/4}$$

$$= \frac{1}{\frac{1}{4} \left(1 + \frac{4 q D_n W_B n_i^2 \exp(V_{BE}/V_T)}{W_B q D_n N_A(0) N_A(0)} \right)^{1/2} + 3/4}$$

$$\beta_F = \frac{\beta_{FO}}{\frac{1}{4} \left(1 + \frac{4 I_F}{I_{C0}} \right)^{1/2} + 3/4}$$

PROBLEM 2:

a)
PROBLEM 2
ASSGN #8
 BY LIAN CHUANXIN



b) Assume the total potential drop across collector is V_c

$$V_c = -\int E dx = -\int p(x) dx$$

$$= \int_0^{w_c} \int_0^x \frac{q}{\epsilon} \frac{J_{kirk}}{q V_{sat}(x')} dx' dx$$

For collector made entirely of A.

$$V_c = \frac{J_{kirk} w_c^2}{2 \epsilon V_{sat}} \Rightarrow J_{kirk, A} = \frac{2 \epsilon V_{sat} V_c}{w_c^2}$$

If A is adjacent to base.

$$V_c = \int_0^{w_c/2} \frac{J_{kirk, AB}}{\epsilon V_{sat}} x dx + \int_{w_c/2}^{w_c} \frac{2 J_{kirk, AB}}{\epsilon V_{sat}} dx$$

$$= \frac{7}{8} \left(\frac{\epsilon V_{sat}}{J_{kirk, AB} w_c} \right)^{-1}$$

$$\Rightarrow J_{kirk, AB} = \frac{8}{7} \frac{\epsilon V_{sat} V_c}{w_c^2} = \frac{4}{7} J_{kirk, A}$$

c)

$$V_c = -\int_{x_B}^{x_C} E dx = \frac{1}{\epsilon} \int_{x_B}^{x_C} x \left[qN(x) - \frac{J_c}{V(x)} \right] dx$$

$$= \frac{1}{2\epsilon_s} \left(2Nd + \frac{J_c}{\sqrt{2} V_{sat}} \right) x_{CB}^2$$

$$- \frac{qNd}{\epsilon} \left(1 + \frac{J_c}{\sqrt{2} V_{sat}} \right) x^2 \dots \text{--- (1)}$$

Where J_c is the collector current

$$J_i = q N_d V_{sat}$$

$$x_{cB} = x_c - x_B$$

$$\text{from } \textcircled{1} \Rightarrow x_{cB} = \frac{x_{c0}}{(1 + J_c/J_i)^{1/2}} \quad \text{where } x_{c0} = \left(\frac{2 \epsilon_s V_c}{q N_d} \right)^{1/2}$$

In is problem $x_{c0} = W_c$

$$x_B = W_B + W_c - x_{cB} = W_B + W_c - \frac{W_c}{(1 + J_c/J_i)^{1/2}}$$

The plot is attached.

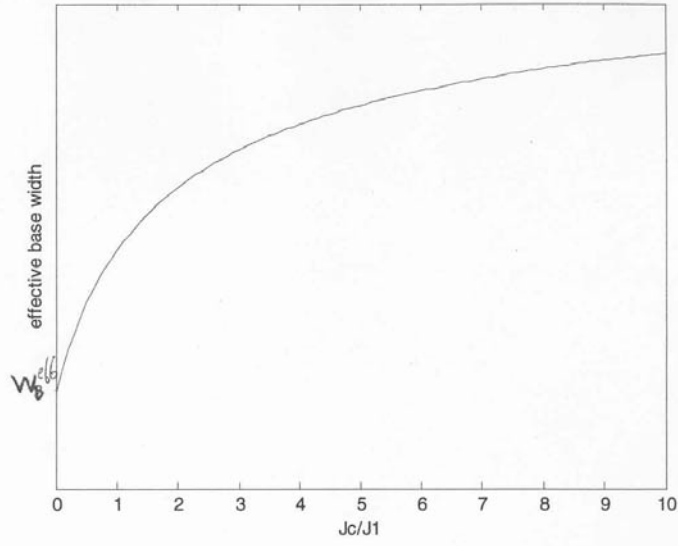
d) if B is adjacent to Base.

$$V_c = \int_0^{1/2 W_c} \frac{2 J_{\text{ink}, BA}}{\epsilon V_{sat}} x dx + \int_{1/2 W_c}^{W_c} \frac{J_{\text{ink}, BA}}{\epsilon V_{sat}} dx$$
$$= \frac{1}{8} \frac{J_{\text{ink}, BA} W_c^2}{\epsilon V_{sat}}$$

$$\Rightarrow J_{\text{ink}, BA} = \frac{8}{5} \frac{\epsilon V_{sat} V_c}{W_c^2} = \frac{4}{5} J_{\text{ink}, A} \quad (\text{good...})$$

e) From Fig 7.8 in MKC, we see as the J_c increases, the max electric field moves to close to collector-subcollector interface,

therefore if A is close to base, the max electric field will then lie in material B so that the breakdown voltage is maximized.



PROBLEM 3:

Problem 3 Method A-ⓐ By Kejia (Albert) Wang
 Ref. from Robert Meyer and Richard Muller, IEEE Trans. on
 Electron Devices, VOL: ED-34, Paso. 1987

NOTE: 3 SOLUTIONS ARE ATTACHED
 Aⓐ & Aⓑ are similar
 B is different
 Another soln in last assignments!

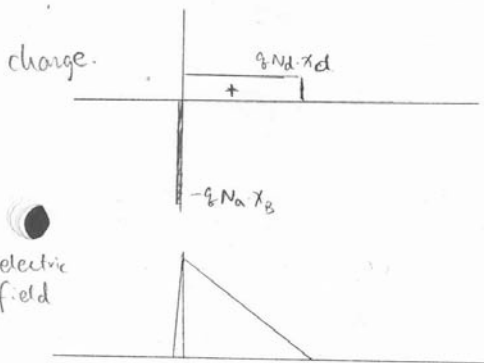
In a npn BJT

Using poisson equation



$$V_{CB} + \phi_i = \frac{qNd x_d^2}{2\epsilon}$$

But we know that there are electrons flowing in the junction, the electrons move at constant velocity v_{sat} and this will introduce a mobile negative charge $Q = I_c/v_i$ in depletion region



$$So V_{CB} + \phi_i = \left[\frac{qNd - Q_c}{2\epsilon_s} \right] x_d'^2$$

where x_d' is depletion width after considering the I_c .

$$\therefore \frac{x_d}{x_d'} = \sqrt{\frac{1}{1 - Q_c/qNd}} \approx 1 + \frac{Q_c}{2qNd} \quad \text{if } Q_c/q \ll Nd$$

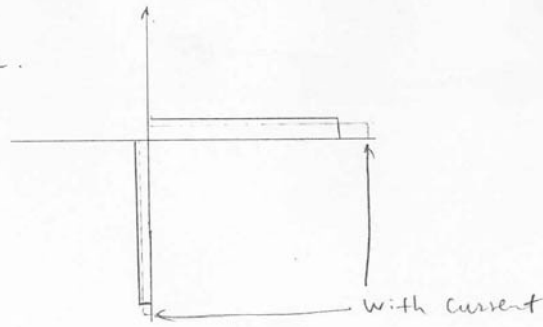
$$\therefore x_d' - x_d = \frac{x_d Q_c}{2qNd}$$

The change of depletion width will cause change of positive charge in collector

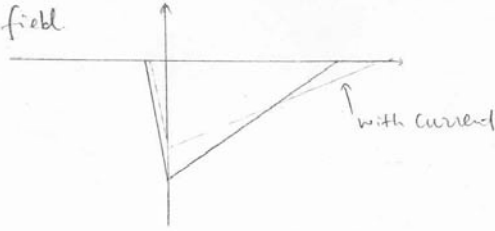
$$\therefore \Delta Q = qNd \Delta x_d = qNd \cdot (x_d' - x_d) = Q_c \frac{x_d}{2}$$

Because depletion region must be charge neutral, so this change in collector side will cause Base depletion region change.

charge.

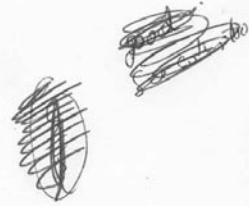


Electric field



so the charge on the base side is support by base lead
and only this contributes to the time constant τ_c

$$\therefore \tau_c = \frac{Q_c \tau_d}{2 J_c} = \frac{Q_c \tau_d}{2 Q_c V_{sat}} = \frac{\tau_d}{2 V_{sat}}$$

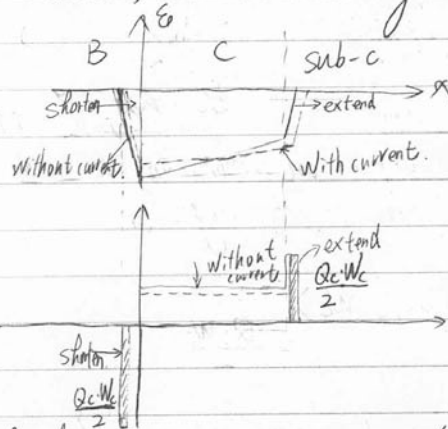


Method A-⑥ (Similar to A-⑤)

By Li Shang.

problem 3. The collector is very lightly doped. The subcollector is heavily doped. Usually, the collector is completely depleted, the whole collector is space charge region.

In the space charge region, the electric field is sufficient large to cause the electron move at the saturation velocity V_{sat} . Therefore, we can see the electron moves at V_{sat} in the collector, which is totally depleted.



Since the electrons move at constant velocity, a mobile charge density Q_c equal to J_c/V_{sat} exists in the space charge region, adding to the negative charge density and subtracting from the positive charge density in the n-region.

Most depletion region is at collector.

$$\therefore V_{CB} + \phi_i = \frac{q N_A W_c^2}{2 \epsilon_s} \quad (\text{completely depleted})$$

When there is current flow in the depletion region.

$$V_{CB} + \phi_i = \left[\frac{q N_A - Q_c}{\epsilon_s} \right] \cdot X_c^2$$

$$\frac{x_c^2}{W_c^2} = \frac{N_d}{N_d - Q_c/\beta} = \frac{N_d}{N_d - Q_c'} \quad Q_c' = Q_c/\beta$$

$$\Rightarrow \frac{x_c}{W_c} = \sqrt{\frac{1}{1 - Q_c'/N_d}}$$

Low injection $\Rightarrow Q_c \ll N_d$ $\therefore (1+x)^{\frac{1}{2}} \sim 1 + \frac{x}{2}$

$$\therefore \frac{x_c}{W_c} \approx 1 + \frac{Q_c'}{2N_d}$$

$$\Rightarrow x_c - W_c = \frac{W_c Q_c}{2\beta N_d}$$

\therefore Due to the depletion width charging, the extra positive charge exposed on donors in the widened space-charge region at collector edge is

$$Q_{\text{extra}} = \int N_d(x_c - W_c) = \frac{Q_c W_c}{2} \quad \dots \text{ (1)}$$

The mobile charge density Q_c caused by electron moving give rise to a total charge per unit area in the space-charge region of $Q_c \cdot W_c$. Eqn (1) shows that $\frac{1}{2}$ of $+Q_c \cdot W_c$ is neutralized by the added positive charge density on the collector/sub-collector side by extending the depletion region.

Since the entire space charge region is charged neutral, there must exist an additional positive charge per unit area of $\frac{Q_c W_c}{2}$ contribute by a shortening of space charge region at the base edge.

The shortening of space charge is caused by the electron flow and supplied by the base lead (in the form of holes)

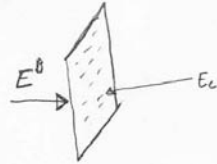
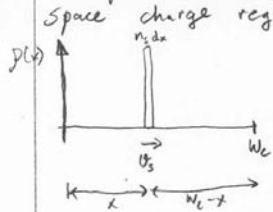
$$\Rightarrow J_c = J_{p, \text{lead}}$$

$$Q = \frac{Q_c}{J} \Rightarrow \tau_c = \frac{Q_c \cdot W_c}{J_c} = \frac{Q_c \cdot W_c}{2 Q_c V_{\text{sat}}} = \frac{W_c}{2 V_{\text{sat}}}$$

Ref: R.G. MEYER and R.S. MULLER, IEEE trans. Electron Device, ED-34, 450
1987.

Method (B) - by Jack Nespor

3. The e^- 's in the depletion are terminated partly on the base and partly on the collector. Why? Consider the following (which is completely analogous to finding the delay through the collector region. In the space charge region.

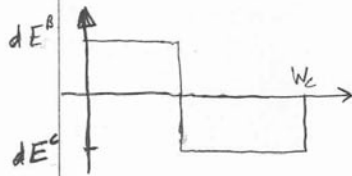


from Gauss' Law we know for a sheet charge the

$$\oint \vec{E} \cdot d\vec{A} = \int \frac{\sigma}{\epsilon}$$

$$E^B A - E^C A = \frac{\sigma A}{\epsilon}$$

$$E^B - E^C = \frac{\sigma}{\epsilon}$$



$$dE^B - dE^C = \frac{q n_s dx}{\epsilon}$$

$x dE^B + dE^C (W-x) = 0$ Current flowing through the depletion cannot produce a change in Voltage

$$dE^C = -dE^B \left(\frac{x}{W-x} \right)$$

$$n_s = \frac{J_c}{q v_{sat}}$$

$$dE^B \left(1 + \frac{x}{W-x} \right) = \frac{q n_s dx}{\epsilon}$$

$$= \frac{J_c dx}{v_{sat} \epsilon}$$

$$dE^B = \frac{dQ_s}{\epsilon} = \frac{J_c dx (W-x)}{v_{sat} \epsilon W}$$

$$Q_B = \frac{J_c}{v_{sat}} \int_0^{W_c} \left(1 - \frac{x}{W_c} \right) dx$$

$$\tau_{BC} = \frac{Q_B}{J_c} = \frac{1}{v_{sat}} \int_0^{W_c} \left(1 - \frac{x}{W_c} \right) dx$$

$$= \frac{W_c}{2 v_{sat}}$$

□

PROBLEM 4:

This question was open-ended, you can read about it from various sources. The numbers are very dynamic – they keep getting updated every year.