EE566 Solid State Devices

Spring 2007
Dept of Electrical Engineering
University of Notre Dame
Instructor: Debdeep Jena (djena@nd.edu, x8835)

Assignment 7
SOLUTIONS

Soln' by Zengxiao Jin

ASSGN (7)
EE 566, Fall 2005)

Problem 1 MKC 6.17

a) We know that

$$\gamma \approx \frac{1}{1+\frac{GN_B\tilde{D}_{pE}}{GN_E\tilde{D}_{nB}}},$$

And now we have $GN_B = N_a x_B A = 2\times10^8\,atoms$, $GN_E = 8\times10^9\,atoms$, $\tilde{D}_{nB} = 18cm^2s^{-1}$,

$\tilde{D}_{pE} = 2cm^2s^{-1}$, so we can get

$$\gamma \approx \frac{1}{1+\frac{GN_B\tilde{D}_{pE}}{GN_E\tilde{D}_{nB}}} = \frac{1}{1+\frac{2\times10^8\times2}{8\times10^9\times18}} = 0.99723$$

b) Obviously,

$$\alpha_T = 1-\frac{x_B^2}{2D_n\tau_n} = 1-\frac{(0.5um)^2}{2\times18cm^2s^{-1}\times10^{-6}s} = 0.99993$$

c) $\alpha_F = \gamma\alpha_T = 0.99716$

$$\beta_F = \frac{\alpha_F}{1-\alpha_F} = \frac{0.99716}{1-0.99716} = 351.1$$

If we use equation $\beta_F \approx Q_{EO}\tilde{D}_{nB}/Q_{BO}\tilde{D}_{pE}$ for calculation, we will get

$$\beta_F = \frac{8\times10^9\times18}{2\times10^8\times2} = 360$$

The error rate for this equation is

$$ER = \frac{360-351.1}{351.1} = 2.5\%$$

So this is a very good approximation.

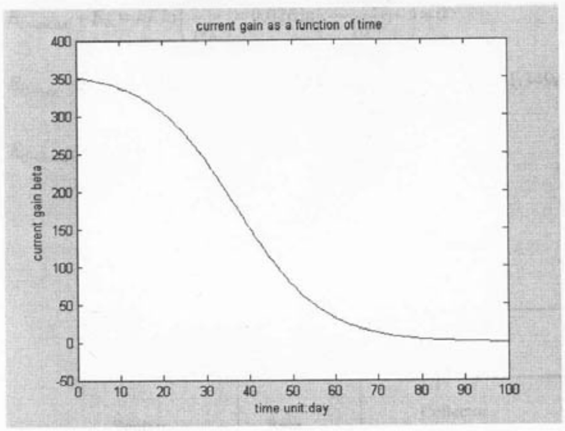
Problem 2 MKC 6.18

We know that

$$\beta_F = \frac{\alpha_F}{1-\alpha_F} = \frac{\gamma\alpha_T}{1-\gamma\alpha_T} = \frac{\gamma\left(1-\frac{x_B^2}{2D_n\tau_n}\right)}{1-\gamma\left(1-\frac{x_B^2}{2D_n\tau_n}\right)} = \frac{\gamma\left(2D_n\tau_n - x_B^2\right)}{2D_n(1-\gamma)\tau_n + \gamma x_B^2}$$

$$= \frac{3.59\times10^{-5}\times e^{-t/t_d} - 2.493\times10^{-9}}{9.972\times10^{-8}\times e^{-t/t_d} + 2.493\times10^{-9}}$$

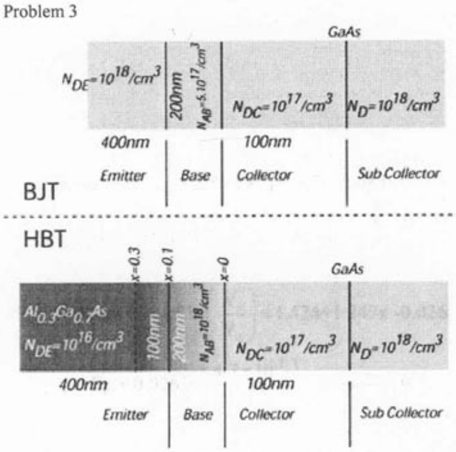
So we can get β_F as a function of time as below,



The time interval until β_F drops to unity is 88.8 days, and at that time the base lifetime is

$$\tau_n = \tau_{n0} \exp\left(-t / t_d\right) = 1.39 \times 10^{-10} s$$

Problem 3



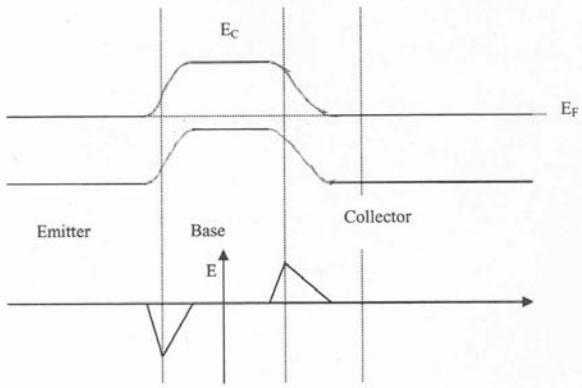
a) The grading of composition from GaAs to $Al_xGa_{1-x}As$ in base and the grading of composition in emitter can reduce the effect of the spike in the conduction band. Plot ρ -E-B diagram of BJT and HBT as below:

For BJT

$$E_{C-emitter} - E_F = kT\ln\left(\frac{N_C}{N_D}\right) = 0.026\ln\left(\frac{4.7\times10^{17}}{10^{18}}\right) \approx 0$$

$$E_{C-base} - E_F = E_g - kT\ln\left(\frac{N_v}{N_A}\right) = 1.424 - 0.026\ln\left(\frac{9\times10^{18}}{5\times10^{17}}\right) = 1.349eV$$

$$E_{C-emitter} - E_F = 0.026\ln\left(\frac{4.7\times10^{17}}{10^{17}}\right) = 0.04eV \approx 0$$



For HBT

$$For\ Al_xGa_{1-x}As\ N_C = 2.5\times10^{19}\left(0.063+0.083x\right)^{3/2}, N_v = 2.5\times10^{19}\times(0.85\text{-}0.14\text{x})^{3/2}$$

$$\Rightarrow E_{c-emitter} - E_F = kT\ln\left(\frac{N_C}{N_D}\right) = 0.026\ln\left(\frac{2.5\times10^{19}\left(0.063+0.083x\right)^{3/2}}{10^{16}}\right)$$

$$E_{c-base} - E_F = E_g - kT\ln\left(\frac{N_v}{N_A}\right) = 1.424\text{+}1.247\text{x -}0.026\ln\left(\frac{2.5\times10^{19}\left(0.063+0.083x\right)^{3/2}}{10^{18}}\right)$$

$$E_{c-emitter} - E_F = 0.026\ln\left(\frac{4.7\times10^{17}}{10^{17}}\right) = 0.04eV \approx 0$$

And we can get the depletion width in emitter

$$X_n^{BE} = \frac{N_{AB}}{N_{AB}+N_{DE}}W_{delp}^{BE} = \frac{N_{AB}}{N_{AB}+N_{DE}}\sqrt{\frac{2\varepsilon_s}{q}\left(\frac{1}{N_{DE}}+\frac{1}{N_{AB}}\right)\left(V_{bi}-2kT\right)} = 25nm$$

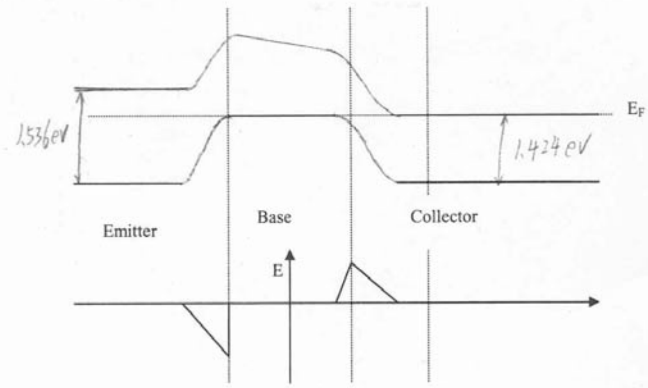
And in base

$$X_p^{BE} = \frac{N_{DE}}{N_{AB}+N_{DE}} W_{delp}^{BE} = \frac{N_{DE}}{N_{AB}+N_{DE}} \sqrt{\frac{2\varepsilon_s}{q}\left(\frac{1}{N_{DE}}+\frac{1}{N_{AB}}\right)(V_{bi}-2kT)} = 50nm$$

$$X_p^{BC} = \frac{N_{DC}}{N_{AB}+N_{DC}} W_{delp}^{BC} = \frac{N_{DC}}{N_{AB}+N_{DC}} \sqrt{\frac{2\varepsilon_s}{q}\left(\frac{1}{N_{DC}}+\frac{1}{N_{AB}}\right)(V_{bi}-2kT)} = 25nm$$

$$W_B' = W_B - X_p^{BE} - X_p^{BC} = 125nm$$

The sketch is as below



The purpose of the heavily doped sub-collector is to decrease the resistance in collector.

To get the quasi-electric field in HBT, we have

$$X_p^{BC} = \frac{N_{DC}}{N_{AB}+N_{DC}} W_{delp}^{BC} = \frac{N_{DC}}{N_{AB}+N_{DC}} \sqrt{\frac{2\varepsilon_s}{q}\left(\frac{1}{N_{DC}}+\frac{1}{N_{AB}}\right)(V_{bi}-2kT)} = 13nm$$

$$Also, X_p^{BE} = 1.4nm$$

$$So, W_B' = W_B - X_p^{BC} - X_p^{BE} = 186nm$$

Also, in emitter, the depletion width is

$$X_n^{BE} = \frac{N_{AB}}{N_{AB}+N_{DE}} W_{delp}^{BE} = \frac{N_{AB}}{N_{AB}+N_{DE}} \sqrt{\frac{2\varepsilon_s}{q}\left(\frac{1}{N_{DE}}+\frac{1}{N_{AB}}\right)(V_{bi}-2kT)} = 140nm$$

which shows the grading region is totally depleted.

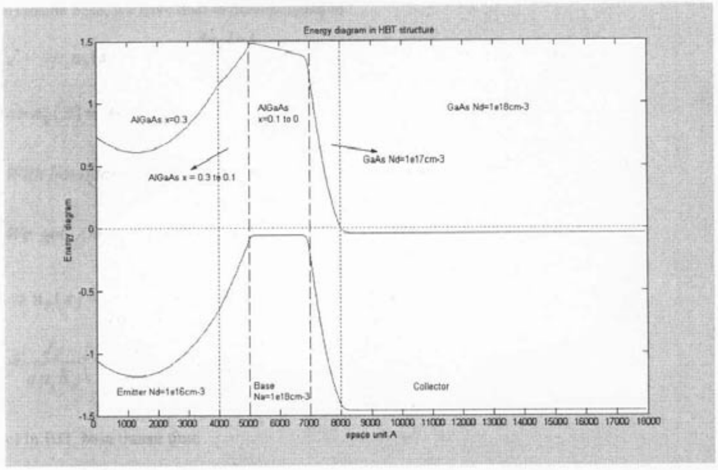
Band gap change over W_B' is $\Delta E_g^{base} \times \frac{186nm}{200nm} \approx \Delta E_g^{base}$

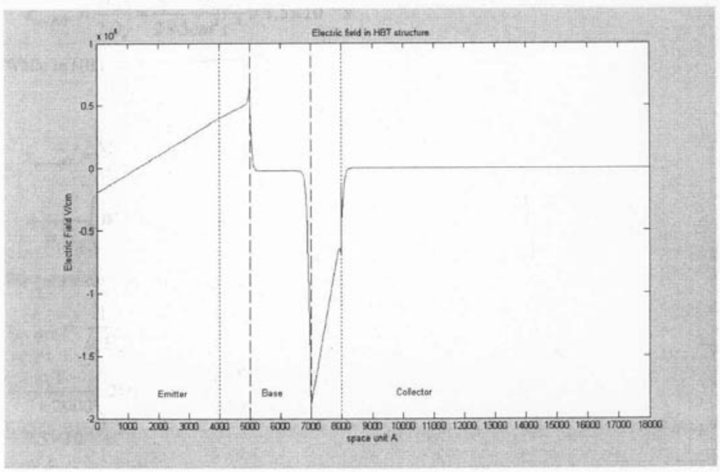
Now since base is doped p-type, E_V is flat, $\Delta E_C = \Delta E_g$,

And we get the quasi electric field

$$E_B = \frac{\Delta E_g^{base}}{qW_B^{depl}} = \frac{0.374 \times 0.1}{186nm} = 2kV/cm$$

Using 1-D Poisson to verify the result





b) Inside base, we have drift-diffusion equation

$$J = q\mu_n n_B(x)E_B + qD_n \frac{dn_B(x)}{dx} = J_c \ \textit{Assume no recombination in the base}$$

$$\Rightarrow n_B(x) = Ae^{\mu_n E_B x/D_n} + \frac{J_C}{q\mu_n E_B} = Ae^{qE_B x/kT} + \frac{J_C}{q\mu_n E_B}$$

With boundary condition $n_B(W_B^{'}) = Ae^{qE_B W_B^{'}/kT} + \frac{J_C}{q\mu_n E_B} = 0$

We get $A = \frac{-J_C}{q\mu_n E_B e^{qE_B W_B^{'}/kT}}$

$$\Rightarrow n_B(x) = \frac{-J_C}{q\mu_n E_B e^{qE_B W_B^{'}/kT}} e^{qE_B x/kT} + \frac{J_C}{q\mu_n E_B}$$

$$= \frac{J_C}{q\mu_n E_B}\left(1 - e^{qE_B(x - W_B^{'})/kT}\right)$$

c) In BJT, base transit time

$$\tau_{tr-BJT} = \frac{W_B^2}{2D_n} \approx \frac{(125nm)^2}{2\times 5cm^2 s^{-1}} = 1.5\times 10^{-11} s$$

While in HBT, we have

$$\tau_{tr-HBT} = \frac{Q_n}{J_n} = \frac{q\int_0^{W_B^{'}} \frac{J_C}{q\mu_n E_B}\left(1 - e^{qE_B(x-W_B^{'})/kT}\right)dx}{J_C}$$

$$= \frac{1}{\mu_n E_B}\left(W_B^{'} - \frac{kT}{qE_B}\left(1 - e^{-qE_B W_B^{'}/kT}\right)\right)$$

For numerical value, we can use the result in a)

$$\tau_{tr-HBT} = \frac{1}{\mu_n E_B}\left(W_B^{'} - \frac{kT}{qE_B}\left(1 - e^{-qE_B W_B^{'}/kT}\right)\right) \approx \tau_{tr}(BJT) * \frac{2kT}{\Delta E_g} \sim 42\%$$

← good assumption here ..

~~$$\approx \frac{1}{7\times 2000}\left(200\times 10^{-7} - \frac{0.026}{2000}\left(1 - e^{2000\times 200\times 10^{-7}/0.026}\right)\right)$$~~

~~$$= 7.3\times 10^{-10} s$$~~

The ratio between the two base transit time is

$$\frac{\tau_{tr-BJT}}{\tau_{tr-HBT}} = \frac{1.5\times10^{-11}}{7.3\times10^{-10}} = 0.02$$

?? τ_{HBT} should be smaller than τ_{BJT} due to build-in $\mathcal{E}$

d) For BJT

$$\gamma_E = \frac{1}{1+\frac{D_p}{D_n}\frac{G_{NB}}{G_{NE}}} = \frac{1}{1+\frac{5}{7}\frac{5\times10^{17}\times125\times10^{-7}}{10^{18}\times375\times10^{-7}}} = 0.89$$

$$\alpha_T = 1-\frac{W_B'^2}{2D_n\tau_n} = 1-\frac{(125nm)^2}{2\times5cm^2s^{-1}\times10^{-8}s} = 0.998$$

$$\beta_F = \frac{\gamma_E\alpha_T}{1-\gamma_E\alpha_T} = 7.9$$

While for HBT, we have

$$\gamma_E = \frac{1}{1+\frac{G_{NB}}{G_{NE}}\frac{n_{iE}^2}{n_{iB}^2}} = \frac{1}{1+\frac{10^{18}\times200\times10^{-7}}{10^{16}\times460\times10^{-7}}e^{-0.374/0.026}} = 0.999997$$

$$\alpha_T = 1-\frac{W_B'^2}{2D_n\tau_n} = 1-\frac{(200nm)^2}{2\times30cm^2s^{-1}\times10^{-8}s} = 0.9993$$

$$\beta_F = \frac{\gamma_E\alpha_T}{1-\gamma_E\alpha_T} = 1421$$

So HBT has a much larger current gain.

e) The pros and cons of BJT and HBT

	pros	cons
BJT	Cheap	Large base resistance Can't have too high frequency
HBT	Large current gain Low base resistance High frequency operation	Complex manufacture

f) A SiGe HBT is similar to a conventional Si bipolar transistor except for the base. SiGe, a material with narrower band-gap than Si, is used as the base material. SiGe HBT has a higher gain, lower RF noise, and low 1/f noise than an identically constructed Si BJT, and higher raw speed can be traded for lower power consumption as well. So SiGe is mainly used in high frequency application. Also, unlike other technologies like GaAs, SiGe has the ability to integrate analog, RF and digital on a single chip using existing CMOS fabs, this leads to massive drive toward BiCMOS technology. SiGe HBTs can be used in making low-cost, lightweight, personal communications devices like digital wireless handsets, as well as other entertainment and information technologies like digital set-top boxes, Direct Broadcast Satellite (DBS), automobile collision avoidance systems, and personal digital assistants. SiGe extends the life of wireless phone batteries, and allows smaller and more durable communication devices.

Problem 4 MKC 6.19

a) We know that in the base, we have continuity equation

$$\frac{\partial n_p}{\partial t} = D_n \frac{\partial^2 n_p}{\partial x^2} - \frac{n_p - n_{p0}}{\tau_n}$$

And considering steady state, we can get an equation for excess density

$$0 = D_n \frac{d^2 n_p^{'}}{dx^2} - \frac{n_p^{'}}{\tau_n}$$

$$\Rightarrow n_p^{'}(x) = A \exp\left(-\frac{x - x_B}{\sqrt{D_n \tau_n}}\right) + B \exp\left(\frac{x - x_B}{\sqrt{D_n \tau_n}}\right)$$

Now we have boundary condition

$$n_p^{'}(0) = n_{p0}\left(\exp\left(qV_{BE}/kT\right) - 1\right)$$

$$n_p^{'}(x_B) = -n_{p0}$$

So we can get

$$n_p(x) = n_p^{'}(x) + n_{p0} = n_{p0} \frac{\left(e^{qV_{BE}/kT} - 1\right) + e^{-x_B/\sqrt{D_n\tau_n}}}{e^{x_B/\sqrt{D_n\tau_n}} - e^{-x_B/\sqrt{D_n\tau_n}}} \exp\left(-\frac{x - x_B}{\sqrt{D_n \tau_n}}\right) - n_{p0} \frac{e^{x_B/\sqrt{D_n\tau_n}} + \left(e^{qV_{BE}/kT} - 1\right)}{e^{x_B/\sqrt{D_n\tau_n}} - e^{-x_B/\sqrt{D_n\tau_n}}} \exp\left(\frac{x - x_B}{\sqrt{D_n \tau_n}}\right) + n_{p0}$$

$$= n_{p0}\left(e^{qV_{BE}/kT} - 1\right) \frac{e^{(x_B - x)/L_n} - e^{(x - x_B)/L_n}}{e^{x_B/L_n} - e^{-x_B/L_n}} + n_{p0} \frac{e^{-x/L_n} - e^{x/L_n}}{e^{x_B/L_n} - e^{-x_B/L_n}} + n_{p0}$$

$$= n_{p0}\left(e^{qV_{BE}/kT} - 1\right) \frac{\sinh\left(\frac{x_B - x}{L_n}\right)}{\sinh\left(\frac{x_B}{L_n}\right)} - n_{p0} \frac{\sinh\left(\frac{x}{L_n}\right)}{\sinh\left(\frac{x_B}{L_n}\right)} + n_{p0}$$

b) Using the expression in (a), we can get the slope of the distribution at x=0 and x=x_B, since we know the relation of current and the slope is

$$J_n(x) = qD_n \frac{dn_p}{dx}$$

So we can get

$$J_n(0) = \left| qD_n \frac{dn_p}{dx} \right|_{x=0} = \frac{n_{p0}}{L_n}\left(e^{qV_{BE}/kT} - 1\right) \coth\left(\frac{x_B}{L_n}\right) + \frac{n_{p0}}{L_n} \csc h\left(\frac{x_B}{L_n}\right)$$

$$J_n(x_B) = \left| qD_n \frac{dn_p}{dx} \right|_{x=x_B} = \frac{n_{p0}}{L_n}\left(e^{qV_{BE}/kT} - 1\right) \operatorname{csch}\left(\frac{x_B}{L_n}\right) + \frac{n_{p0}}{L_n} \cot h\left(\frac{x_B}{L_n}\right)$$

The difference of the current is caused by recombination, and we can get the expression as

$$J_{RB} = J_n(0) - J_n(x_B) = \frac{n_{p0}}{L_n}\left(e^{qV_{BE}/kT} - 1\right)\left(\coth\left(\frac{x_B}{L_n}\right) - \operatorname{csch}\left(\frac{x_B}{L_n}\right)\right) + \frac{n_{p0}}{L_n}\left(\csc h\left(\frac{x_B}{L_n}\right) - \coth\left(\frac{x_B}{L_n}\right)\right)$$

$$= \frac{n_{p0}}{L_n}\left(e^{qV_{BE}/kT} - 2\right)\left(\coth\left(\frac{x_B}{L_n}\right) - \operatorname{csch}\left(\frac{x_B}{L_n}\right)\right)$$

c) $\gamma \approx \dfrac{1}{1 + \dfrac{GN_B\tilde{D}_{pE}}{GN_E\tilde{D}_{nB}}} = \dfrac{1}{1 + \dfrac{10^{17} \times 1um \times 8}{10^{19} \times 1um \times 3}} = 0.974$

$$\alpha_T = 1 - \left|\frac{J_{RB}}{J_n(0)}\right| = 1 - \left|\frac{\left(e^{qV_{BE}/kT} - 2\right)\left(\coth\left(\frac{x_B}{L_n}\right) - \operatorname{csch}\left(\frac{x_B}{L_n}\right)\right)}{\left(e^{qV_{BE}/kT} - 1\right)\coth\left(\frac{x_B}{L_n}\right) + \csc h\left(\frac{x_B}{L_n}\right)}\right|$$

$$= 1 - \left|\frac{\left(e^{qV_{BE}/kT} - 2\right)\left(\coth(0.18) - \operatorname{csch}(0.18)\right)}{\left(e^{qV_{BE}/kT} - 1\right)\coth(0.18) + \csc h(0.18)}\right|, \frac{x_B}{L_n} = \frac{x_B}{\sqrt{D_n\tau_n}} = 0.18$$

$$\approx 1 - \left|\frac{\coth(0.18) - \operatorname{csch}(0.18)}{\coth(0.18)}\right| \quad assume \ \frac{qV_{BE}}{kT} >> 1$$

$$= 0.984$$

$$\alpha_F = \gamma\alpha_T = 0.9584$$

$$\beta_F = \frac{\alpha_F}{1 - \alpha_F} = 23.04$$