EE 566
Solid State Devices
Assignment 5
Solutions

PROBLEM 1

First we list some fundamental physical constants:

$h := 6.62606876 \cdot 10^{-34} \cdot \text{joule} \cdot \text{sec}$ $q := 1.602176462 \cdot 10^{-19} \cdot \text{cou}$ $\text{eV} := 1.602176462 \cdot 10^{-19} \text{joule}$

$m_0 := 9.10938188 \cdot 10^{-31} \cdot \text{kg}$ $k := 8.617 \cdot 10^{-5} \frac{\text{eV}}{\text{K}}$ $\text{hbar} := \frac{h}{2\pi}$ $c := 3 \cdot 10^8 \cdot \frac{\text{m}}{\text{sec}}$ $\text{meV} := .001 \text{eV}$ $\text{nm} := 10^{-9} \text{m}$

Then we can list the properties of GaAs since the well is made of this material. The AlGaAs outside serves to make it an almost infinite potential well:

$m_e := .063 \cdot m_0$ $m_{hh} := .51 \cdot m_0$ $m_{lh} := .082 \cdot m_0$ $E_g := 1.424 \text{eV}$ $L := 8 \cdot \text{nm}$

Now we calculate the total effective mass for holes:

$$m_p := \left(m_{hh}^{\frac{3}{2}} + m_{lh}^{\frac{3}{2}} \right)^{\frac{2}{3}} \qquad m_p = 0.532\, m_0$$

Then we use the basic formula to calculate the first eigenvalue for electrons and holes inside the quantum well:

$$E(n, m) := \frac{n^2 \pi^2 \cdot \text{hbar}^2}{2 \cdot m \cdot L^2}$$

$E_{e1} := E(1, m_e)$ $\boxed{E_{e1} = 93.261 \text{ meV}}$ for electrons

$E_{p1} := E(1, m_p)$ $\boxed{E_{p1} = 11.051 \text{ meV}}$ for holes

The total gap becomes:

$$E_{gtotal} := E_g + E_{e1} + E_{p1} \qquad E_{gtotal} = 1.528 \text{eV}$$

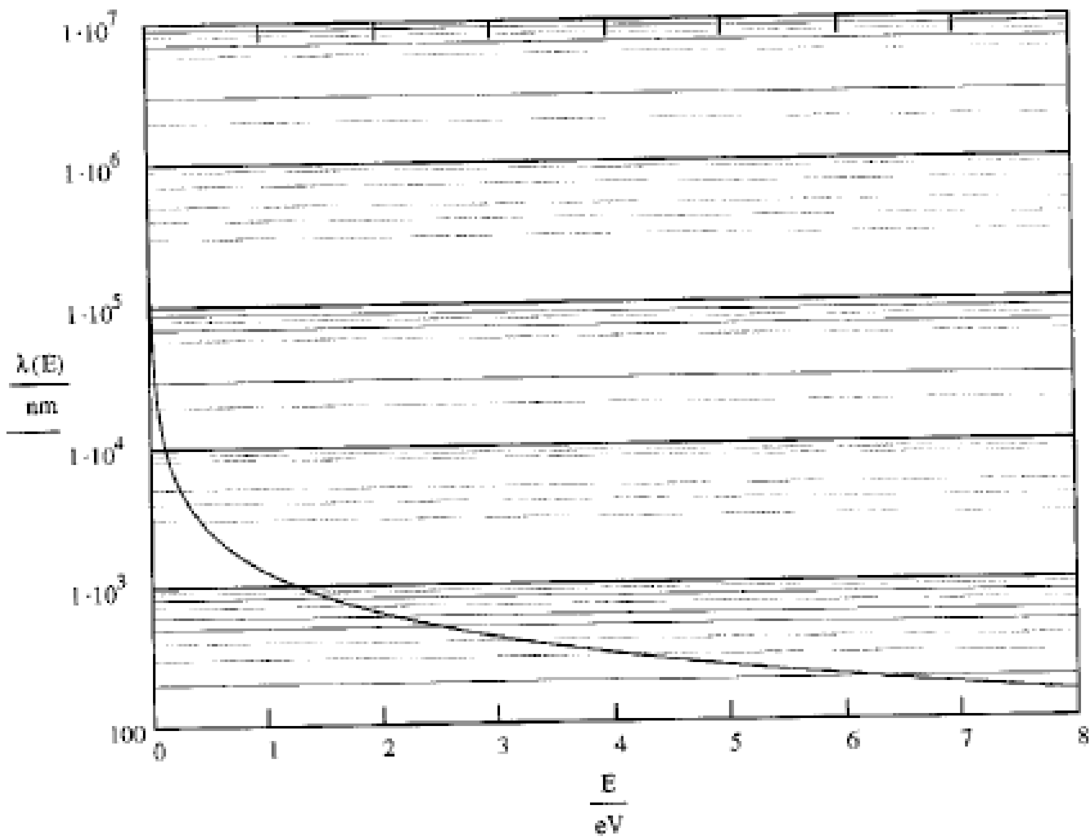
And finally we can find the wavelength of emitted light:

$$\lambda_{emitted} := \frac{h \cdot c}{E_{gtotal}} \qquad \boxed{\lambda_{emitted} = 811.811 \text{ nm}}$$

This light would be in the infrared part of the spectrum.

$$\lambda(E) := \frac{h \cdot c}{E} \quad E := 0\text{eV}, .001\text{eV} .. 8.0\text{eV}$$

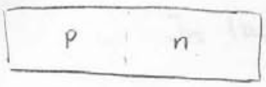
Just becuase I can here is a plot of wavelength in nm versus the Bandgap in eV.



PROBLEM 2: Next Page

Problem I. 5.11

(a)



$1\,\Omega\text{-cm}$ $\quad 0.2\,\Omega\text{-cm}$

$\tau_n = 10^{-6}\,s \quad \tau_p = 10^{-8}\,s$

From Fig. 1.15.

$\rho_p = 1\,\Omega\cdot cm, \quad p = 1.5\times10^{16}\ cm^{-3}$

$\rho_n = 0.2\,\Omega\cdot cm, \quad n = 2.9\times10^{16}\ cm^{-3}$

ASSGN5

$$V_{bi} = \frac{kT}{q}\ln\frac{N_d\cdot N_A}{n_i^2}$$

suppose fully ionized, $N_d = n$, $N_A = p$

$$\therefore V_{bi} = \frac{kT}{q}\ln\frac{np}{n_i^2} = \frac{1.38\times10^{-23}\times300}{1.6\times10^{-19}}\ln\frac{1.5\times10^{16}\times2.9\times10^{16}}{(1.45\times10^{10})^2}$$

$$= 0.722(V)$$

(b) $$n_{po} = \frac{n_i^2}{p} = \frac{(1.45\times10^{10})^2}{1.5\times10^{16}} = 1.39\times10^{4}\ cm^{-3}$$

$$p_{no} = \frac{n_i^2}{n} = \frac{(1.45\times10^{10})^2}{2.9\times10^{16}} = 7.25\times10^{3}\ cm^{-3}$$

with applied bias:

$$n_p = n_{po}\cdot(\exp\frac{qV}{kT} - 1) = 1.35\times10^{14}\ cm^{-3}$$

$$p_n = p_{no}\,(\exp\frac{qV}{kT} - 1) = 7.06\times10^{13}\ cm^{-3}$$

(c) $$J_0 = q\,n_i^2\left(\frac{D_p}{N_d L_p} + \frac{D_n}{N_a L_n}\right) = q\,n_i^2\left(\frac{1}{N_d}\sqrt{\frac{D_p}{\tau_p}} + \frac{1}{N_a}\sqrt{\frac{D_n}{\tau_n}}\right)$$

$$= 1.6\times10^{-19}\times(1.45\times10^{+10})^2\left(\frac{1}{2.9\times10^{16}}\sqrt{\frac{12.3}{10^{-8}}} + \frac{1}{1.5\times10^{16}}\sqrt{\frac{34.6}{10^{-6}}}\right)$$

$$= 5.387\times10^{-11}\ A/cm^2$$

on p-side

$$J_n\ (\text{at } x_p) = q D_n \frac{n_i^2}{N_A L_n} \left(e^{qV_a/kT} - 1\right)$$

$$= 1.6\times10^{-19} \times 34.6 \times \frac{(1.45\times10^{10})^2}{1.5\times10^{16} \times \sqrt{34.6\times10^{-6}}} (e^{23} - 1)$$

$$= 0.1285\ A/cm^2$$

on n-side

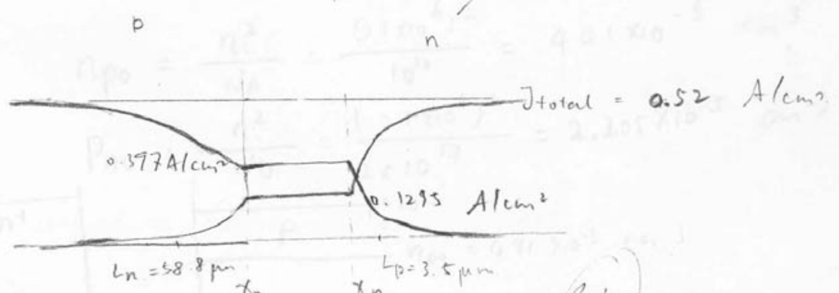
$$J_p\ (\text{at } x_n) = q D_p \frac{n_i^2}{N_D L_p} \left(e^{qV_a/kT} - 1\right)$$

$$= 1.6\times10^{-19} \times 12.3 \times \frac{(1.45\times10^{10})^2}{2.9\times10^{16} \times \sqrt{12.3\times10^{-8}}} (e^{23} - 1)$$

$$= 0.397\ A/cm^2$$

$$J_{total} = J_0 \left(e^{qV/kT} - 1\right) = 5.387\times10^{-11} (e^{23} - 1)$$

$$= 0.52\ A/cm^2$$



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(d) on n-side,

$$J_p = J_p\ (\text{at } x_n) \cdot \exp\left(-\frac{x - x_n}{L_p}\right)$$

when $J_p = J_n$, $\rightarrow$ $J_p = \frac{1}{2} J_{total}$

$$0.397 \times \exp\left(-\frac{x - x_n}{L_p}\right) = \frac{1}{2} \times 0.52$$

$$x - x_n = 1.48\ \mu m$$

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1). $W=\sqrt{2\frac{\epsilon_s}{q}(V_{bi}-V_a)(\frac{1}{N_a}+\frac{1}{N_d})}$

$=\sqrt{\frac{2\times 11.7\times 8.85\times 10}{1.6\times 10^{-19}}\times(0.7382-0.589)\times 10^{-16}\times(\frac{1}{1.5}+\frac{1}{3.0})}$

$\approx 0.135\ \mu m$ $\qquad N_A\cdot x_p = N_D\cdot x_n$

$\Rightarrow x_n=0.045,\ x_p=0.09\ \mu m$

when $J_{majo}=J_{tot}-J_{mino} \Rightarrow J_{mino}=\frac{1}{2}J_{tot}=0.247\ A/cm^2$

$\therefore J_p(x)=0.3790\cdot e^{-\frac{x-x_n}{L_p}}$

$=0.247$

$\Rightarrow x=1.4\ \mu m$ from the x_n in the n-type region.

problem 2

a) $J_0=q(\frac{D_n}{L_n}n_{p0}+\frac{D_p}{L_p}p_{n0})=qn_i^2(\frac{D_n}{L_nN_a}+\frac{D_p}{L_pN_d})$ $\qquad D_n\simeq 200\ cm^2/s \quad D_p\simeq 10\ cm^2/s$

$n_i\simeq 2.1\times 10^6\ cm^{-3}$

$L_n=\sqrt{D_n\tau_n}=0.0063\ cm$

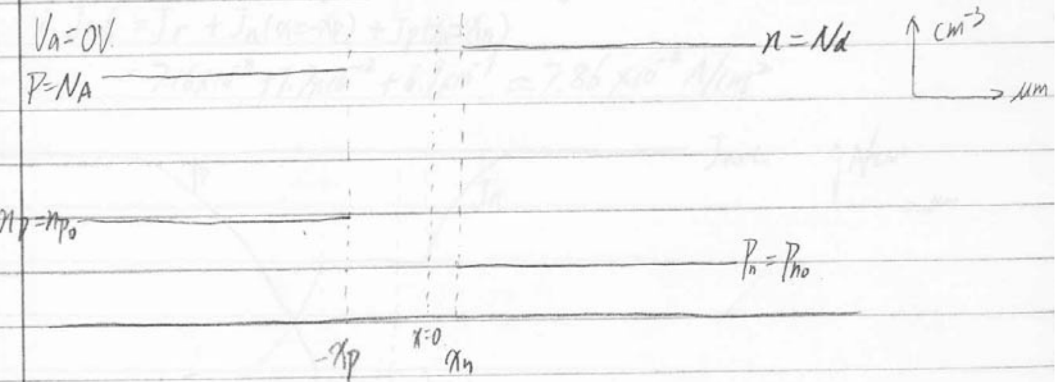
$L_p=\sqrt{D_n\tau_p}=0.0014\ cm$

$\therefore J_0=2.2365\times 10^{-17}\ A/cm^2$

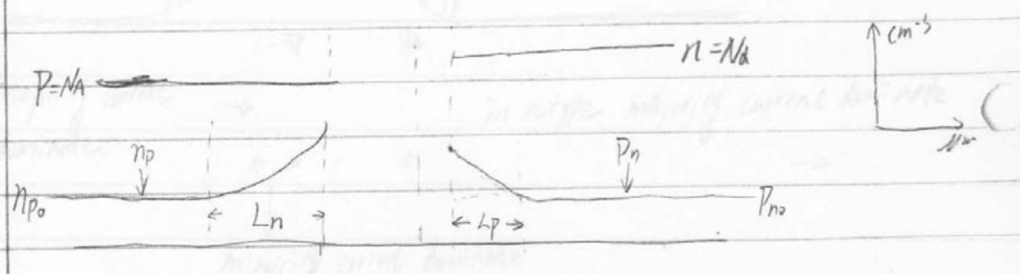
b) in the p-side $\quad P=N_A \quad n_p=n_{p0}\cdot e^{\frac{qV_a}{kT}}\ e^{-x/L_n}=\frac{n_i^2}{N_A}\cdot e^{\frac{qV_a}{kT}}\cdot e^{\frac{(x+x_p)}{L_n}}$

in the n-side $\quad n=N_D \quad P_n=P_{n0}\cdot e^{\frac{qV_a}{kT}}\cdot e^{-x/L_p}=\frac{n_i^2}{N_D}\cdot e^{\frac{qV_a}{kT}}\cdot e^{\frac{-(x-x_n)}{L_p}}$

$V_a=0V$



$V_a=0.5V$



c). Outside depletion region in n-type

$$J_p = q n_i^2 \frac{D_p}{N_d L_p} (e^{qV_a/kT} - 1) e^{-\frac{x - x_n}{L_p}} \approx 7.7\times10^{-12} e^{-\frac{x-x_n}{L_p}} \qquad x > x_n$$

Outside depletion region in p-type

$$J_n = q n_i^2 \frac{D_p}{N_a L_n} (e^{qV_a/kT} - 1) e^{\frac{x+x_p}{L_n}} = 6.9\times10^{-9} e^{\frac{x+x_p}{L_n}} \qquad x < -x_p$$

$$\begin{cases} W = \sqrt{\frac{2\varepsilon_s}{q}(V_{bi} - V_a)\left(\frac{1}{N_a} + \frac{1}{N_d}\right)} \\ V_{bi} = \frac{kT}{q} \ln \frac{N_a N_d}{n_i^2} \end{cases}$$

$\Rightarrow W = 0.81\ \mu m$

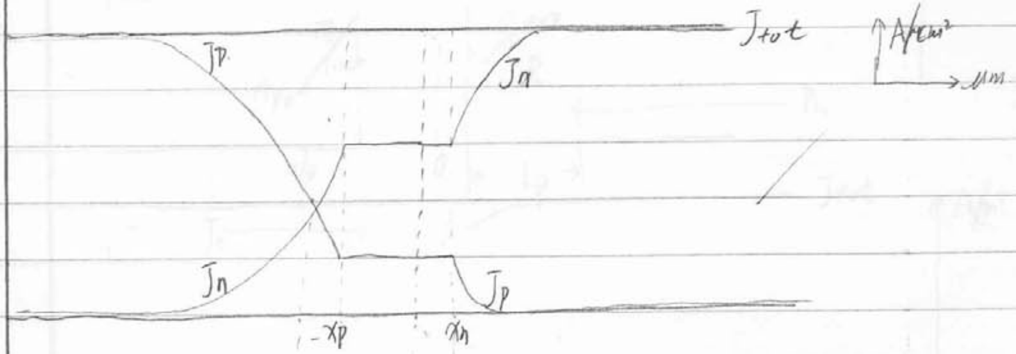
At the juction $x=0$, $E(0) = \frac{2(V_{bi} - V_a)}{W} = 16.75\ kV/cm$

$$J_{recombination} = \frac{q\pi n_i}{2\tau} \cdot \frac{kT}{q E(0)} \exp\left(\frac{qV_a}{2kT}\right) = 7.16\times10^{-8}\ A/cm^2$$

Inside the depletion region, the minority current is constant

$\therefore J_{tot} = J_r + J_n(x=-x_p) + J_p(x=x_n)$

$= 7.16\times10^{-8} + 7.7\times10^{-12} + 6.9\times10^{-9} \simeq 7.86\times10^{-8}\ A/cm^2$



← majority current → dominate.

in n-type majority current dominate →

minority current dominate

d). In the p-type region. $L_n = \sqrt{D_n \tau_n} = \sqrt{200 \times 0.2\times10^{-6}} = 0.0063\ cm \simeq 63\ um >> W_B$

$\therefore \Delta n(x) = \frac{n_i^2}{N_a}\left(\frac{x+x_p}{W_B - x_p}+1\right)\left(e^{qV_a/kT}-1\right)$ $\quad n(x) = \Delta n(x) + n_{p0}$

$\because N_A x_p = N_D x_n$

$\therefore x_p \simeq W = 0.8\ um$, $J_n(x=-x_p) >> J_p(x=x_n)$

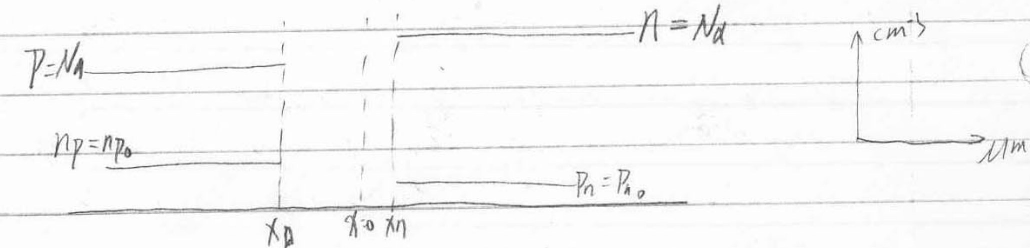
$J_n = qD_n \frac{dn(x)}{dx} = \frac{q n_i^2 D_n}{N_a(W_B - x_p)}\left(e^{\frac{qV_a}{kT}}-1\right)$ where $x_p \simeq W = 0.8\ um$.

$\Rightarrow J_n(x=-x_p) \simeq 3.3\times10^{-7}\ A/cm^2$

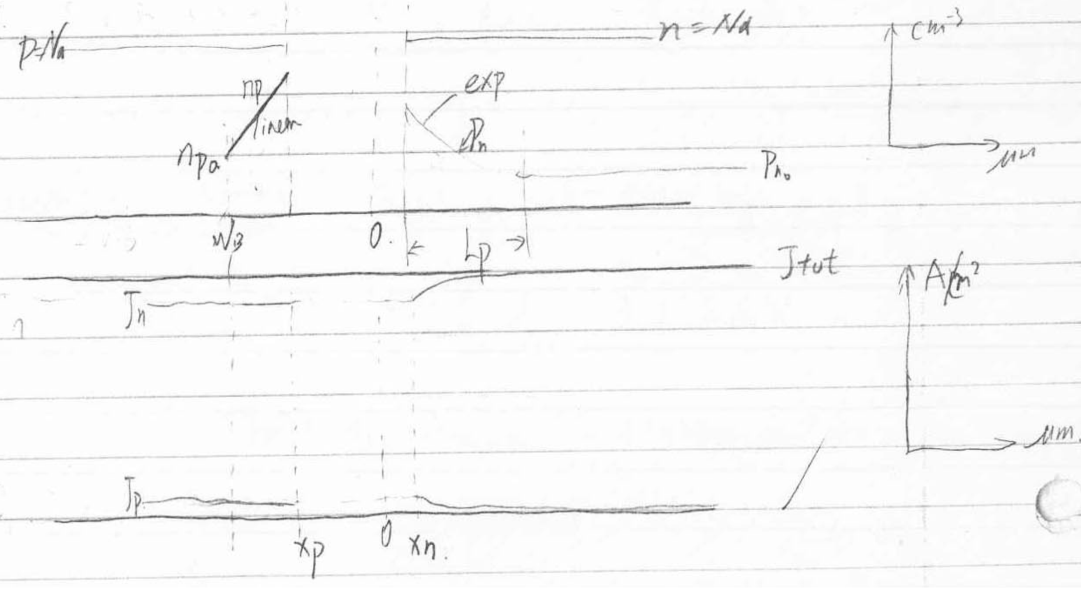
J_{rec} is not changed. $J_{rec} = 7.16\times10^{-8}\ A/cm^2$

$J = J_n + J_p + J_{rec} \simeq 4.0\times10^{-7}\ A/cm^2$

$V_a = 0V$



$V_a = 0.5V$



e. The current in long base diode is much smaller than the current in short base diode.

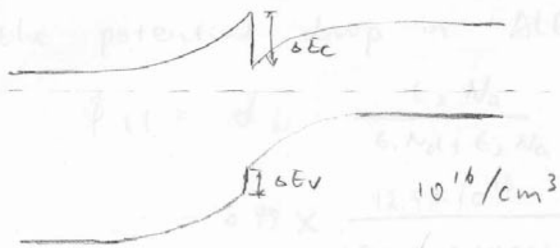
The reason is in long base diode $W_B \simeq L$, therefore most minority carrier has been combined before they reach the contact. While in short diode, $W_B << L$. Therefor most minority carrier can get through the base without recombination. When they reach the contact, the current increase.

PROBLEM 4

For $Al_xGa_{1-x}As$ / GaAs heterojunction:

when $x < 0.45$, electron affinity: $\chi = 4.07 - 1.1x$ eV

GaAs, $\chi = 4.07$ eV



Since it is $n^+ p$ junction, electrons injected from n^+ to p region

$$J_n = -\frac{qD_n}{L_n}\frac{n_{i1}^2}{N_{a1}}\exp\left(\frac{\Delta E_g}{kT}\right)\left[\exp\left(\frac{qV_a}{kT}\right)-1\right]$$

to increase by 100, we can keep the doping same and introducing ΔE_g

$$\exp\left(\frac{\Delta E_g}{kT}\right) = 0.119 \text{ eV}$$

$$E_{gAlGaAs} - E_{gGaAs} = 1.155x + 0.37x^2 = 0.119 \text{ eV}$$

$$x = 0.1$$

$$\phi_i = \chi_2 - \chi_1 + \frac{E_{g2}}{q} - \frac{kT}{q} \ln \frac{N_{c1} N_{v2}}{N_{d1} N_{a2}}$$

$$= -1.1 \times 0.1 + 1.42 - \frac{kT}{q} \ln \frac{1.9 \times 10^{19} \times 9.5 \times 10^{18}}{10^{17} \times 10^{16}}$$

$$= -0.11 + 1.42 - 0.312$$

$$= 0.99 \text{ eV}$$

the potential drop in AlGaAs region:

$$\phi_{i1} = \phi_i \cdot \frac{\epsilon_2 N_a}{\epsilon_1 N_d + \epsilon_2 N_a}$$

$$= 0.99 \times \frac{12.9 \times 10^{16}}{(12.9 - 2.84 \times 0.1) \times 10^{17} + 12.9 \times 10^{16}}$$

$$= 0.99 \times \frac{12.9}{12.6 \times 10 + 12.9}$$

$$= 0.09 \text{ eV}$$

$$\therefore \phi_{i2} = 0.9 \text{ eV}$$

In AlGaAs,

$$\frac{q N_d x_n^2}{2 \epsilon_1} = 0.09 \text{ eV}, \quad x_n = \sqrt{\frac{0.09 \times 2 \times 12.6 \times \epsilon_0}{1.6 \times 10^{-19} \times 10^{23}}}$$

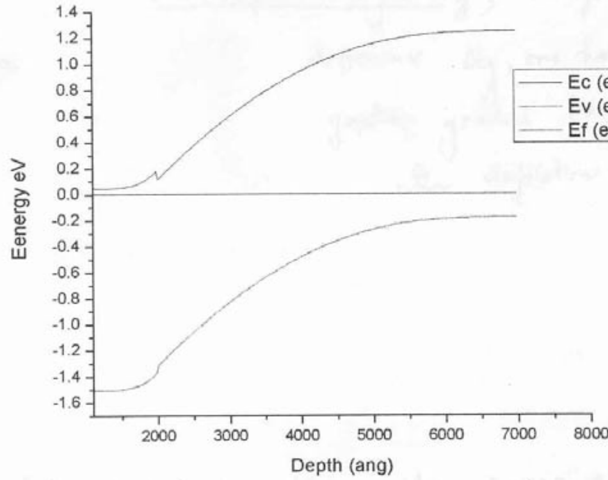
$$x_n = 3.54 \times 10^{-8} \text{ m} = 35.4 \text{ nm}$$

In GaAs

$$\frac{q N_d x_p^2}{2 \epsilon_2} = 0.9 \text{ eV}, \quad x_p = \sqrt{\frac{0.9 \times 2 \times 12.9 \times \epsilon_0}{1.6 \times 10^{-19} \times 10^{22}}}$$

$$x_p = 3.58 \times 10^{-7} \text{ m} = 358 \text{ nm}$$

With I-V posim, for $Al_{0.1}Ga_{0.9}As$ / GaAs with doping of $1\times10^{17}/cm^3$ and $1\times10^{16}\ cm^{-3}$, we can see the ΔE_c is not very big.



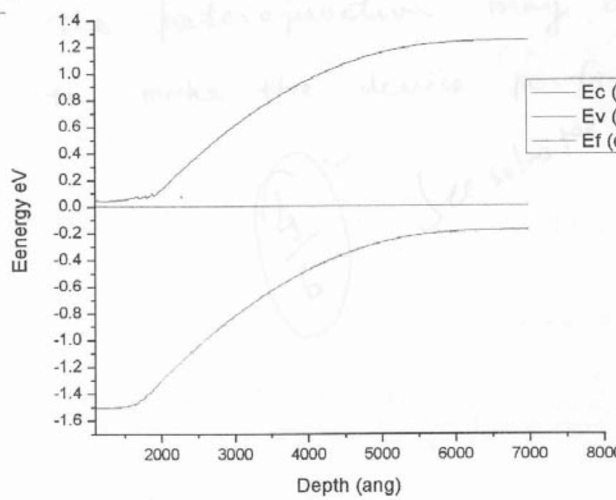
If I add 3 gradient layer to the structure, the ΔE_c kink could be removed:

GaAs $\quad N_a = 1\times10^{16}/cm^3$

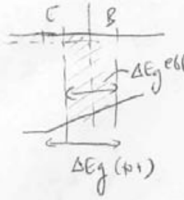
$Al_{0.01}Ga_{0.99}As$	30nm	$N_d = 1\times10^{17}/cm^3$
$Al_{0.04}Ga_{0.96}As$	30 nm	
$Al_{0.07}Ga_{0.93}As$	30 nm	

$Al_{0.1}Ga_{0.9}As$

try 'graded' structure too....



(c) Since the $e^{\frac{\Delta E_g}{kT}}$ factor depends upon the ΔE_g across the E·B depletion region only, to get all the bandgap difference ΔE_g, one has to ensure that the ~~grading~~ graded layer lies entirely inside the depletion region.



(d). for heterojunction, the current is high

The disadvantage is : require epitaxy growth, thus increasing cost.

The heterojunction may contain defects to make the device performance worse.