EE566 Solid State Devices

Spring 2007
Dept of Electrical Engineering
University of Notre Dame
Instructor: Debdeep Jena (djena@nd.edu, x8835)

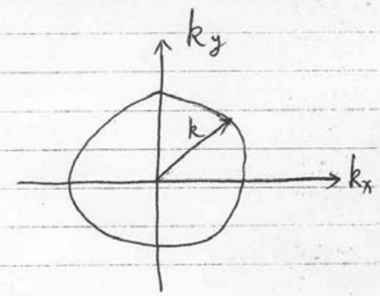
Assignment 2
SOLUTIONS

Problem 1:

1\. Lian→

Assume the side of the 2D semiconductor is L
Then the density of states for 2D is

$$g(E) = N(E)/E/L^2$$



$$N(E) = 2\cdot\frac{1}{4}\cdot\frac{\pi^2 k^2}{(\pi/L)^2}$$

$$= \frac{1}{2}\frac{k^2L^2}{\pi}$$

$$E = \frac{p^2}{2m^*} = \frac{\hbar^2k^2}{2m^*} \Rightarrow k^2 = \frac{2m^*E}{\hbar^2}$$

$$\therefore N(E) = \frac{m^*EL^2}{\pi\hbar^2}$$

$$\therefore g(E) = \frac{m^*}{\pi\hbar^2}$$

$$n_{2d} = \int_{E_c}^{\infty} g(E)\, f(E)\, dE$$

$$= \int_{E_c}^{\infty} \frac{m^*}{\pi\hbar^2}\cdot\frac{1}{1+e^{(E-E_F)/kT}}\, dE$$

$$= \frac{m^*}{\pi\hbar^2}kT\int_{\frac{E_c-E_F}{kT}}^{\infty}\frac{1}{1+e^{(E-E_F)/kT}}\, d\left(\frac{E-E_F}{kT}\right)$$

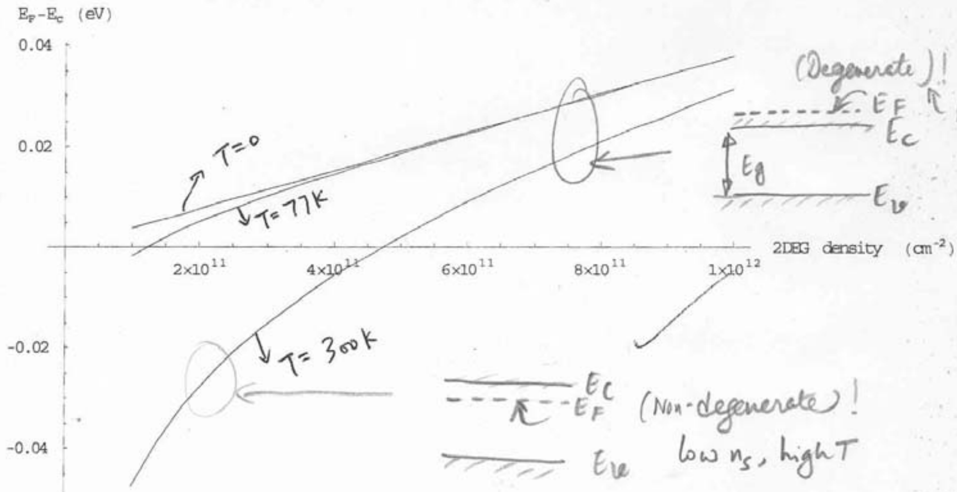
$$= \frac{m^*}{\pi\hbar^2}kT\cdot\ln\left(1+e^{\frac{E_F-E_c}{kT}}\right)$$

Let $N_c^{2d} = \frac{m^*}{\pi\hbar^2}kT$, $\eta = \frac{E_F-E_c}{kT}$

Then $n_{2d} = N_c^{2d}(1+e^{\eta})$

At $T=77K, 300K$: $E_F - E_c = kT \ln\left[\exp\left(\frac{n_{2d}\,\pi\hbar^2}{m^* kT}\right) - 1\right]$

At $T=0$, $E_F - E_c = \frac{n_{2d}\,\pi\hbar^2}{m^*}$



$$n_{2d} = \frac{m^* kT}{\pi\hbar^2} \ln\left(1 + e^{\frac{E_F - E_c}{kT}}\right)$$

From the above figure, it is shown that at low temperature the 2DEG carrier distribution is DEGENERATE.

$\therefore$ $E_F > E_c$ and for small T, $\frac{E_F - E_c}{kT} \gg 1$

then $n_{2d} = \frac{m^* kT}{\pi\hbar^2} \cdot \ln\left(e^{\frac{E_F - E_c}{kT}}\right) = \frac{m^*}{\pi\hbar^2}(E_F - E_c)$

$\therefore$ at low temperature, n_{2d} is independent of temperature.

6/6

Problem 2:

So plot figure as follows. (fig. 1).

2DEG carrier distribution is degenerate

6/6

2 a) At room temperature, assume $n = N_D(x)$

When equilibrium condition is satisfied. $\vec{J} = 0$

i.e. $\vec{J} = qn\mu\vec{\mathcal{E}} + qD_n\frac{dn}{dx} = 0$

$$\Rightarrow \mathcal{E}(x,T) = \frac{-D_n}{\mu n}\frac{dn}{dx}$$

$$= \frac{-D_n}{\mu N_D(x)} \cdot \frac{dN_D(x)}{dx}$$

Solution

Jing →

From $\frac{D_n}{\mu} = \frac{kT}{q}$

We get the relationship between $\mathcal{E}$ and T is

$$\mathcal{E}(x,T) = -\frac{kT}{qN_D(x)}\frac{dN_D(x)}{dx}$$

b) As temperature increase, the magnitue of $\mathcal{E}$ increases ~~proportional~~ to the temperature. The direction of $\mathcal{E}$ will not change. — All we can say is that $\mathcal{E}\uparrow$ as $T\uparrow$.

$\mathcal{E} = f(N_D, n)$ which change with temperature too.

c) For constant doping $\frac{dN_D(x)}{dx} = 0$. $\Rightarrow \mathcal{E}(x,T) = 0$.

d) $$\mathcal{E} = -\frac{kT}{qN_0 e^{-(\frac{x}{\lambda})^2}} \cdot N_0 e^{-(\frac{x}{\lambda})^2}\left(-\frac{2x}{\lambda^2}\right)$$

$$= +\frac{2kTx}{q\lambda^2}$$

Please refer to fig. 2.

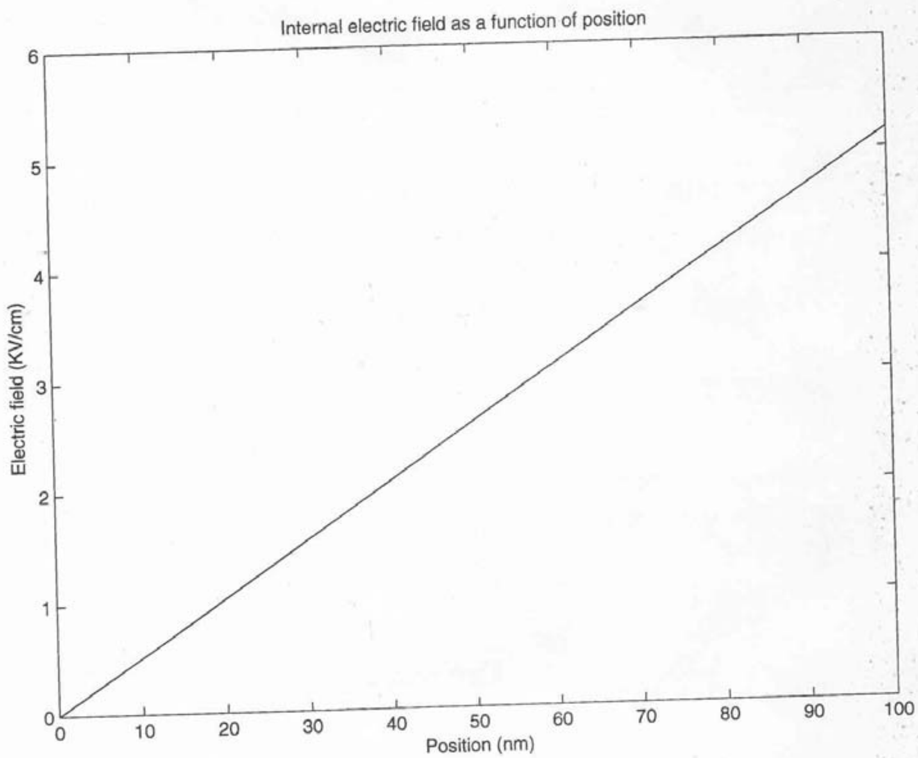

Internal electric field as a function of position
Electric field (KV/cm)
Position (nm)
0
1
2
3
4
5
6
0
10
20
30
40
50
60
70
80
90
100

Fig 2.

Problem 3: (3.7, MKC)

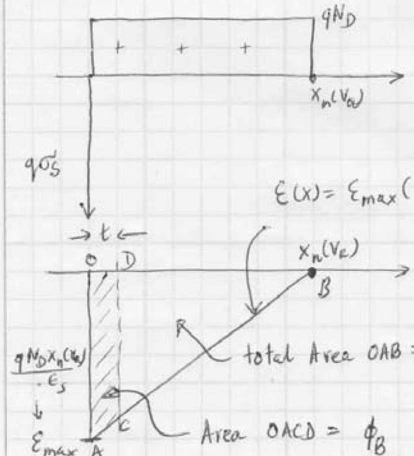
Assignment 3 · EE 566 SOLID STATE DEVICES, SPRING 2005 02/16/05

SOLUTIONS

PROBLEM 1 (3.7, MKC).

See the charge-field-band diagram.

Electric field: $\mathcal{E}(0) = \mathcal{E}_{max}$



$$\frac{1}{2}\mathcal{E}_{max}\, x_n(V_a) = \phi_i - V_a$$

$$OACD \rightarrow \frac{1}{2}\left[\mathcal{E}_{max} + \mathcal{E}(t)\right]\cdot t = \phi_B$$

$$\mathcal{E}(x) = \mathcal{E}_{max}\left(1 - \frac{x}{x_n(V_a)}\right) \longrightarrow \mathcal{E}(t) = \mathcal{E}_{max}\left(1 - \frac{t}{x_n(V_a)}\right).$$

$$\therefore\ \mathcal{E}_{max}\left[2 - \frac{t}{x_n(V_a)}\right] = \frac{2\phi_B}{t}$$

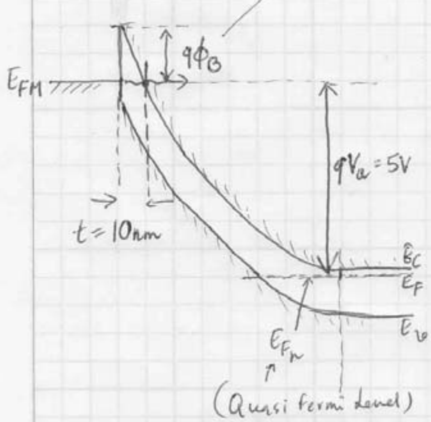
Also, $$x_n(V_a) = \frac{\epsilon_s \mathcal{E}_{max}}{qN_D} = \frac{2(\phi_i - V_a)}{\mathcal{E}_{max}}$$

Therefore, we get

$$\mathcal{E}_{max}\left[2 - \frac{t\cdot\mathcal{E}_{max}}{2(\phi_i - V_a)}\right] = \frac{2\phi_B}{t}$$

$$\mathcal{E}_{max}^2 - \frac{4(\phi_i - V_a)}{t}\mathcal{E}_{max} + \frac{4\phi_B(\phi_i - V_a)}{t^2} = 0$$

$$\therefore\ \mathcal{E}_{max} = \frac{2(\phi_i - V_a)}{t}\left[1 \pm \sqrt{1 - \frac{\phi_B}{\phi_i - V_a}}\right]$$



→ ONLY REASONABLE SOLN:

$$\mathcal{E}_{max} = 6.72 \times 10^5\ V/cm.$$

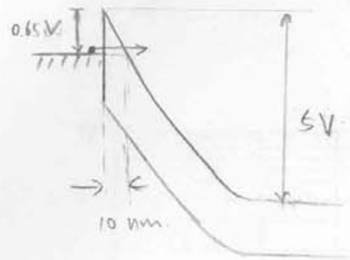
(other is $\mathcal{E}'_{max} = 1.93\times 10^7\ V/cm \gg \mathcal{E}_{breakdown}$)

$$\Rightarrow \boxed{N_D = \frac{\epsilon_s \mathcal{E}_{max}^2}{2q(\phi_i - V_a)} \sim 3\times 10^{17}/cm^3}$$

b) From fig 1-15 in MKC, for $N_D \sim 3\times 10^{17}/cm^3$, $\boxed{\rho \sim 5\times 10^{-2}\ \Omega\cdot cm}$

c) Shown above.

Problem 4: (3.17, MKC)



$$\frac{df}{dV} = -\frac{1}{4\pi\sqrt{LC}}\,\frac{1}{C}\,\frac{dC}{dV} = 0.22 \times 10^{6}\ \text{Hz/V}$$

$$\because \quad N(x_d) = \frac{(C/A)^3}{q\epsilon_s\, d(C/A)/dV}, \qquad \frac{C^3}{A^2 N(x_d) q \epsilon_s} = \frac{dC}{dV}$$

$$\frac{-1}{4\pi\sqrt{LC}}\,\frac{1}{C}\,\frac{C^3}{A^2 N(x_d)\, q\epsilon_s} = -0.22 \times 10^{6}$$

$$\frac{C^2}{A^2 4\pi\sqrt{LC}\ q\epsilon_s \times 0.22\times10^{6}} = N(x_d)$$

$$C = A\cdot \epsilon_s / x_d$$

$$\therefore \quad \frac{A^2\epsilon_s^2/x_d^2}{A^2 4\pi\sqrt{L\cdot A\epsilon_s/x_d}\cdot q\epsilon_s \times 0.22\times10^{6}} = N(x_d)$$

$$\frac{(11\times 8.854\times10^{-12})^{\frac{1}{2}}/x_d^2}{4\times3.14\times\sqrt{2\times10^{-3}\times10^{-7}/x_d}\times1.6\times10^{-19}\times0.22\times10^{6}} = N(x_d)$$

$$1.58\times10^{12}\ x_d^{-\frac{3}{2}} = N(x_d)$$

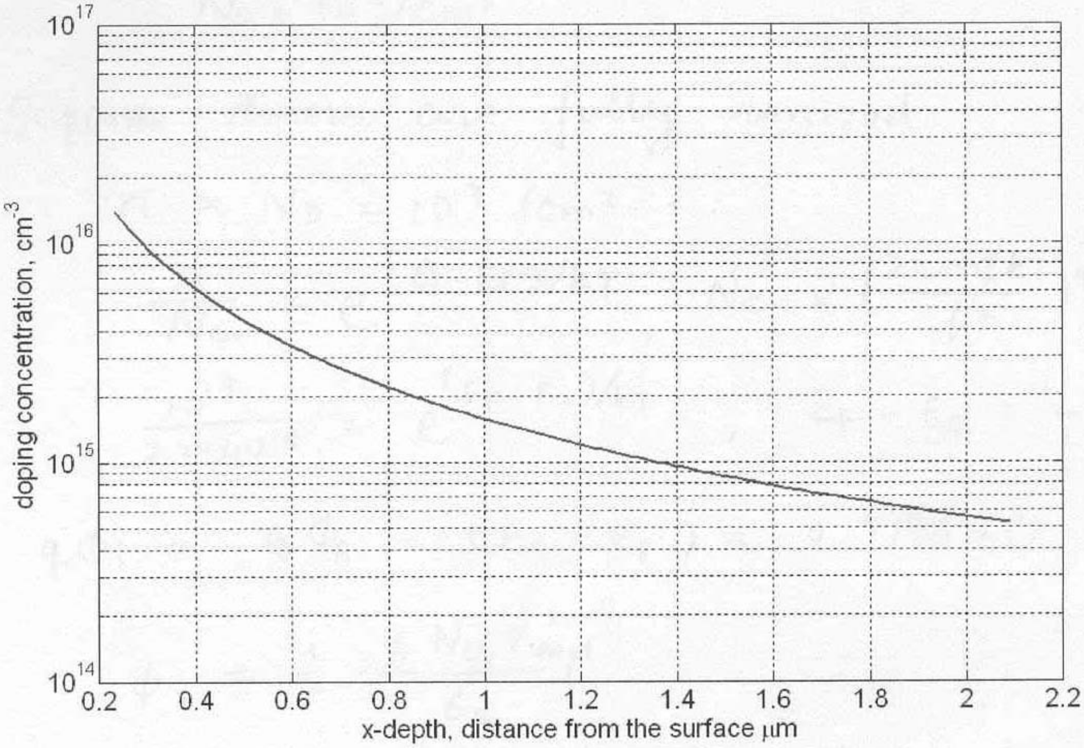
at $V = 0$, $\quad C = 41.8\times10^{-12} = A\cdot\frac{\epsilon_s}{x_d}$, $x_d = 2.33\times10^{-7}$ m

$= 233$ nm

$C = 4.65\times10^{-12} = A\cdot\frac{\epsilon_s}{x_d}$, $x_d = 2.09\times10^{-6}$ m

$= 2.09$ μm

The doping profile is plotted on the next page.



10^{17}
10^{16}
10^{15}
10^{14}
doping concentration, cm^{-3}
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
2.2
x-depth, distance from the surface μm