EE566 Solid State Devices

Spring 2006
Dept of Electrical Engineering
University of Notre Dame
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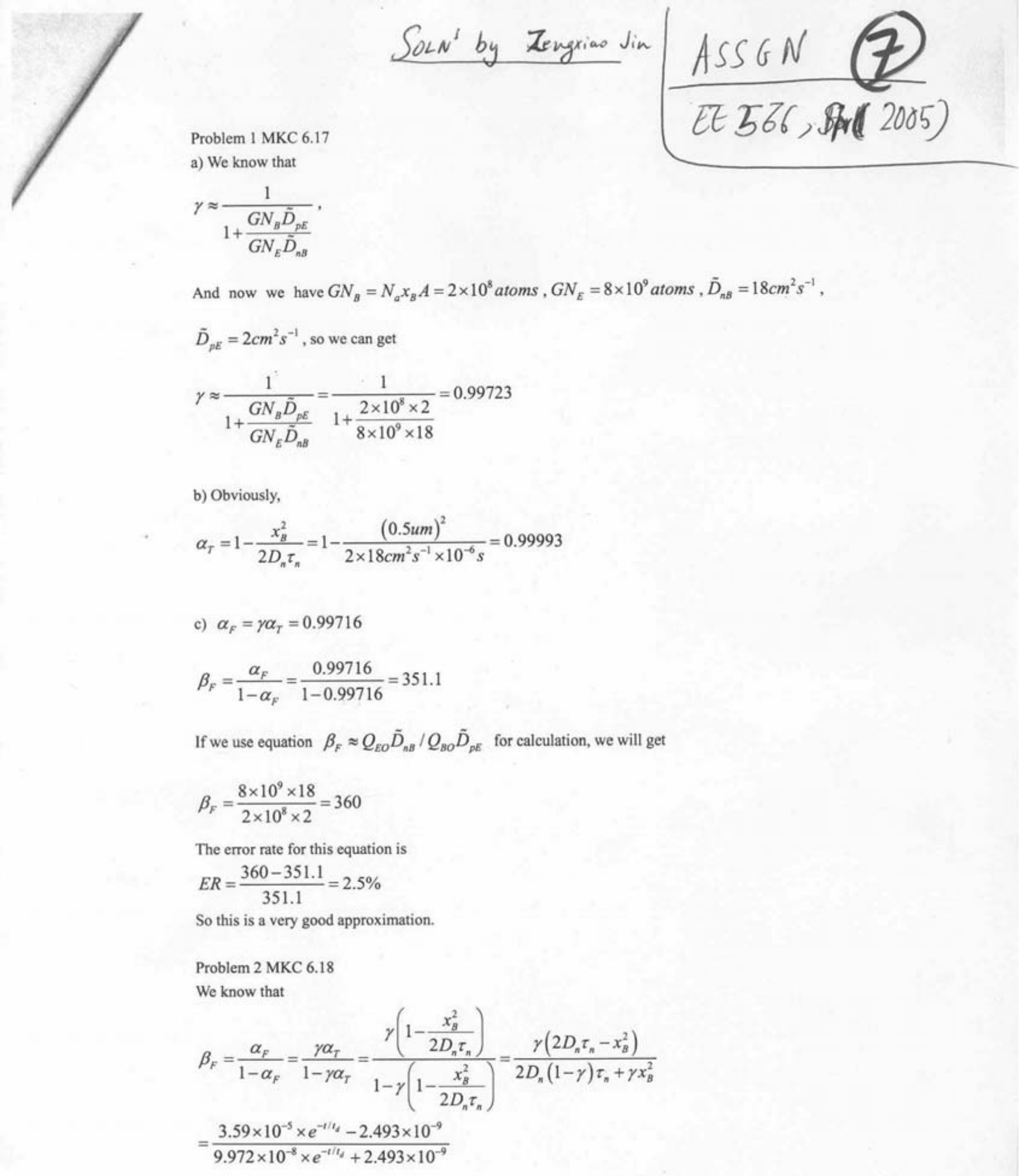
Assignment 7

SOLUTIONS



Soln' by Zengxiao Jin

ASSGN ⑦
EE 566, Fall 2005)

Problem 1 MKC 6.17

a) We know that

$$\gamma \approx \frac{1}{1+\frac{GN_B \tilde{D}_{pE}}{GN_E \tilde{D}_{nB}}},$$

And now we have $GN_B = N_a x_B A = 2\times10^8\,atoms$, $GN_E = 8\times10^9\,atoms$, $\tilde{D}_{nB} = 18cm^2s^{-1}$,

$\tilde{D}_{pE} = 2cm^2s^{-1}$, so we can get

$$\gamma \approx \frac{1}{1+\frac{GN_B \tilde{D}_{pE}}{GN_E \tilde{D}_{nB}}} = \frac{1}{1+\frac{2\times10^8\times2}{8\times10^9\times18}} = 0.99723$$

b) Obviously,

$$\alpha_T = 1-\frac{x_B^2}{2D_n\tau_n} = 1-\frac{(0.5um)^2}{2\times18cm^2s^{-1}\times10^{-6}s} = 0.99993$$

c) $\alpha_F = \gamma\alpha_T = 0.99716$

$$\beta_F = \frac{\alpha_F}{1-\alpha_F} = \frac{0.99716}{1-0.99716} = 351.1$$

If we use equation $\beta_F \approx Q_{EO}\tilde{D}_{nB} / Q_{BO}\tilde{D}_{pE}$ for calculation, we will get

$$\beta_F = \frac{8\times10^9\times18}{2\times10^8\times2} = 360$$

The error rate for this equation is

$$ER = \frac{360-351.1}{351.1} = 2.5\%$$

So this is a very good approximation.

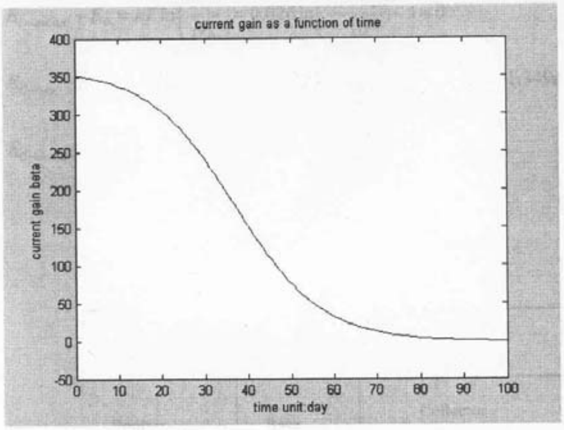
Problem 2 MKC 6.18

We know that

$$\beta_F = \frac{\alpha_F}{1-\alpha_F} = \frac{\gamma\alpha_T}{1-\gamma\alpha_T} = \frac{\gamma\left(1-\frac{x_B^2}{2D_n\tau_n}\right)}{1-\gamma\left(1-\frac{x_B^2}{2D_n\tau_n}\right)} = \frac{\gamma\left(2D_n\tau_n - x_B^2\right)}{2D_n(1-\gamma)\tau_n + \gamma x_B^2}$$

$$= \frac{3.59\times10^{-5}\times e^{-t/t_d} - 2.493\times10^{-9}}{9.972\times10^{-8}\times e^{-t/t_d} + 2.493\times10^{-9}}$$

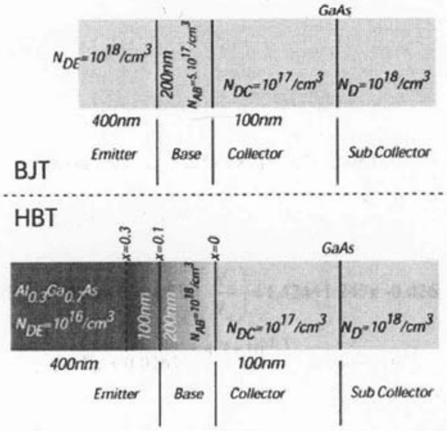
So we can get β_F as a function of time as below,



The time interval until β_F drops to unity is 88.8 days, and at that time the base lifetime is

$$\tau_n = \tau_{n0}\exp\left(-t/t_d\right) = 1.39\times10^{-10}\,s$$

Problem 3



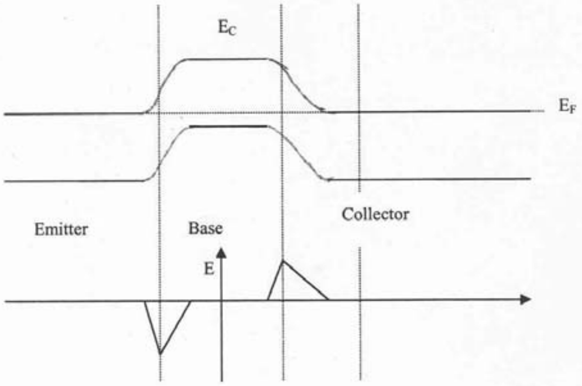
a) The grading of composition from GaAs to $Al_xGa_{1-x}As$ in base and the grading of composition in emitter can reduce the effect of the spike in the conduction band. Plot ρ -E-B diagram of BJT and HBT as below:

For BJT

$$E_{C-emitter} - E_F = kT\ln\left(\frac{N_C}{N_D}\right) = 0.026\ln\left(\frac{4.7\times10^{17}}{10^{18}}\right) \approx 0$$

$$E_{C-base} - E_F = E_g - kT\ln\left(\frac{N_v}{N_A}\right) = 1.424 - 0.026\ln\left(\frac{9\times10^{18}}{5\times10^{17}}\right) = 1.349eV$$

$$E_{C-emitter} - E_F = 0.026\ln\left(\frac{4.7\times10^{17}}{10^{17}}\right) = 0.04eV \approx 0$$



For HBT

$$For\ Al_xGa_{1-x}As\ N_C = 2.5\times10^{19}(0.063+0.083x)^{3/2}, N_v = 2.5\times10^{19}\times(0.85\text{-}0.14\text{x})^{3/2}$$

$$\Rightarrow E_{c-emitter} - E_F = kT\ln\left(\frac{N_C}{N_D}\right) = 0.026\ln\left(\frac{2.5\times10^{19}(0.063+0.083x)^{3/2}}{10^{16}}\right)$$

$$E_{c-base} - E_F = E_g - kT\ln\left(\frac{N_v}{N_A}\right) = 1.424 + 1.247\text{x} - 0.026\ln\left(\frac{2.5\times10^{19}(0.063+0.083x)^{3/2}}{10^{18}}\right)$$

$$E_{c-emitter} - E_F = 0.026\ln\left(\frac{4.7\times10^{17}}{10^{17}}\right) = 0.04eV \approx 0$$

And we can get the depletion width in emitter

$$X_n^{BE} = \frac{N_{AB}}{N_{AB}+N_{DE}}W_{delp}^{BE} = \frac{N_{AB}}{N_{AB}+N_{DE}}\sqrt{\frac{2\varepsilon_s}{q}\left(\frac{1}{N_{DE}}+\frac{1}{N_{AB}}\right)(V_{bi}-2kT)} = 25nm$$

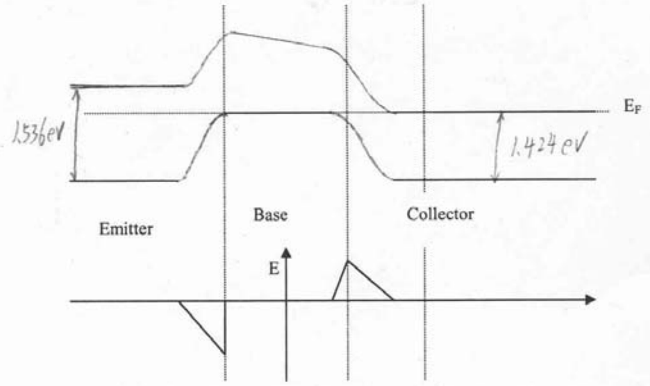
And in base

$$X_p^{BE}=\frac{N_{DE}}{N_{AB}+N_{DE}}W_{delp}^{BE}=\frac{N_{DE}}{N_{AB}+N_{DE}}\sqrt{\frac{2\varepsilon_s}{q}\left(\frac{1}{N_{DE}}+\frac{1}{N_{AB}}\right)\left(V_{bi}-2kT\right)}=50nm$$

$$X_p^{BC}=\frac{N_{DC}}{N_{AB}+N_{DC}}W_{delp}^{BC}=\frac{N_{DC}}{N_{AB}+N_{DC}}\sqrt{\frac{2\varepsilon_s}{q}\left(\frac{1}{N_{DC}}+\frac{1}{N_{AB}}\right)\left(V_{bi}-2kT\right)}=25nm$$

$$W_B^{'}=W_B-X_p^{BE}-X_p^{BC}=125nm$$

The sketch is as below



The purpose of the heavily doped sub-collector is to decrease the resistance in collector.

To get the quasi-electric field in HBT, we have

$$X_p^{BC}=\frac{N_{DC}}{N_{AB}+N_{DC}}W_{delp}^{BC}=\frac{N_{DC}}{N_{AB}+N_{DC}}\sqrt{\frac{2\varepsilon_s}{q}\left(\frac{1}{N_{DC}}+\frac{1}{N_{AB}}\right)\left(V_{bi}-2kT\right)}=13nm$$

$$Also, X_p^{BE}=1.4nm$$

$$So, W_B^{'}=W_B-X_p^{BC}-X_P^{BE}=186nm$$

Also, in emitter, the depletion width is

$$X_n^{BE}=\frac{N_{AB}}{N_{AB}+N_{DE}}W_{delp}^{BE}=\frac{N_{AB}}{N_{AB}+N_{DE}}\sqrt{\frac{2\varepsilon_s}{q}\left(\frac{1}{N_{DE}}+\frac{1}{N_{AB}}\right)\left(V_{bi}-2kT\right)}=140nm$$

which shows the grading region is totally depleted.

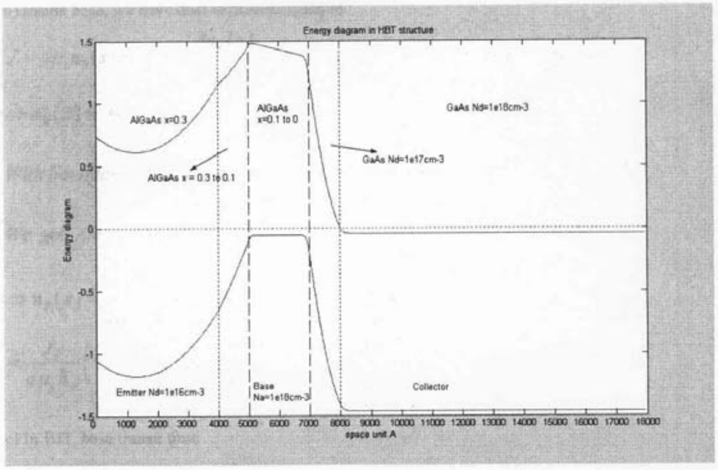
Band gap change over $W_B^{'}$ is $\Delta E_g^{base}\times\frac{186nm}{200nm}\approx\Delta E_g^{base}$

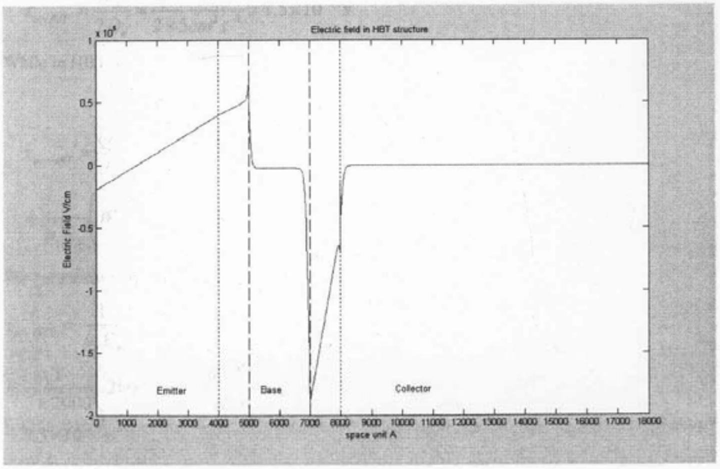
Now since base is doped p-type, E_V is flat, $\Delta E_C = \Delta E_g$,

And we get the quasi electric field

$$E_B = \frac{\Delta E_g^{base}}{qW_B^{depl}} = \frac{0.374 \times 0.1}{186nm} = 2kV/cm$$

Using 1-D Poisson to verify the result





b) Inside base, we have drift-diffusion equation

$$J = q\mu_n n_B(x) E_B + qD_n \frac{dn_B(x)}{dx} = J_c \textit{ Assume no recombination in the base}$$

$$\Rightarrow n_B(x) = Ae^{\mu_n E_B x / D_n} + \frac{J_C}{q\mu_n E_B} = Ae^{qE_B x/kT} + \frac{J_C}{q\mu_n E_B}$$

$$\textit{With boundary condition } n_B\left(W_B^{'}\right) = Ae^{qE_B W_B^{'}/kT} + \frac{J_C}{q\mu_n E_B} = 0$$

$$\textit{We get } A = \frac{-J_C}{q\mu_n E_B e^{qE_B W_B^{'}/kT}}$$

$$\Rightarrow n_B(x) = \frac{-J_C}{q\mu_n E_B e^{qE_B W_B^{'}/kT}} e^{qE_B x/kT} + \frac{J_C}{q\mu_n E_B}$$

$$= \frac{J_C}{q\mu_n E_B}\left(1 - e^{qE_B\left(x - W_B^{'}\right)/kT}\right)$$

c) In BJT, base transit time

$$\tau_{tr-BJT} = \frac{W_B^2}{2D_n} \approx \frac{(125nm)^2}{2\times 5cm^2 s^{-1}} = 1.5\times 10^{-11} s$$

While in HBT, we have

$$\tau_{tr-HBT} = \frac{Q_n}{J_n} = \frac{q\int_0^{W_B^{'}} \frac{J_C}{q\mu_n E_B}\left(1 - e^{qE_B\left(x - W_B^{'}\right)/kT}\right)dx}{J_C}$$

$$= \frac{1}{\mu_n E_B}\left(W_B^{'} - \frac{kT}{qE_B}\left(1 - e^{-qE_B W_B^{'}/kT}\right)\right)$$

For numerical value, we can use the result in a)

$$\tau_{tr-HBT} = \frac{1}{\mu_n E_B}\left(W_B^{'} - \frac{kT}{qE_B}\left(1 - e^{-qE_B W_B^{'}/kT}\right)\right) \approx \tau_{tr}(BJT) * \frac{2kT}{\Delta E_g} \sim 42\%$$

← good assumption here ..

~~$\approx \frac{1}{7\times 2000}\left(200\times 10^{-7} - \frac{0.026}{2000}\left(1 - e^{-2000\times 200\times 10^{-7}/0.026}\right)\right)$~~

~~$= 7.3\times 10^{-10} s$~~

The ratio between the two base transit time is

$$\frac{\tau_{tr-BJT}}{\tau_{tr-HBT}}=\frac{1.5\times10^{-11}}{7.3\times10^{-10}}=0.02$$

?? τ_{HBT} should be smaller than τ_{BJT} due to build-in $\mathcal{E}$

d) For BJT

$$\gamma_E=\frac{1}{1+\frac{D_p}{D_n}\frac{G_{NB}}{G_{NE}}}=\frac{1}{1+\frac{5}{7}\frac{5\times10^{17}\times125\times10^{-7}}{10^{18}\times375\times10^{-7}}}=0.89$$

$$\alpha_T=1-\frac{W_B'^2}{2D_n\tau_n}=1-\frac{(125nm)^2}{2\times5cm^2s^{-1}\times10^{-8}s}=0.998$$

$$\beta_F=\frac{\gamma_E\alpha_T}{1-\gamma_E\alpha_T}=7.9$$

While for HBT, we have

$$\gamma_E=\frac{1}{1+\frac{G_{NB}}{G_{NE}}\frac{n_{iE}^2}{n_{iB}^2}}=\frac{1}{1+\frac{10^{18}\times200\times10^{-7}}{10^{16}\times460\times10^{-7}}e^{-0.374/0.026}}=0.999997$$

$$\alpha_T=1-\frac{W_B'^2}{2D_n\tau_n}=1-\frac{(200nm)^2}{2\times30cm^2s^{-1}\times10^{-8}s}=0.9993$$

$$\beta_F=\frac{\gamma_E\alpha_T}{1-\gamma_E\alpha_T}=1421$$

So HBT has a much larger current gain.

e) The pros and cons of BJT and HBT

	pros	cons
BJT	Cheap	Large base resistance Can't have too high frequency
HBT	Large current gain Low base resistance High frequency operation	Complex manufacture

f) A SiGe HBT is similar to a conventional Si bipolar transistor except for the base. SiGe, a material with narrower band-gap than Si, is used as the base material. SiGe HBT has a higher gain, lower RF noise, and low 1/f noise than an identically constructed Si BJT, and higher raw speed can be traded for lower power consumption as well. So SiGe is mainly used in high frequency application. Also, unlike other technologies like GaAs, SiGe has the ability to integrate analog, RF and digital on a single chip using existing CMOS fabs, this leads to massive drive toward BiCMOS technology. SiGe HBTs can be used in making low-cost, lightweight, personal communications devices like digital wireless handsets, as well as other entertainment and information technologies like digital set-top boxes, Direct Broadcast Satellite (DBS), automobile collision avoidance systems, and personal digital assistants. SiGe extends the life of wireless phone batteries, and allows smaller and more durable communication devices.

Problem 4 MKC 6.19

a) We know that in the base, we have continuity equation

$$\frac{\partial n_p}{\partial t} = D_n \frac{\partial^2 n_p}{\partial x^2} - \frac{n_p - n_{p0}}{\tau_n}$$

And considering steady state, we can get an equation for excess density

$$0 = D_n \frac{d^2 n_p'}{dx^2} - \frac{n_p'}{\tau_n}$$

$$\Rightarrow n_p'(x) = A\exp\left(-\frac{x - x_B}{\sqrt{D_n\tau_n}}\right) + B\exp\left(\frac{x - x_B}{\sqrt{D_n\tau_n}}\right)$$

Now we have boundary condition

$$n_p'(0) = n_{p0}\left(\exp\left(qV_{BE}/kT\right) - 1\right)$$

$$n_p'(x_B) = -n_{p0}$$

So we can get

$$n_p(x) = n_p'(x) + n_{p0} = n_{p0}\frac{\left(e^{qV_{BE}/kT} - 1\right) + e^{-x_B/\sqrt{D_n\tau_n}}}{e^{x_B/\sqrt{D_n\tau_n}} - e^{-x_B/\sqrt{D_n\tau_n}}}\exp\left(-\frac{x - x_B}{\sqrt{D_n\tau_n}}\right) - n_{p0}\frac{e^{x_B/\sqrt{D_n\tau_n}} + \left(e^{qV_{BE}/kT} - 1\right)}{e^{x_B/\sqrt{D_n\tau_n}} - e^{-x_B/\sqrt{D_n\tau_n}}}\exp\left(\frac{x - x_B}{\sqrt{D_n\tau_n}}\right) + n_{p0}$$

$$= n_{p0}\left(e^{qV_{BE}/kT} - 1\right)\frac{e^{(x_B - x)/L_n} - e^{(x - x_B)/L_n}}{e^{x_B/L_n} - e^{-x_B/L_n}} + n_{p0}\frac{e^{-x/L_n} - e^{x/L_n}}{e^{x_B/L_n} - e^{-x_B/L_n}} + n_{p0}$$

$$= n_{p0}\left(e^{qV_{BE}/kT} - 1\right)\frac{\sinh\left(\dfrac{x_B - x}{L_n}\right)}{\sinh\left(\dfrac{x_B}{L_n}\right)} - n_{p0}\frac{\sinh\left(\dfrac{x}{L_n}\right)}{\sinh\left(\dfrac{x_B}{L_n}\right)} + n_{p0}$$

b) Using the expression in (a), we can get the slope of the distribution at x=0 and x=x$_B$, since we know the relation of current and the slope is

$$J_n(x) = qD_n\frac{dn_p}{dx}$$

So we can get

$$J_n(0) = \left|qD_n\frac{dn_p}{dx}\right|_{x=0} = \frac{n_{p0}}{L_n}\left(e^{qV_{BE}/kT} - 1\right)\coth\left(\frac{x_B}{L_n}\right) + \frac{n_{p0}}{L_n}\csc h\left(\frac{x_B}{L_n}\right)$$

$$J_n(x_B) = \left|qD_n\frac{dn_p}{dx}\right|_{x=x_B} = \frac{n_{p0}}{L_n}\left(e^{qV_{BE}/kT} - 1\right)\mathrm{csch}\left(\frac{x_B}{L_n}\right) + \frac{n_{p0}}{L_n}\cot h\left(\frac{x_B}{L_n}\right)$$

The difference of the current is caused by recombination, and we can get the expression as

$$J_{RB}=J_n(0)-J_n(x_B)=\frac{n_{p0}}{L_n}\left(e^{qV_{BE}/kT}-1\right)\left(\coth\left(\frac{x_B}{L_n}\right)-\operatorname{csch}\left(\frac{x_B}{L_n}\right)\right)+\frac{n_{p0}}{L_n}\left(\csc h\left(\frac{x_B}{L_n}\right)-\coth\left(\frac{x_B}{L_n}\right)\right)$$

$$=\frac{n_{p0}}{L_n}\left(e^{qV_{BE}/kT}-2\right)\left(\coth\left(\frac{x_B}{L_n}\right)-\operatorname{csch}\left(\frac{x_B}{L_n}\right)\right)$$

c) $\gamma\approx\dfrac{1}{1+\dfrac{GN_B\tilde{D}_{pE}}{GN_E\tilde{D}_{nB}}}=\dfrac{1}{1+\dfrac{10^{17}\times 1um\times 8}{10^{19}\times 1um\times 3}}=0.974$

$$\alpha_T=1-\left|\frac{J_{RB}}{J_n(0)}\right|=1-\left|\frac{\left(e^{qV_{BE}/kT}-2\right)\left(\coth\left(\frac{x_B}{L_n}\right)-\operatorname{csch}\left(\frac{x_B}{L_n}\right)\right)}{\left(e^{qV_{BE}/kT}-1\right)\coth\left(\frac{x_B}{L_n}\right)+\csc h\left(\frac{x_B}{L_n}\right)}\right|$$

$$=1-\left|\frac{\left(e^{qV_{BE}/kT}-2\right)\left(\coth(0.18)-\operatorname{csch}(0.18)\right)}{\left(e^{qV_{BE}/kT}-1\right)\coth(0.18)+\csc h(0.18)}\right|,\frac{x_B}{L_n}=\frac{x_B}{\sqrt{D_n\tau_n}}=0.18$$

$$\approx 1-\left|\frac{\coth(0.18)-\operatorname{csch}(0.18)}{\coth(0.18)}\right|\ assume\ \frac{qV_{BE}}{kT}>>1$$

$$=0.984$$

$$\alpha_F=\gamma\alpha_T=0.9584$$

$$\beta_F=\frac{\alpha_F}{1-\alpha_F}=23.04$$

Problem 5 (Textbook problem 6.30)

(Problem 6·30)

As indicated in the example, the emitter efficiency γ is typically the factor that limits the size of the common-emitter current gain β_F in a BJT. Furthermore, as described above, both bandgap narrowing and Auger recombination limit the improvement in γ that can be obtained by increasing the emitter Gummel number.

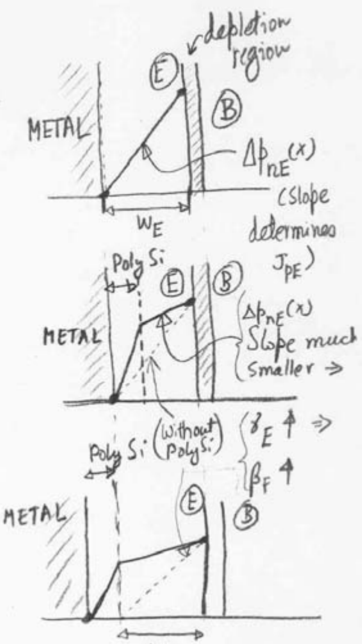
We saw in the discussion leading to Equation 6.2.9 that the gain of a modern transistor is limited by reverse injection of holes from the base into the emitter and that this injection is proportional to the hole-density gradient at the edge of the emitter quasi-neutral region. Figure 6.6*b* showed that the slope of the hole density in the emitter region depends on the concentration at the surface of the emitter region when $x_E \ll L_p$ the hole diffusion length. For a typical metal contact to the emitter region, the excess hole density at the contact is virtually zero, and the hole-density gradient in the emitter is maximum, limiting the emitter injection efficiency and the transistor gain. If the hole density at the surface of the single-crystal emitter can be increased, the gradient decreases; the reverse hole injection decreases, and the transistor gain increases markedly.

The desired increase in hole density can be achieved by placing a layer of n^+ polycrystalline silicon (polysilicon—Sec. 2.6) between the metal contact and the heavily doped, single-crystal emitter region. The hole density at the single-crystal/polysilicon interface can be high, reducing the hole-density gradient in the emitter and the reverse hole injection from the base. The resulting increase in emitter injection efficiency can increase the transistor gain β_F by as much as an order of magnitude. !!

Several physical mechanisms have been suggested to explain this improvement. The dominant mechanism has not been firmly determined and probably depends on the details of the fabrication process. One mechanism suggests that a very thin barrier ($\leq$1 nm), probably of residual silicon dioxide, blocks the flow of holes from the single-crystal emitter into the polysilicon, without severely impeding the flow of electrons from the polysilicon into the single-crystal emitter. [This mechanism relies on the lower barrier for electrons than holes ($E_{c,\mathrm{oxide}} - E_{c,\mathrm{Si}} < E_{v,\mathrm{Si}} - E_{v,\mathrm{oxide}}$)]. Another explanation relies on the shorter diffusion length of minority-carrier holes in polycrystalline silicon (with its highly imperfect crystal structure) than in single-crystal silicon. In both cases, the gradient of the minority-carrier hole density in the single-crystal emitter decreases, reducing hole injection from the base into the emitter and improving the emitter injection efficiency. The fabrication of this "polysilicon-emitter" bipolar transistor will be discussed in Sec. 6.5.

The theory that we developed in this section applies to dc bias conditions, and the equations have been derived for total currents. For application to amplifiers, we need equations that express the response to incremental changes in voltage around a dc bias point. We will consider such *small-signal* variations in Chapter 7, where we derive several equivalent circuits that describe transistor behavior in a way useful for circuit design. We will also consider frequency effects in transistors at that time. Our analysis thus far has considered only the dc (and low-frequency) case.

Before resuming our discussion of transistor action, we should point out that the analysis of hole injection into the emitter that we carried out for the case of nonuniform doping applies in general to *pn* junctions in integrated circuits. For example, the injected hole current given by Equation 6.2.18 is analogous to the result for electron injection into the *p*-type region in the diffused *pn*-junction diode considered in Sec. 5.6 (Figure 5.19*b*).



Advantage → $\beta_F \uparrow$

The disadvantage is obviously that there are more minority carriers stored in the emitter

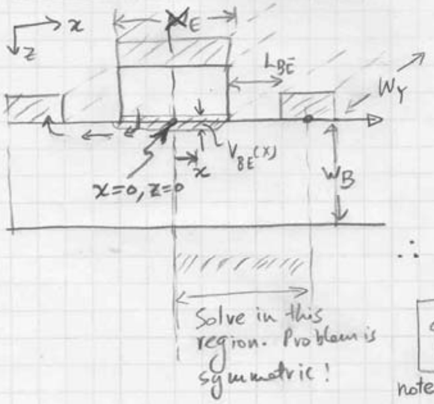
$$\left(\int_{W_E} Q_p(x)\,dx \uparrow\right) \Rightarrow \quad \tau_E = \frac{Q_{pE}\uparrow}{J_{pE}\downarrow} \Rightarrow \tau_E \uparrow.$$

→ Slower!

Problem 6 (Textbook Problem 6.36)

ⓐ

PROBLEM 6 (6.36, Textbook)



ⓐ Do not assume short-base.

$$\boxed{\Delta n_{pB}(x,z) \approx n_{po}\left(e^{qV_{BE}(x)}\right)\frac{\sinh\left(\frac{W_B - z}{L_n}\right)}{\sinh\left(\frac{W_B}{L_n}\right)}}$$

$$\therefore \quad q(x) = q\int_{z=0}^{W_B} dz\, \Delta n_{pB}(x,z)$$

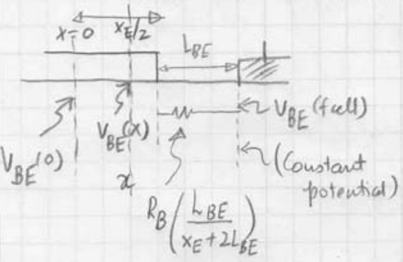
$$\boxed{q(x) = q n_{po} \cdot L_n \tanh\left(\frac{W_B}{2L_n}\right) \cdot e^{\frac{qV_{BE}(x)}{kT}}}$$

note that for short-base, $W_B << L_n$, $\tanh(x) \sim x$

$$\& \quad q_{SB}(x) \simeq n_{po}\frac{W_B}{2}e^{\frac{qV_{BE}(x)}{kT}}.$$

The vertical base resistance is neglected.

ⓑ Assume the emitter-end of the junction is @ V = 0 volts, & the base contacts are @ V_{BE}(full). The total current is I_B. Forget the collector for now. $\frac{1}{2}I_B$ flows into each base contact, & the total base resistance in each half is $R_B/2$.



$$V_{BE}(x) = V_{BE}(\text{full}) - \frac{1}{2}I_B R_B\left(\frac{L_{BE}}{x_E + 2L_{BE}}\right)$$

$$- W_y\int_{x_E/2}^{x}\left(\frac{dx'}{x_E + 2L_{BE}}\right)R_B\left[\int_0^{W_B}\left(\frac{q(x')}{t_N}\right)dz\right]$$

Total recombin. current

↑ Effective J(x)!

$$W_y\int_0^{W_B}\frac{q(x)}{t_N}dz = I(x).$$

$$\therefore \quad V_{BE}(x) = \overbrace{V_{BE}(\text{full}) - \frac{1}{2}I_B R_B\left(\frac{L_{BE}}{x_E + 2L_{BE}}\right)}^{V_{BE}(x_E/2)} - \left(\frac{W_B W_y}{x_E + 2L_{BE}}\right)\left(\frac{R_B}{t_N}\right)q n_{po} L_n \tanh\left(\frac{W_B}{2L_n}\right) * \int_{x_E/2}^{x} dx'\, e^{\frac{qV_{BE}(x')}{kT}}$$

$$\Rightarrow \quad \frac{\partial V_{BE}(x)}{\partial x} = -\left\{\frac{W_B W_y R_B q n_{po} L_n \tanh\left(\frac{W_B}{2L_n}\right)}{(x_E + 2L_{BE})t_N}\right\} e^{\frac{qV_{BE}(x)}{kT}}$$

Problem 6 contd...

∴ The differential equation relating $V_{BE}(x)$ to other known quantities is

$$\frac{\partial V_{BE}(x)}{\partial x} = -\left\{ \frac{\overset{\text{Voltage}}{R_B\, q/t_N}}{\dfrac{(x_E + 2L_{BE})}{W_B W_y L_n \tanh\left(\frac{W_B}{2L_n}\right)\cdot n_{p0}}} \right\} e^{\frac{qV_{BE}(x)}{kT}}$$

length

↑ dimensionless

← units of an effective "electric field". Call it F_0.

$$\boxed{\frac{\partial V_{BE}(x)}{\partial x} = -F_0\, e^{\frac{qV_{BE}(x)}{kT}}}$$

↑ depends only on geometry, t_N, and known qtys.

Solve using boundary conditions →

$$\int_{V_{BE}(x_E/2)}^{V_{BE}(0)} \frac{dV_{BE}(x)}{e^{\frac{qV_{BE}(x)}{kT}}} = -F_0 \int_{x_E/2}^{x} dx' \quad \rightarrow \underline{\text{Easy to solve!}}$$

$$\Rightarrow \quad \boxed{V_{BE}(x) = \frac{kT}{q} \ln\left[\frac{1}{\frac{F_0}{kT/q}\left(\frac{x_E}{2} - |x|\right) + e^{-\frac{qV_{BE}(x_E/2)}{kT}}} \right]}$$

for $|x| < x_E/2$.

here

$$\begin{cases} V_{BE}(x_E/2) = V_{BE}(\text{full}) - \frac{1}{2} I_B R_B \left(\frac{L_{BE}}{x_E + 2L_{BE}}\right). \\ F_0 = \left(\dfrac{R_B q/t_N}{x_E + 2L_{BE}}\right) \cdot \left(W_B W_y L_n n_{p0} \tanh\left(\frac{W_B}{2L_n}\right)\right) \end{cases}$$

ⓒ

Problem 6 contd...

ⓒ $$\Delta n_{B}(x,z) \simeq n_{p_0}\, e^{\frac{qV_{BE}(x)}{kT}}\, \frac{\sinh\left(\frac{W_B - z}{L_n}\right)}{\sinh\left(\frac{W_B}{L_n}\right)}$$

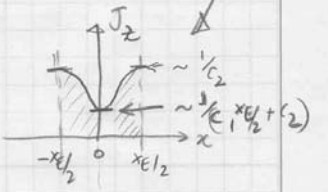
$$e^{\frac{qV_{BE}(x)}{kT}} = \frac{1}{\frac{F_0}{kT/q}\left(\frac{x_E}{2} - |x|\right) + e^{-\frac{qV_{BE}(x_E/2)}{kT}}}$$

$$\therefore \quad \boxed{\Delta n_{pB}(x,z) = \left(\frac{n_{p0}}{\frac{F_0}{kT/q}\left(\frac{x_E}{2} - |x|\right) + e^{-\frac{qV_{BE}(x_E/2)}{kT}}}\right) * \frac{\sinh\left(\frac{W_B - z}{L_n}\right)}{\sinh\left(\frac{W_B}{L_n}\right)}}$$

$$\boxed{q(x) = \frac{q\, n_{p0} L_n \tanh\left(\frac{W_B}{2L_n}\right)}{\frac{F_0}{kT/q}\left(\frac{x_E}{2} - |x|\right) + e^{-\frac{qV_{BE}(x_E/2)}{kT}}}}$$

$$\boxed{J_z(x) = \frac{q(x) W_B}{t_N} = \frac{W_B q\, n_{p0} L_n \tanh\left(\frac{W_B}{2L_n}\right)/t_N}{\frac{F_0}{kT/q}\left(\frac{x_E}{2} - |x|\right) + \exp\left(-\frac{qV_{BE}\left(\frac{x_E}{2}\right)}{kT}\right)} \sim \frac{1}{c_1\left(\frac{x_E}{2} - |x|\right) + c_2}}$$

This clearly shows that the total current is lower!

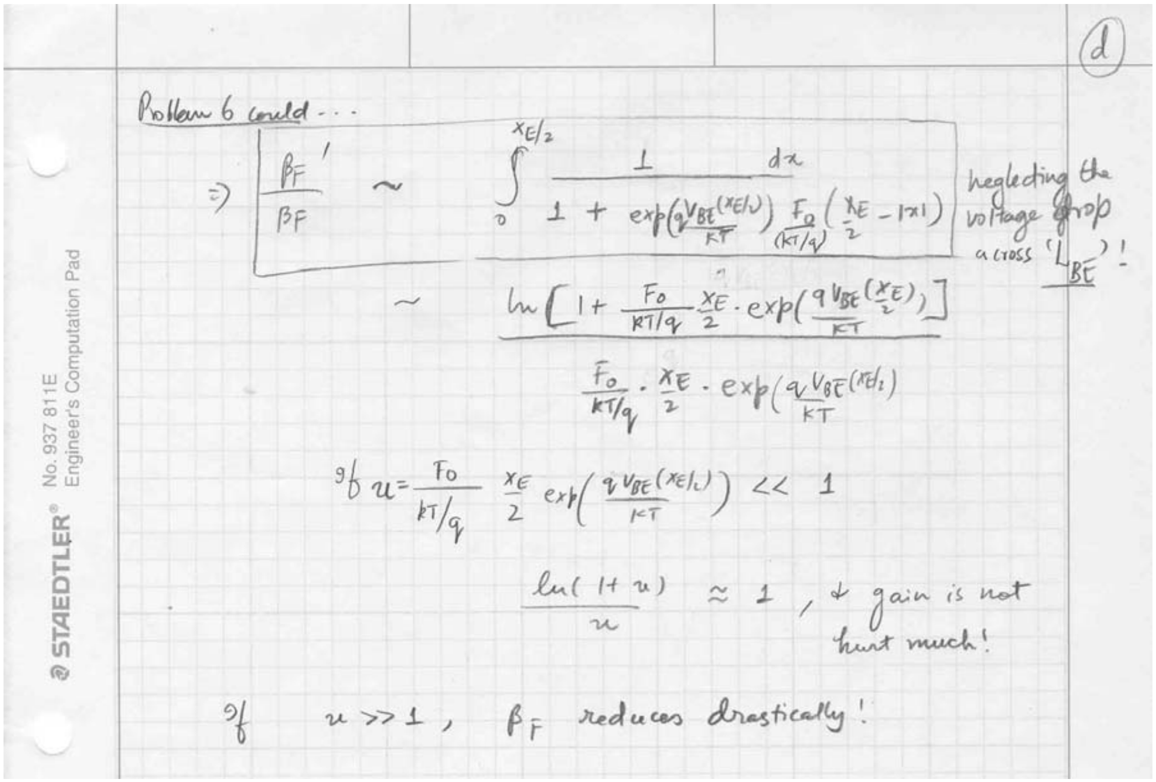


ⓓ $$\gamma_E \to 1 \Rightarrow J_{pE} \sim 0$$

$$\frac{\beta_F(\text{base resistance})}{\beta_F(\text{ideal})} \sim \frac{I_c'}{I_c}$$

$$I_c' = 2\int_0^{x_E/2} dx\, J_z(x)$$

$$I_c = 2\int_0^{x_E/2} dx\, J_z(x) * \left[\frac{\frac{F_0}{kT/q}\left(\frac{x_E}{2} - |x|\right) + e^{-\frac{qV_{BE}(x_E/2)}{kT}}}{e^{-\frac{qV_{BE}(x_E/2)}{kT}}}\right]$$



ⓓ

Problem 6 contd . . .

$$\Rightarrow \boxed{\frac{\beta_F'}{\beta_F} \sim \int_0^{x_E/2} \frac{1}{1 + \exp\left(\frac{qV_{BE}(x_E/2)}{kT}\right) \frac{F_0}{(kT/q)}\left(\frac{x_E}{2} - |x|\right)} dx}$$

neglecting the voltage drop across 'L_{BE}'!

$$\sim \frac{\ln\left[1 + \frac{F_0}{kT/q} \frac{x_E}{2} \cdot \exp\left(\frac{qV_{BE}(\frac{x_E}{2})}{kT}\right)\right]}{\frac{F_0}{kT/q} \cdot \frac{x_E}{2} \cdot \exp\left(\frac{qV_{BE}(x_E/2)}{kT}\right)}$$

If $u = \frac{F_0}{kT/q} \frac{x_E}{2} \exp\left(\frac{qV_{BE}(x_E/2)}{kT}\right) << 1$

$\frac{\ln(1+u)}{u} \approx 1$, & gain is not hurt much!

If $u >> 1$, β_F reduces drastically!

The real reduction in gain can be found accurately by setting $L_{BE}=0$ in the expressions derived above.