EE566 Solid State Devices

Spring 2005
Dept of Electrical Engineering
University of Notre Dame
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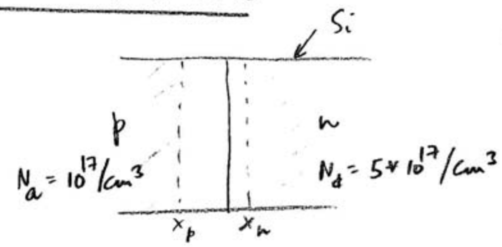
Assignment 3
SOLUTIONS

Problem 1

PROBLEM 4.3 (Textbook) (Debdeep Jena)



All values here are calculated using the depletion approx. The exact values are compared in the next page.

Built-in voltage: $V_{bi} = \frac{kT}{q} \ln\left(\frac{N_A N_D}{n_i^2}\right) = (26\,mV) * \ln\left(\frac{10^{17} * 5*10^{17}}{10^{20}}\right)$

$$\boxed{V_{bi} = 0.88\,Volt}.$$

Total depletion width: $W_{depl} = \left[\frac{2\epsilon_s}{q}\left(\frac{N_A+N_D}{N_A N_D}\right) V_{bi}\right]^{\frac{1}{2}}$

$$\boxed{W_{depl} \simeq 113.3\,nm} \Rightarrow x_n = \frac{N_A}{N_D+N_A} W_{depl} = \frac{1}{6} W_{depl}$$

$$\Rightarrow \boxed{\begin{array}{l} x_n = 18.9\,nm \\ x_p = 94.4\,nm \end{array}}$$

Maximum Electric field: (F_{max})

$$\frac{1}{2}|F_{max}|\, W_{depl} = V_{bi} \quad \Rightarrow \boxed{|F_{max}| \simeq 155.3\,\frac{kV}{cm}}.$$

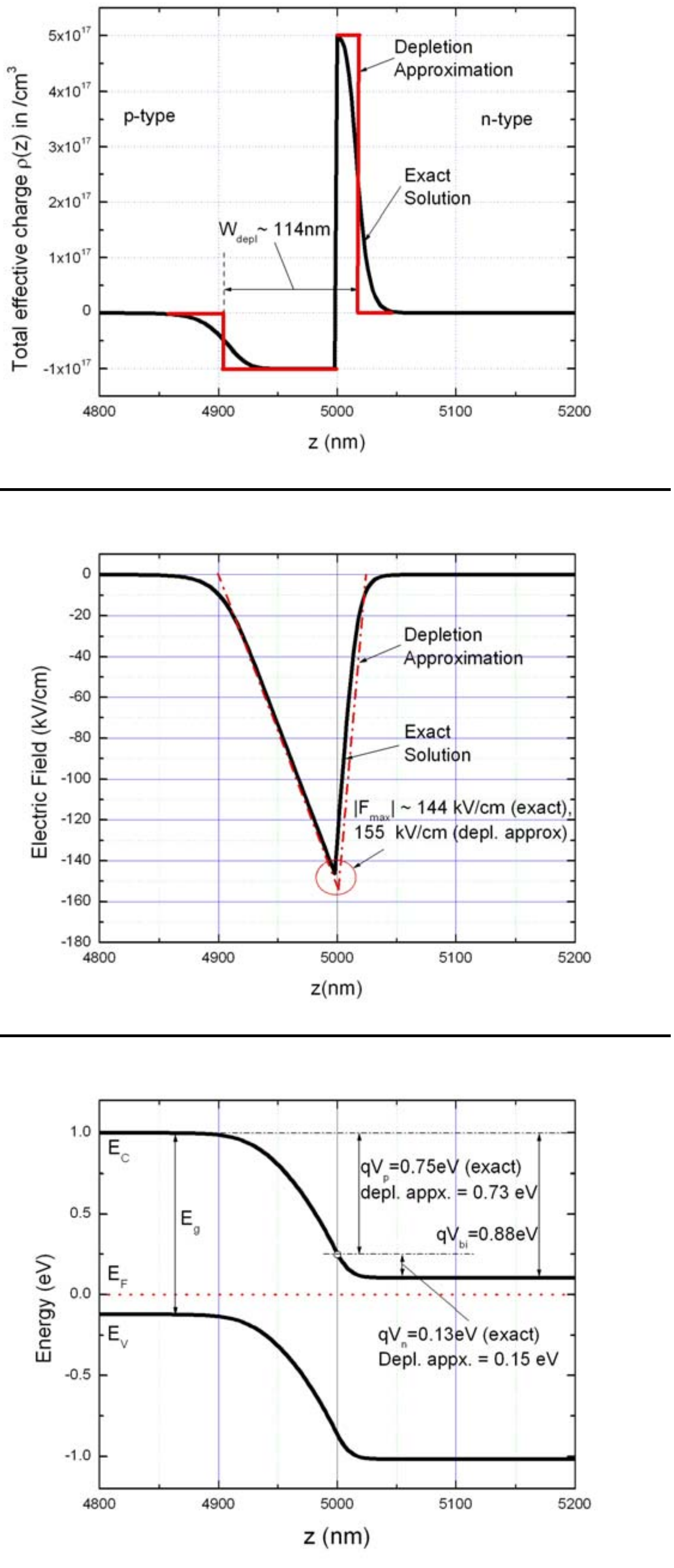
Voltage distribution:

Voltage drop in the n-side: $V_n = \frac{1}{2} F_{max} x_n = \boxed{0.15\,Volt}$

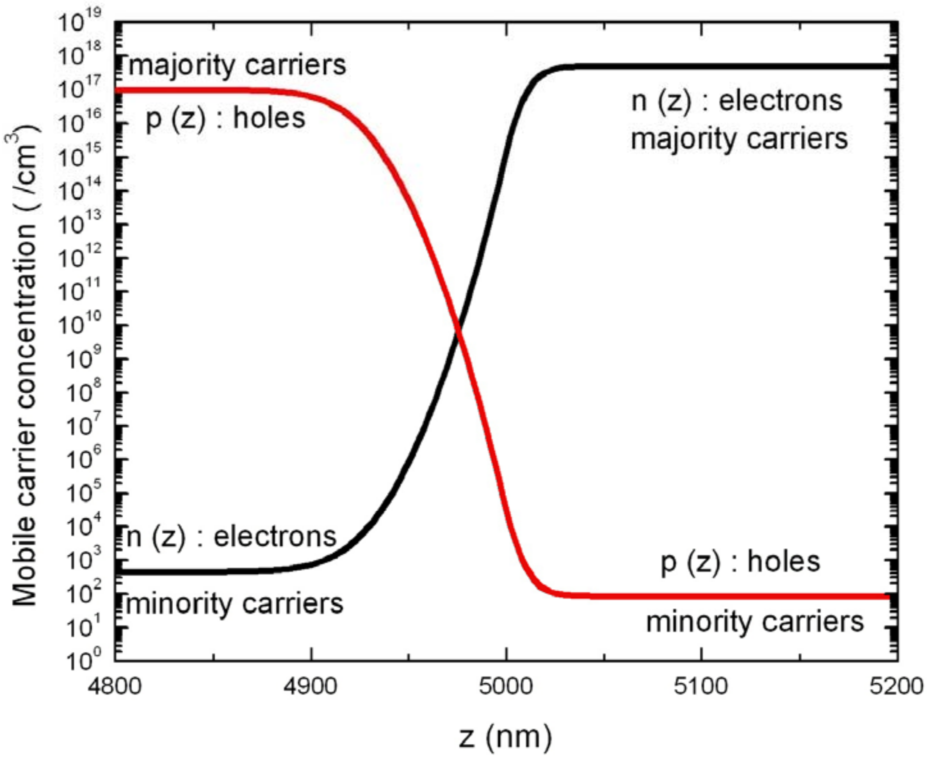
" " " p-side: $V_p = \frac{1}{2} F_{max} x_p = \boxed{0.73\,Volt}$

Total $V_{bi} = 0.88$ Volt.

Compare each of these values with those got from the exact 1-D Poisson Soln in the next page ⟶.



The Charge-Field-Band diagram for the Si p-n junction. Note carefully the differences between the values calculated by the depletion approximation, and those got by the exact solution of Poisson equation. The simulation is done using the 1D Poisson simulator.



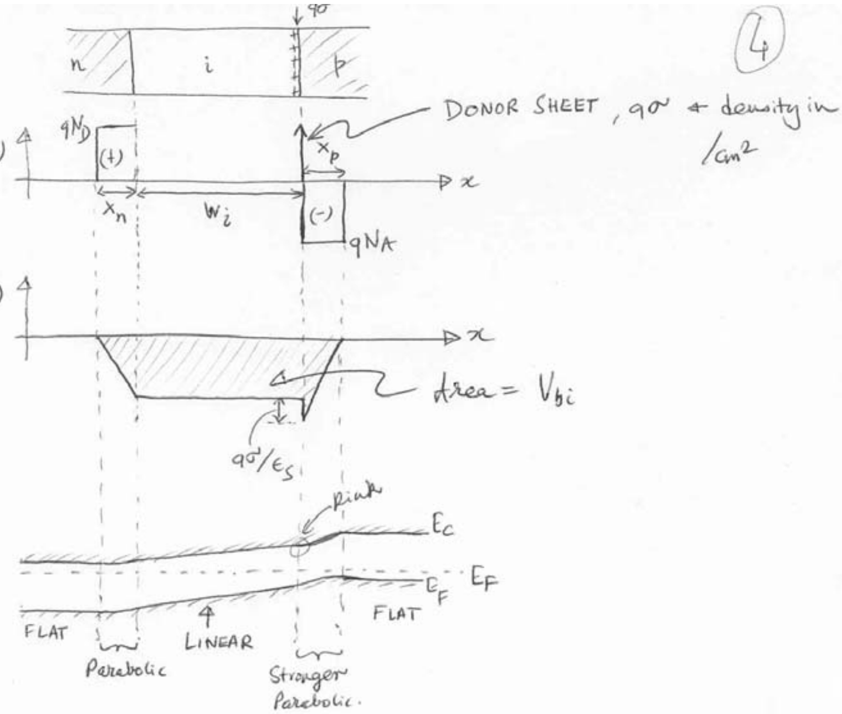
Mobile carrier densities in the p-n junction. The majority and minority carriers in each region are clearly labeled.

It is clear that by neglecting the Gummel correction, the maximum electric field calculated is higher than the actual field. The depletion region thickness is also larger from a simple depletion approximation. The carrier profiles have the highest error in the depletion approximation; the error in assuming an abrupt depletion edge is of the order of several Debye lengths for the mobile carrier concentration.

However, for most important quantities, the depletion approximation works reasonably well.

Problem 2

(c)



Charge Neutrality: $qN_A x_p = qN_D x_n + q\sigma$.

$$\therefore \quad x_p = x_n + \frac{\sigma}{N_A} \;\rightarrow\; V_{bi} = \frac{qN_A}{\epsilon_s}x_p^2 + \left(\frac{W_i q N_A}{\epsilon_s} - \frac{q\sigma}{\epsilon_s}\right)x_p + \frac{1}{2}\frac{q\sigma^2}{\epsilon_s N_A} - \frac{W_i q\sigma}{\epsilon_s}$$

(a) — AREA UNDER $F(x)-x$ plot.

$$C_{p-i-n} = C_{p-n}$$

$$\text{(b)} \quad \frac{\epsilon_s}{x_n + x_p + W_i} = \frac{\epsilon_s}{W} \leftarrow \left(\frac{2\epsilon_s}{q}\cdot\frac{2}{N_A}\cdot V_{bi}\right)^{1/2}$$

with i-layer + sheet doping   No i-layer.

Solve (a) & (b) together to get -

$$\boxed{W_i = (\sqrt{2}-1)\frac{\sigma}{N_A}} \Rightarrow W_i \approx (\sqrt{2}-1) \times \frac{10^{11}\times 5}{10^{17}} \text{ cm}$$

$$\boxed{W_i \approx 20.7 \text{ nm}}$$

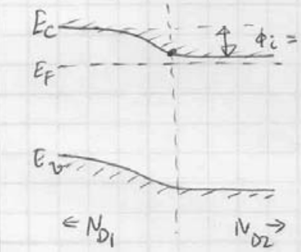
Problem 3

ASSIGNMENT 4 - EE 566 - Solid State Devices, Spring 2005. - (Solution).

Problem 1 -

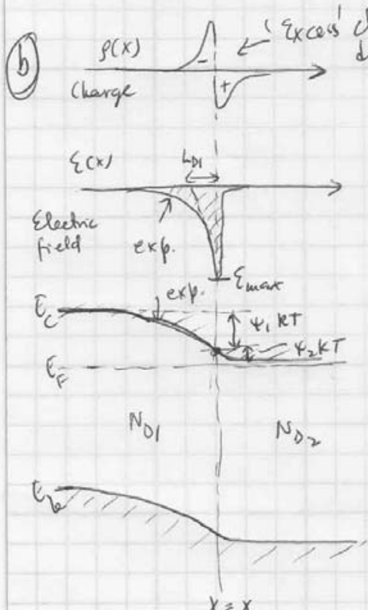
(a) Potential barrier is $\phi_i = \frac{kT}{q}\ln\left(\frac{N_{D2}}{N_{D1}}\right) = 0.12$ Volt

$$\boxed{\phi_i = 120\,\text{meV}}$$



for electron flow from region $1 \to 2$, there is no barrier. However, for electron flow from $2 \to 1$, there is a potential barrier $\phi_i \simeq 120$ meV. However, this barrier is very small, & any applied bias greater than 120 mV will make the current ohmic in both directions. So, there is rectification, but it is VERY WEAK.

(b)



(c) The exact solution to Poisson Eq^n is

$$\frac{\partial^2\psi(x)}{\partial x^2} = \frac{q^2 N_D(x)}{\epsilon_S kT}\left[1 - e^{-\psi(x)}\right]$$

which yields for Electric field (Done in class!)

$$|\mathcal{E}(x)|^2 = \frac{2kT\,N_D}{\epsilon_S}\left[\psi(x) + e^{-\psi(x)} - 1\right]$$

Denoting total band bending in 'N_{D1}' region as $\psi_1 kT$ & in 'N_{D2}' region as $\psi_2 kT$, and equating the field at $x = x_0$, the interface,

$$\begin{cases} \frac{2kT\,N_{D1}}{\epsilon_S}\left[-|\psi_1| + e^{|\psi_1|} - 1\right] = \frac{2kT\,N_{D2}}{\epsilon_S}\left[|\psi_2| + e^{-|\psi_2|} - 1\right] \\ \text{also,}\quad |\psi_1| + |\psi_2| = \frac{\phi_i}{kT} = \frac{120\,\text{meV}}{26\,\text{meV}} \text{ @300K.} \end{cases}$$

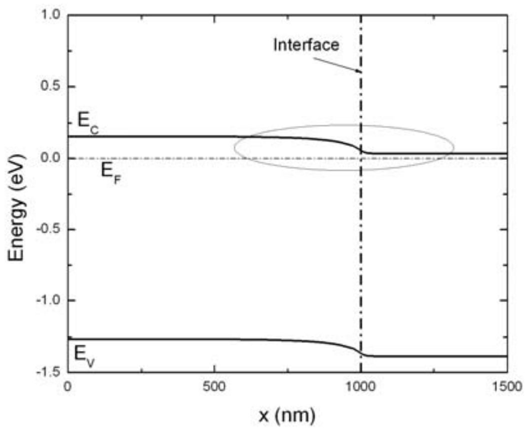
Solve simultaneously to get $\boxed{\begin{array}{l}|\psi_1| = 3.6517 \\ |\psi_2| = 0.9636\end{array}}$

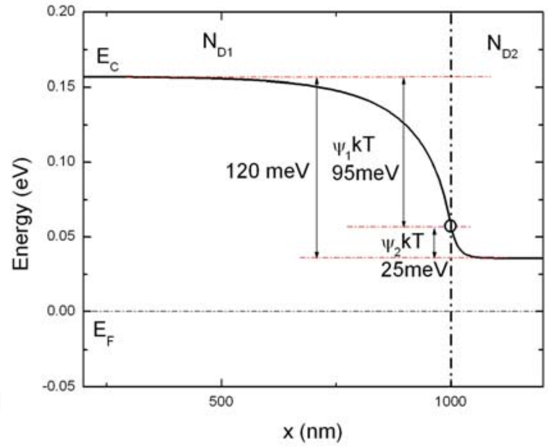
$\Rightarrow$ 95 meV drops in 'N_{D1}'

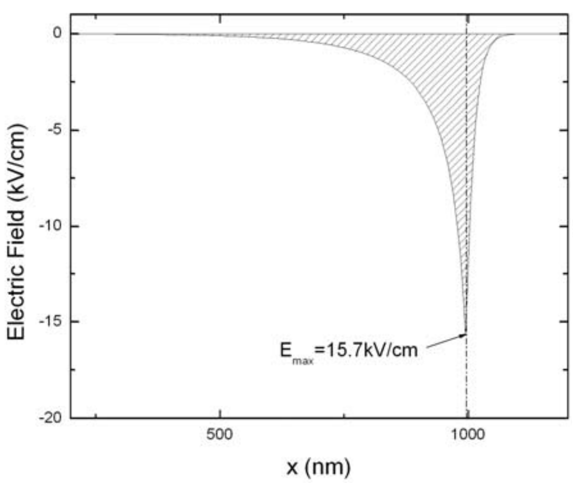
25 meV drops in 'N_{D2}', &

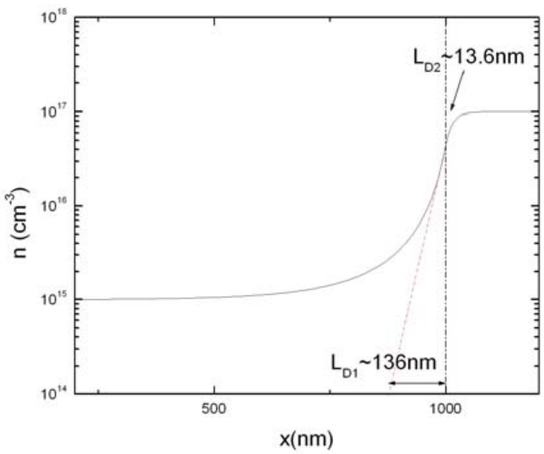
$$\mathcal{E}_{max}\,(\text{Exact}) = \left\{\frac{2kT\,N_{D1}}{\epsilon_S}\left(-|\psi_1| + e^{-|\psi_1|} - 1\right)\right\}^{1/2} \simeq \boxed{15.7\ \text{kV/cm}}$$

The simulated band diagrams using 1D Poisson are shown below. As can be seen, the band bending, the maximum electric field at the interface, and the Debye lengths match our exact calculations very well. This example illustrates that only for very simple structures can the Poisson equation be solved exactly. Most of the simulation for real devices is done numerically.



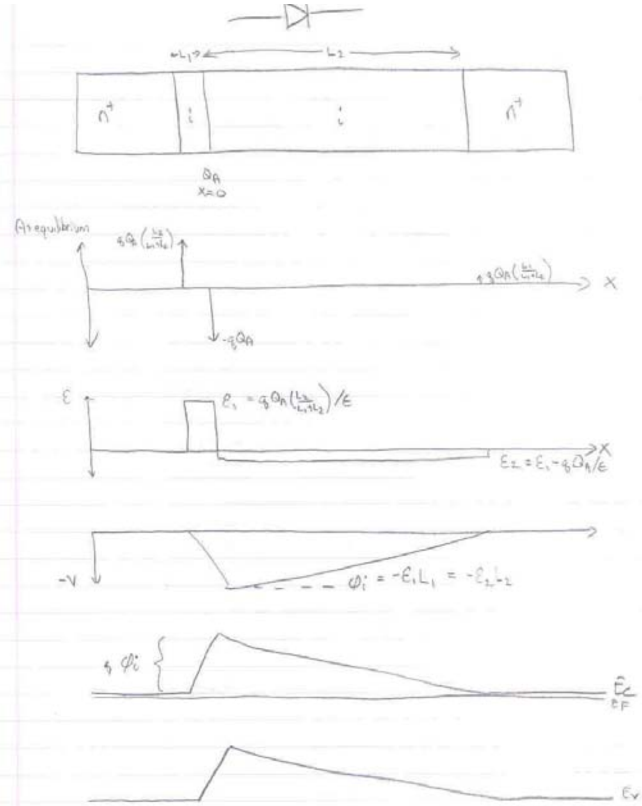






Problem 4

1)



Note: The depletion region is treated as a δ function since it is doped n^+

Gauss' Law gives: $\epsilon(\mathcal{E}_2 - \mathcal{E}_1) = \int_{x=0^-}^{x=0^+} \rho \, dx = -q Q_A$

(i) $$\Rightarrow \boxed{(\mathcal{E}_2 - \mathcal{E}_1) = -q Q_A / \epsilon}$$

Since the device is at equilibrium, the total voltage across it must be zero.

Voltage is given by

(ii) $$V = \int_{-L_1}^{L_2} \mathcal{E}\,dx = \boxed{\mathcal{E}_1 L_1 + \mathcal{E}_2 L_2 = 0}$$

Solving (i) and (ii) for $\mathcal{E}_1$ and $\mathcal{E}_2$ gives:

(iii) $$\boxed{\begin{aligned} \mathcal{E}_1 &= \left(\frac{L_2}{L_1+L_2}\right)\frac{qQ_A}{\epsilon} \\ \mathcal{E}_2 &= -\left(\frac{L_1}{L_1+L_2}\right)\frac{qQ_A}{\epsilon} \end{aligned}}$$

Lastly we solve for φ_i which is the height of the peak in voltage.

(iv) $$\boxed{\varphi_i = \mathcal{E}_1 \cdot L_1 = -\mathcal{E}_2 \cdot L_2 = (L_1 L_2 / L_1 + L_2)\, q Q_A / \epsilon}$$

a) In the case where $L_1 = .1\mu m$, $L_2 = 1\mu m$, $Q_A = 5 \times 10^{11} cm^{-2}$, $N_D = N_c = 4.4 \times 10^{17} cm^{-3}$, $\epsilon = 13.1\epsilon_0$ we first check our assumption about the charge distribution. The depletion width in n^+_{left} + n^+_{right} can be determined from the charge neutrality condition:

$$q\,(W_{left} + W_{Right})\, N_D = q Q_A$$

$$\Rightarrow \quad (W_{left} + W_{Right}) = Q_A / N_D = .01\mu m \ll L_1, L_2$$

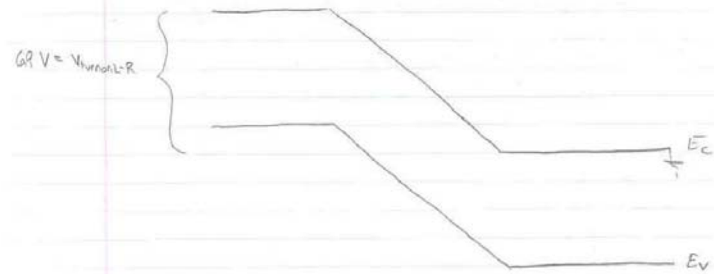
This is significantly smaller than L_1, L_2 and we can proceed without δ function assumption and find

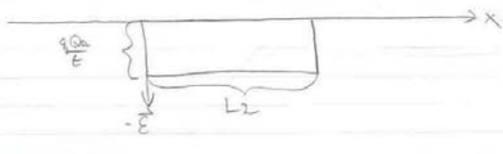
$$\boxed{\varphi_i = .627\,V}$$

To achive turn on we must flatband one side or the other. Assume apply a positive voltage to the left side, the positive charge is reduced, and appears on the right side as increased charge. At flat band, $\mathcal{E}_1 = 0$, but $(\mathcal{E}_2 - \mathcal{E}_1)$ remains fixed by Q_A. So we find

$$V_{turn\,on\,L\text{-}R} = -(\mathcal{E}_2 - \mathcal{E}_1) \cdot L_2 = \frac{qQ_A}{\epsilon} \cdot L_2 = \boxed{6.9\,V = V_{BR}}$$

1) Continued:

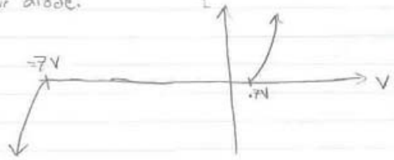




Similarly, we can flat band in the opposite direction and

$$V_{turnon R-L} = \frac{qQ_A}{\epsilon} L_1 = \boxed{.69\ V = V_{to}}$$

So if we define the turn on voltage to be $V_{turnon R-L}$ and the break down voltage as $V_{turnon L-R}$, we get the following I-V curve for our diode:



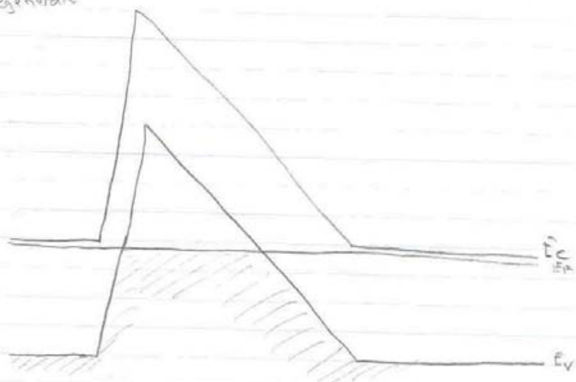
b) The turn on voltage voltage is directly proportional to Q_A, so we want Q_A as large as possible. To keep the valence band non-degenerate we set $\phi_i = E_g/q$

$$\Rightarrow \quad \phi_i = 1.42V = \left(\frac{L_1 L_2}{L_1 + L_2}\right) \frac{q Q_{A,max}}{\epsilon}$$

$$\Rightarrow \quad Q_{A,Max} = 1.42V\left(\frac{L_2+L_1}{L_1L_2}\right)\frac{\epsilon}{q} = \boxed{1.13\times10^{12}\,cm^{-2} = Q_{A,Max}}$$

$$\Rightarrow \quad V_{to,max} = 1.56V \qquad V_{BR,max} = 15.6V$$

c) Setting $Q = 2Q_{A,Max}$ gives $\varphi_i = 2.84\,V$ and the valence band is degenerate



Now we have essentially created a tunnel diode. We will see current immediately when we apply a voltage as e^- tunnel to the empty valence states in the degenerate region and tunnel from there to the empty conduction band states at the negative end of the diode. The new IV curve looks like: