EE566 Solid State Devices

Spring 2006
Dept of Electrical Engineering
University of Notre Dame
Instructor: Debdeep Jena (djena@nd.edu, x8835)

Assignment 2
SOLUTIONS

Some commentary on the solutions: (The detailed solutions are in the following pages – by Khalifa). In this page, I point out the common errors committed, and clarify some points.

Problem 1:

For 2DEGs, the carrier density approaches a constant value as T approaches 0K: $n_{2d} \to \frac{m^*}{\pi\hbar^2}(E_F - E_C)$ as $T \to 0K$. In fact, for carrier densities exceeding $10^{12}/cm^2$, the low-temperature limit works well at room temperature as well. For numerical evaluation, one can always use the exact form.

Though carrier transport in BJTs and diodes is 1-dimensional, the carriers are free to move in all 3 dimensions. Therefore, 3D statistics has to be used for such structures. However, in nanotubes, nanowires, and in general quantum wires, one needs to use 1-D statistics for carriers.

Problem 2:

This problem is designed to illustrate the powerful concept of "quasi-neutrality". As many of you have realized, on the surface, by assuming $n(x) \sim N_D(x)$, the problem is rather simple to solve by balancing the drift and diffusion currents. However, it is clear that the electric field obtained has to originate from a charge imbalance in the doped region; Poisson's equation tells us that $F(x) = \frac{q}{\varepsilon_S}\int_{-\infty}^{x} dx[N_D(x) - n(x)]$ is zero! The exact solution has to be numerical - you can attempt it, but be assured that the answer you will be get will be VERY close to what is got by assuming "quasi-neutrality". You know that the semiconductor is not charge neutral at every point, but since the field is very small, it is close to neutral, i.e., quasi-neutral.

Problem 3:

Simple problem, no explanation needed.

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a)

$$n_{2D} = \int_{E_c}^{E_{top}} g_{2D}(E) f(E) dE$$

but $f(E) = \dfrac{1}{1+e^{\frac{E-E_F}{kT}}}$ and $g_{2D}(E) = \dfrac{m^*}{\pi\hbar^2}\theta(E-E_C)$

$$n_{2D} = \int_{E_c}^{E_{top}} \frac{m^*}{\pi\hbar^2}\frac{1}{1+e^{\frac{E-E_F}{kT}}}\theta(E-E_C)dE$$

little error is introduced by letting $E_{top} \to \infty$

$$n_{2D} = \int_{E_c}^{\infty} \frac{m^*}{\pi\hbar^2}\frac{1}{1+e^{\frac{E-E_F}{kT}}}dE = \int_{E_c}^{\infty} \frac{m^*}{\pi\hbar^2}\frac{1}{1+e^{\frac{E-E_c}{kT}}e^{-\frac{E_F-E_c}{kT}}}dE$$

let $\zeta = \dfrac{E-E_c}{kT} \Rightarrow d\zeta = \dfrac{dE}{kT}$ OR $dE = kTd\zeta$

let $\eta = \dfrac{E_F-E_c}{kT}$

$\zeta \to 0$ as $E \to 0$ and $\zeta \to \infty$ as $E \to \infty$

$$n_{2D} = \int_0^{\infty} \frac{m^*}{\pi\hbar^2}\frac{kT}{1+e^{\zeta-\eta}}d\zeta = \frac{m^* kT}{\pi\hbar^2}\int_0^{\infty}\frac{1}{1+e^{\zeta-\eta}}d\zeta$$

$$n_{2D} = \underbrace{\frac{m^* kT}{\pi\hbar^2}}_{N_c^{2D}}\underbrace{\frac{1}{\Gamma(0+1)}\int_0^{\infty}\frac{\zeta^0}{1+e^{\zeta-\eta}}d\zeta}_{\Im_0(\eta)} = \underbrace{\frac{m^* kT}{\pi\hbar^2}}_{N_c^{2D}}\ln(1+e^{\eta})$$

$$\therefore n_{2D} = N_c^{2D}\Im_0(\eta) = N_c^{2D}\ln(1+e^{\eta})$$

We can make a plot of $E_F - E_c$ as a function of the 2DEG density by noting:

$$n_{2D} = N_c^{2D}\Im_0(\eta) = N_c^{2D}\ln(1+e^{\eta})$$

$$\frac{n_{2D}}{N_c^{2D}} = \ln(1+e^{\eta}) \Rightarrow 1+e^{\eta} = e^{\frac{n_{2D}}{N_c^{2D}}} \Rightarrow e^{\eta} = e^{\frac{n_{2D}}{N_c^{2D}}} - 1$$

$$\therefore \eta = \ln\left(e^{\frac{n_{2D}}{N_c^{2D}}} - 1\right) \qquad \text{OR} \qquad E_F - E_c = kT\ln\left(e^{\frac{n_{2D}}{N_c^{2D}}} - 1\right)$$

at lwo Temperatures, $N_c^{2D} = \dfrac{m^* kT}{\pi\hbar^2}$ becomes very small

since n_{2D} is given to be in the range $10^{11} - 10^{12}\text{cm}^{-2}$, $e^{\frac{n_{2D}}{N_c^{2D}}}$ becomes >> 1

$$\therefore E_F - E_c \approx kT\ln\left(e^{\frac{n_{2D}}{N_c^{2D}}}\right) = kT\frac{n_{2D}}{N_c^{2D}} = \frac{kTn_{2D}}{\frac{m^* kT}{\pi\hbar^2}} = \frac{n_{2D}\pi\hbar^2}{m^*} \qquad \text{OR} \qquad n_{2D} \approx \frac{m^*(E_F - E_c)}{\pi\hbar^2}$$

which shows that at low temperatures, n_{2D} becomes independent of temperature

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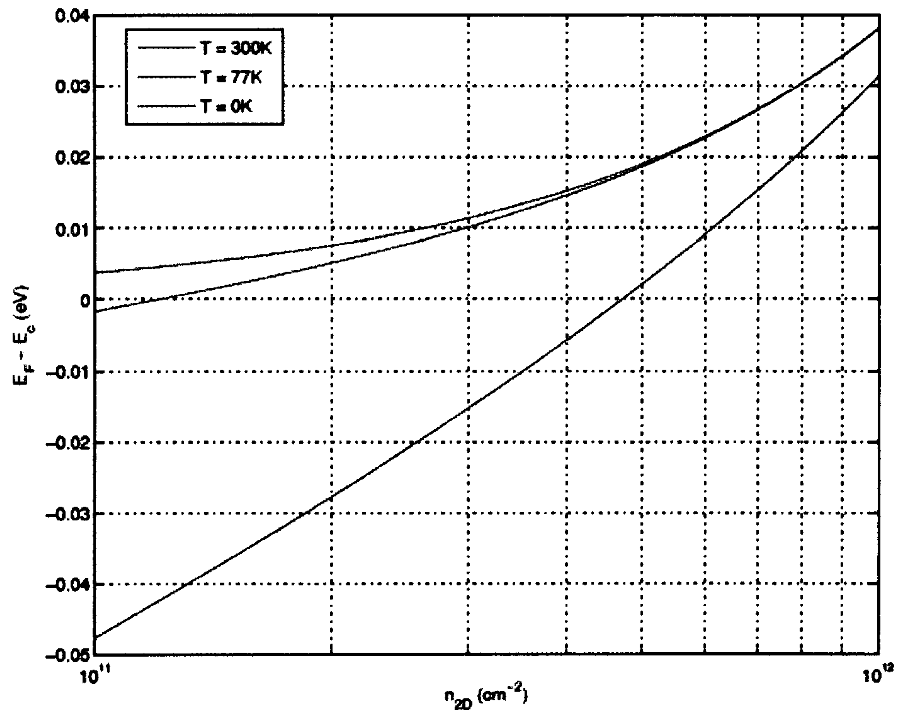
The following figure shows how $E_F - E_c$ varies as a function of the 2DEG density



Examples of devices where electron motion is 2-dimentional:
Devices which utilizes quantum well such us quantum well lasers, quantum well modulators, and (quantum well injection transit
time) QWITT diode oscillator

b)

$$n_{1D} = \int_{E_c}^{E_{top}} g_{1D}(E) f(E) dE$$

but $f(E) = \dfrac{1}{1+e^{\frac{E-E_F}{kT}}}$ and $g_{1D}(E) = \sqrt{\dfrac{2m^*}{\pi^2\hbar^2(E-E_C)}}$

$$n_{2D} = \int_{E_c}^{E_{top}} \sqrt{\frac{2m^*}{\pi^2\hbar^2(E-E_C)}} \frac{1}{1+e^{\frac{E-E_F}{kT}}} dE$$ and noting that little error is introduced by letting $E_{top} \to \infty$

$$n_{1D} = \int_{E_c}^{\infty} \sqrt{\frac{2m^*}{\pi^2\hbar^2(E-E_C)}} \frac{1}{1+e^{\frac{E-E_F}{kT}}} dE = \sqrt{\frac{2m^*}{\pi^2\hbar^2}} \int_{E_c}^{\infty} \frac{(E-E_C)^{-1/2}}{1+e^{\frac{E-E_c}{kT}} e^{-\frac{E_F-E_c}{kT}}} dE$$

let $\zeta = \dfrac{E-E_c}{kT} \Rightarrow d\zeta = \dfrac{dE}{kT}$ OR $dE = kT d\zeta$

let $\eta = \dfrac{E_F - E_c}{kT}$

$\zeta \to 0$ as $E \to 0$ and $\zeta \to \infty$ as $E \to \infty$

$$n_{1D} = \sqrt{\frac{2m^*}{\pi^2\hbar^2}} \int_{E_c}^{\infty} \frac{(kT\zeta)^{-1/2}}{1+e^{\zeta-\eta}} dE = \sqrt{\frac{2m^* kT}{\pi\hbar^2}} \frac{1}{\sqrt{\pi}} \int_{E_c}^{\infty} \frac{\zeta^{-1/2}}{1+e^{\zeta-\eta}} dE = \underbrace{\sqrt{\frac{2m^* kT}{\pi\hbar^2}}}_{N_c^{1D}} \underbrace{\frac{1}{\Gamma\left(-\frac{1}{2}+1\right)} \int_{E_c}^{\infty} \frac{\zeta^{-1/2}}{1+e^{\zeta-\eta}} dE}_{\Im_{-1/2}(\eta)} = N_c^{1D} \Im_{-1/2}(\eta)$$

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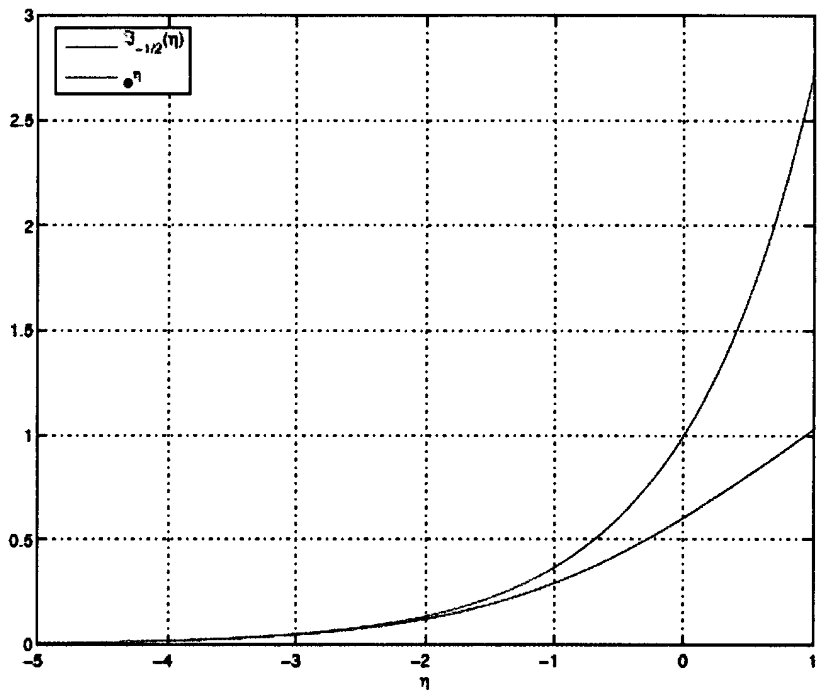
As is evident from the figure below, $\Im_{-1/2}(\eta)$ is closely approximated by e^{η} under the non-degeneracy condition $\eta \le -2 \Rightarrow n_{1D} = N_c^{1D}\Im_{-1/2}(\eta) \approx N_c^{1D}e^{\eta} = N_c^{1D}e^{\frac{E_F - E_c}{kT}}$



Examples of devices where electron motion is 1-dimentional:
Carbon nano-tubes and Carbon nano-wire

Part	Problem 2	EE 60566	Khalifa Al-Hosani

a) Under equilibrium, the total carrier currents inside a semiconductor must be identical to zero:

$$J_n = J_{n|drift} + J_{n|drift} = 0$$

$$q\mu_n nE(x,T) + qD_n \frac{dn}{dx} = 0 \Rightarrow E(x,T) = -\frac{D_n}{\mu_n}\frac{1}{n(x)}\frac{dn}{dx}$$

$$\frac{D_n}{\mu_n} = \frac{kT}{q}\frac{n}{dn/d\eta_c} \text{ where } \eta_c = \frac{E-E_F}{kT}$$

$$\text{Assuming non-degeneracy: } n \to N_c e^{\eta_c}, \frac{n}{dn/d\eta_c} \to 1$$

$$\therefore \frac{D_n}{\mu_n} = \frac{kT}{q} \quad \text{and} \quad E(x,T) = -\frac{kT}{q}\frac{1}{n(x)}\frac{dn}{dx}$$

Assuming Total ionization for the dopant: $n(x) = N_d(x)$

$$\therefore E(x,T) = -\frac{kT}{q}\frac{1}{N_D(x)}\frac{dN_D(x)}{dx}$$

b) The magnitude of the electric field is directly proportional to the temperature (e.g. as temperature increase, the magnitude of the electric field increase , but the direction of the electric field remains the same)

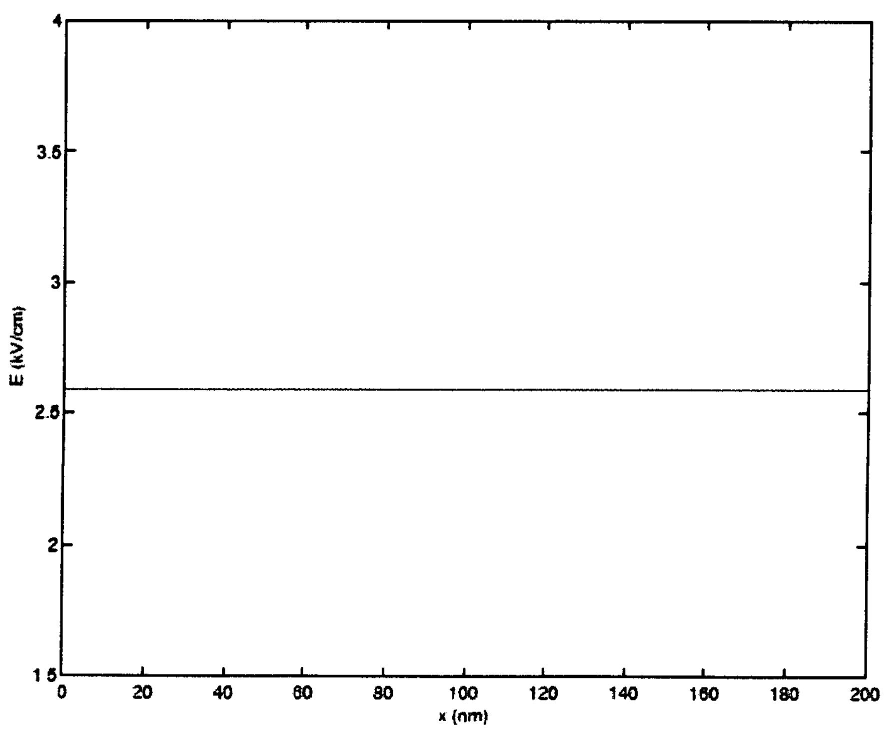
c) For a constant doping profile $N_d(x) = \text{constant} = N_d$: $\frac{dN_D(x)}{dx} = 0$ and thus $E(x,T) = 0$

d)

$$N_d(x) = N_0 e^{-x/\lambda} \Rightarrow \frac{d}{dx}N_d(x) = -\frac{N_0}{\lambda}e^{-x/\lambda}$$

$$E(x,T) = -\frac{kT}{q}\frac{1}{N_D(x)}\frac{dN_D(x)}{dx} = -\frac{kT}{q}\frac{1}{N_0 e^{-x/\lambda}}\frac{-N_0}{\lambda}e^{-x/\lambda}$$

$$E(x,T) = \frac{kT}{q}\frac{1}{\lambda} = 2.5875 \times 10^5 \frac{V}{m} = 2.5875 \frac{kV}{cm}$$



Part	Problem 3	EE 60566	Khalifa Al-Hosani
	$J_{drift}=qn\vartheta_d=(1.6\times10^{-19}C)\left(10^{16}\frac{1}{cm^3}\right)\left(\frac{10^6 cm^3}{1\ m^3}\right)\left(10^7\frac{cm}{s}\right)\left(\frac{1\ m}{100\ cm}\right)=1.6\times10^8\frac{C}{m^2\cdot s}=1.6\times10^8\frac{A}{m^2}$ $J_{drift}=\left(1.6\times10^8\frac{A}{m^2}\right)\left(\frac{1\ m^2}{10^4 cm^2}\right)=1.6\times10^4\frac{A}{cm^2}$ $\lvert J_{diff}\rvert=\lvert J_{drift}\rvert=1.6\times10^4\frac{A}{cm^2}\Rightarrow\left\lvert qD_n\frac{dn}{dx}\right\rvert=1.6\times10^4\frac{A}{cm^2}$ $\left\lvert\frac{dn}{dx}\right\rvert=\frac{1.6\times10^4\frac{A}{cm^2}}{qD_n}=\frac{\left(1.6\times10^4\frac{A}{cm^2}\right)\left(\frac{C/s}{A}\right)}{(1.6\times10^{-19}C)\left(100\frac{cm^2}{s}\right)}=10^{21}cm^{-4}$		