Fundamentals of Electromagnetic Fields and Waves: I

Fall 2006, EE 30348, Electrical Engineering, University of Notre Dame

2nd Mid Term Exam (11/09/2006)

Note: Please show your steps clearly and sketch figures wherever necessary. Points will be awarded for correct steps shown in the solutions.

Fundamental Constants:

$\epsilon_0 \approx \frac{1}{36\pi} \times 10^{-9}$F/m, $\mu_0 = 4\pi \times 10^{-7}$ H/m, $c = \frac{1}{\sqrt{\epsilon_0 \mu_0}} \approx 3 \times 10^8$m/s, $\eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} \approx 377\Omega$.

Note: There are Three problems in this exam, worth 20 Points. Answer all.
All symbols have their usual meanings. Good luck!!

Problem 1 (4 Points): The Dielectric Stack

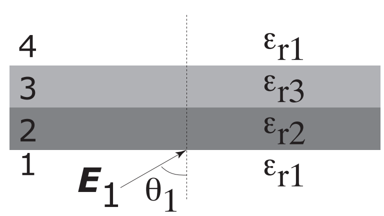

Figure 1: The dielectric stack (Problem 1)

Figure 1 above shows a stack of 2 dielectric layers, surrounded by air. The four regions are labeled 1-4, with 1 & 4 being air, and 2 & 3 are dielectrics. The relative dielectric constants of each layer are shown in the figure.

a) If the electric field $\mathbf{E_1}$ in air in region 1 below the stack points in the direction shown, find expressions for the field (both direction and magnitude) in the other three layers.

b) Show that the field in region 3 is *independent* of the dielectric constant in region 2.

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Problem 2 (6 Points): Communication between Submarines and Aircrafts

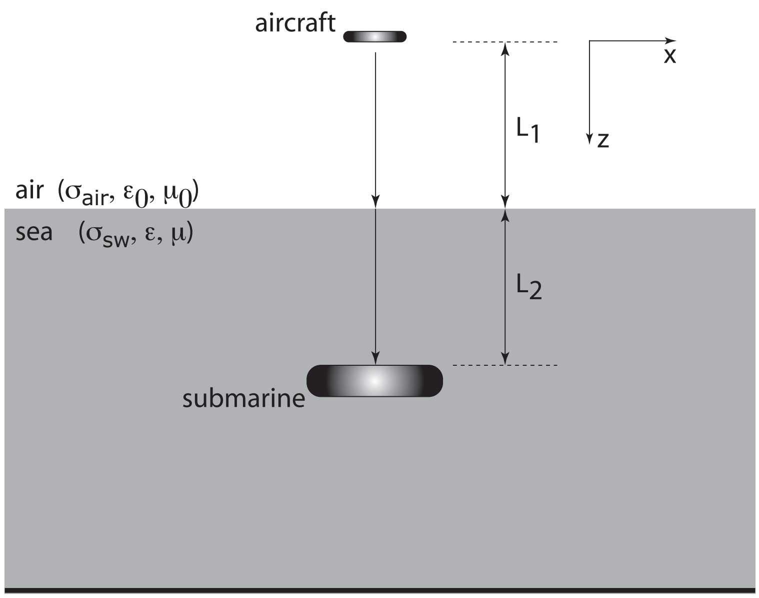

Figure 2: Deep-sea Communication (Problem 2)

Figure 2 above shows a submarine at a depth L_2 from the surface of the sea. An aircraft at a height L_1 above the surface of the sea wants to warn the submarine of an impending attack. The material properties of air and sea water are indicated in Figure 2. The aircraft uses an antenna that generates electromagnetic waves that travel towards the submarine. The electric field component of the wave generated at the aircraft antenna ($z = 0$) is given by

$$\mathbf{E}(z = 0, t) = E_0 \cos(\omega t)\mathbf{a_x} \tag{1}$$

in V/m. Now the submarine also has a receiving antenna, but it can only detect electromagnetic waves whose electric field magnitude is *larger* than a small value given by E_1 in V/m. Obviously $E_0 \geq E_1$ for any communication to occur. Assume that the electromagnetic plane wave travels perpendicular to the surface of the sea as indicated by arrows, and neglect any reflection at the air/water interface.

a) Assume first that the air is not conductive ($\sigma_{air} = 0$ S/m). Find the depth L_2, such that if the submarine was any deeper, there is no communication possible. Your answer should be expressed in terms of all quantities defined in the problem.

b) Suppose it starts raining, and the air becomes conductive with conductivity σ_{air}. Find the new limit on the depth before they lose contact. Is it larger or smaller than your answer to part a)?

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Problem 3 (10 Points): Energy flow in a capacitor

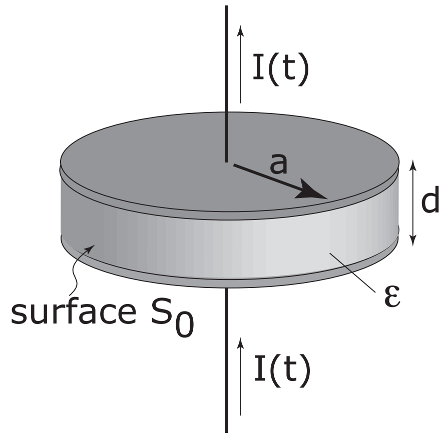

Figure 3: Power/Energy flow in a capacitor (Problem 3)

Figure 3 shows a parallel-plate capacitor (two infinitely conductive metallic discs) filled tightly with a cylindrical (radius a & height d) dielectric material of dielectric constant ϵ. The plates are large ($a >> d$), and therefore it is safe to neglect any fringing fields in this problem. The capacitor is connected to infinitely long wires that carry a time-dependent current given by

$$I(t) = I_0 \cos \omega t \tag{2}$$

in the direction shown. Answer the following questions:

a) Find the electric field $\mathbf{E}$(t) vector inside the dielectric at any time t.
Hint: First find the sheet-charges $\sigma_s(t)$ on the plates using the fact that the total charge on a metal plate at time t is given by $Q(t) = \int_0^t I(t')dt'$. Also, the total charge in the circuit is conserved. Then apply Gauss's law to get $\mathbf{E}$(t).

b) Find the *total electric power* stored in the dielectric cylinder at any time t.

c) Find the magnetic field intensity $\mathbf{H}$(t) at time t on the curved cylindrical surface S_0 of the capacitor, as indicated in the figure.
Hint: Use Ampere's law.

d) Find the total power flowing across the entire surface S_0 at time t. What is it's relation to the total power stored in the capacitor you found in part a)?

e) Explain why *all* the power flows from the sidewalls (S_0), and none from the caps of the cylindrical dielectric.

End.