Fundamentals of Electromagnetic Fields and Waves: I

Fall 2005, EE 30348, Electrical Engineering, University of Notre Dame

Mid Term Exam: II - Solutions

Please show your steps clearly and sketch figures wherever necessary. Points will be awarded for correct steps shown in the solutions.

Fundamental constants:

$\epsilon_0 \approx \frac{1}{36\pi} \times 10^{-9}$ F/m.

$\mu_0 = 4\pi \times 10^{-7}$ H/m.

$\eta_0 = \sqrt{\mu_0/\epsilon_0} \approx 120\pi\ \Omega$.

Problem 1 (8 Points)

An infinitely long conducting wire of radius a carries a current I. As an engineer, you are required to design the space around the wire such that the magnetic field B at a distance $2r_0$ from the wire is *twice* in magnitude of that at a distance r_0 from the wire. How can you achieve that? Give examples of any materials you think might be useful.

Solution: From Ampere's law, the magnetic field at a distance ρ from the axis of the wire is given by

$$\oint_c \mathbf{H} \cdot d\mathbf{l} = I \rightarrow \mathbf{B}(\rho) = \frac{\mu_0 \mu_r I}{2\pi\rho}\mathbf{a}_\phi \tag{1}$$

Therefore, the magnetic field at point 2 with $\rho_2 = 2r_0$ is $2\times$ that at point 1 with $\rho_1 = r_0$ if the relative permeabilities of the material are in the ratio

$$\frac{|\mathbf{B}(2r_0)|}{|\mathbf{B}(r_0)|} = \frac{\mu_{r2}/2r_0}{\mu_{r1}/r_0} = 2 \rightarrow \mu_{r2} = 4\mu_{r1}, \tag{2}$$

i.e., the relative permeability at the location $\rho = 2r_0$ is four times that at $\rho = r_0$.

Problem 2 (4 Points)

Explain what is meant by the *skin depth* of a material medium. Given that the electric field of an electromagnetic wave in a material medium is given by $\hat{E}(z,t) = E_m^+ e^{-\alpha z} e^{j(\omega t - \beta z)}$, where $\alpha = \omega\sqrt{\mu\epsilon/2}[\sqrt{1 + (\frac{\sigma}{\omega\epsilon})^2} - 1]^{1/2}$, explain what the skin depth is for a metal, which might be approximated as an infinitely conductive material ($\sigma \rightarrow \infty$).

Solution: The skin depth of a conductive material is the distance over which the electric (or magnetic) field of an electromagnetic wave propagating in it drops to $1/e$ of its value at the starting point. Evidently, for an infinitely conductive material, the skin depth is zero since as $\sigma \rightarrow \infty$, $\alpha \rightarrow \infty$, and the skin depth becomes $\delta = 1/\alpha \to 0$. On the other hand for a material with a very high (but finite) conductivity, the skin depth can be found with the approximation $\sigma/\omega\epsilon \gg 1$ to be $\delta \approx \sqrt{2/(\omega\mu\sigma)}$.

Problem 3 (8 Points)

The earth's surface receives close to 1000 W/m^2 solar energy on a normal sunny day. Assuming (crudely) that the earth's atmosphere is characterized by the material parameters of free space, (i.e., $\epsilon = \epsilon_0$ and $\mu = \mu_0$), and that the power is averaged over a period of the electromagnetic waves (EM) from the sun,

a) find the magnitude of the electric field in the EM wave,

Solution: The power given is equal to the time averaged power carried by an electromagnetic wave: $P_{av} = (E_m^+)^2/2\eta_0 = 1000$ W/m^2. Since the impedance of free space is $\eta_0 \approx 120\pi\ \Omega$, the magnitude of the electric field is given by

$$E_m^+ = \sqrt{2\eta_0 P_{av}} = \sqrt{2 \times 120\pi \times 1000} = 868.3V/m. \tag{3}$$

b) find the magnitudes of the magnetic field intensity (H) and the magnetic field (B). Make sure you use to correct units in your calculation.

Solution: Evidently the magnetic field intensity is $H_m^+ = E_m^+/\eta_0 = 2.3$ A/m, and the corresponding magnitude of the magnetic field is found by using either $B_m^+ = \mu_0 H_m^+$ or $B_m^+ = E_m^+/c$; in either case, we get the result $B_m^+ = 2.9 \times 10^{-6}$ Tesla.