Fundamentals of Electromagnetic Fields and Waves: I

Fall 2005, EE 30348, Electrical Engineering, University of Notre Dame

Final Exam

- Please show your steps clearly and sketch figures wherever necessary.
- Points will be awarded for correct steps shown in the solutions.

Fundamental constants: $\epsilon_0 \approx \frac{1}{36\pi} \times 10^{-9}$F/m, $\mu_0 = 4\pi \times 10^{-7}$H/m.

Problem 1

Answer the following short questions.

a) (**1 Point**) A fictitious insulating material has a dielectric constant of $\epsilon = 4\epsilon_0$ and permittivity of $\mu = 9\mu_0$. What is the velocity of an electromagnetic wave in the material?

b) (**2 Points**) The electric field intensity of a uniform plane wave in a conductive medium is given by

$$\mathbf{E}(z,t) = 151e^{-0.435z}\cos(2\pi \times 10^6 t - 0.435z + 0.41)\mathbf{a_x}$$

in V/m. Find the frequency, the wavelength, the phase velocity , and the skin depth of the conductive medium for the plane wave.

c) (**1 Point**) Two small metal spheres are kept 4 meters apart in air. One of them is connected to a battery of +4 volts, and the other is connected to ground. Find the line integral $\int_c \mathbf{E} \cdot d\mathbf{l}$ for two contours: one a straight line connecting the spheres, and the second a semi-circle of radius 2 meters connecting the spheres. Do not worry about the sign of your answer.

d) (**2 Points**) The electrostatic potential distribution in a region of dielectric constant ϵ is given by

$$\Phi(x,y,z) = V_0 \sin(\frac{\pi x}{L})\sin(\frac{\pi y}{L})\sin(\frac{\pi z}{L})$$

in Volts. Find the volume charge density $\rho_v(x,y,z)$ in that space.

Problem 2 (**5 Points**)

Consider a parallel plate capacitor, with the plates of area A and the separation between them d. Let the region between the plates be filled with an insulator of dielectric constant ϵ. If a voltage V is applied across the plates, show that the electrostatic energy stored in the capacitor is $\frac{1}{2}CV^2$. Use the result that the electrostatic energy stored in a volume is given by $W_E = \frac{1}{2}\int_{vol} \mathbf{E} \cdot \mathbf{D} dv$. Neglect all fringing fields.

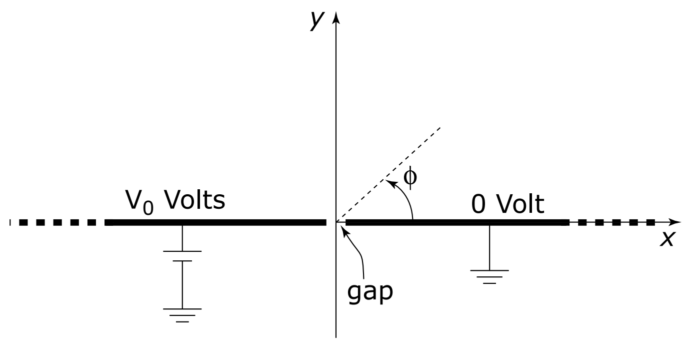

Figure 1: Problem (3)

Problem 3 (8 Points)

Refer to Figure 1. Two infinite conducting planes (extending over $-\infty \leq z \leq +\infty$) are separated by a small air gap. The plane on the right ($x > 0$) is kept at a potential $\Phi_1 = 0$ Volt, and that on the left ($x < 0$) is kept at a potential $\Phi_2 = V_0$ Volts. The charge density for the region $0 < \phi < \pi$ as well as $\pi < \phi < 2\pi$ is zero (ϕ is the angle in cylindrical coordinates, as shown in the figure).

(a) Solve Laplace's equation in the region $0 < \phi < \pi$, and find the electrostatic potential $\Phi(\phi)$ in this region. Look up the expressions for any operators you need in the handout.

(b) Use your result in part (a) to find the Electric Field $\mathbf{E}$ in the region $0 < \phi < \pi$. Sketch some representative field lines.

(c) Calculate the charge density on each conducting plate.

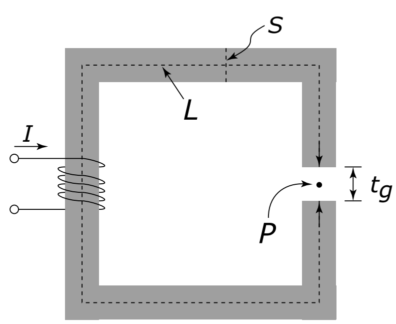

Figure 2: Problem (4)

Problem 4 (6 Points)

Refer to the magnetic circuit shown in Figure 2. The ferromagnetic core has a permeability μ, and the geometrical dimensions are shown (the cross-sectional area s is assumed to be uniform throughout the length of the core). You are given a current source which can deliver a current of I Amperes, and a long wire. Design the windings (i.e., how many turns N would you need) to create a magnetic field B_0 at point P in the air gap? Find the answer in terms of the variables given. Neglect any flux leakage from the magnetic core and fringing fields in the air gap.