EE566 Solid State Devices

Spring 2005
Dept of Electrical Engineering
University of Notre Dame
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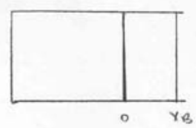
Assignment 8

SOLUTIONS

EE566, Spring '05

ASSGN 8

$5\times10^3\ cm^{-2}$



$N_a = 5\times10^{16}\ cm^{-3}$
$D_n = 20\ cm^2 s^{-1}$
$x_B = 5\ \mu m$

$\beta_F = I_C / I_B$

Soln by Albert Wang

alternative method (similar) by Nestor attached..

Here, we ignore the recombination current in Base, so $\alpha_T \approx 1$

$$\beta = \frac{\gamma\alpha_T}{1-\gamma\alpha_T} \approx \frac{\gamma}{1-\gamma} = \frac{\frac{1}{1+|I_{pE}/I_E|}}{\frac{|I_{pE}/I_E|}{1+|I_{pE}/I_E|}} = \frac{|I_E|}{|I_{pE}|}$$

Since the base current is dominant by the reverse injection into emitter,

$$I_{pE} \propto e^{qV_{BE}/V_t}$$

4/4

$$\beta/\beta_{F0} = \frac{|I_{nE}|/|I_{pE}|}{|I_{nE0}|/|I_{pE0}|} = \frac{|I_{nE}|}{|I_{nE0}|}\ \frac{e^{qV_{BE0}/V_T}}{e^{qV_{BE}/V_T}}$$

I_{nE0}, I_{pE0}, V_{BE0} are the values in the middle bias range

$I_{nE} \propto \frac{dn}{dx}$, if we ignore the recombination in Base, the minority distribution is linear in Base.

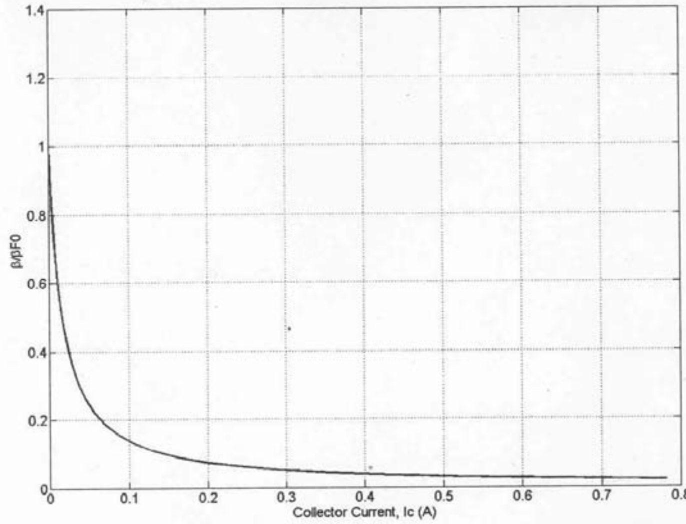
$I_{nE} \propto \frac{n_p(x_B)-n_p(0)}{x_B}$, at Base-collector junction, $n=0$

$$\therefore I_{nE} \propto \frac{-n_p(0)}{x_B}$$

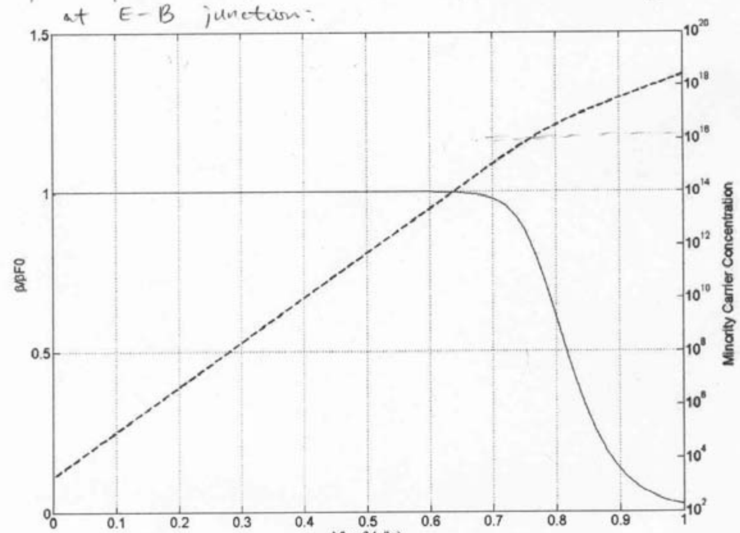
$$\therefore \frac{|I_{nE}|}{|I_{nE0}|} = \frac{n_p(0)}{n_{p0}(0)} = \frac{\frac{N_a(0)}{2}\left[\left(1+\frac{4n_i^2\exp\left(\frac{qV_{BE}}{kT}\right)}{N_a^2(0)}\right)^{\frac{1}{2}}-1\right]}{\frac{n_i^2}{N_a(0)}\exp\left(\frac{qV_{BE0}}{kT}\right)}$$

$$\therefore \beta/\beta_{F0} = \frac{\frac{N_a(0)}{2}\left[\left(1+\frac{4n_i^2\exp\left(\frac{qV_{BE}}{kT}\right)}{N_a^2(0)}\right)^{\frac{1}{2}}-1\right]}{\frac{n_i^2}{N_a(0)}\ e^{qV_{BE}/V_T}}$$

Plot in Matlab, we can see as the I_c increase the gain decreased.



if we plot the β and n_{po} injected minority carrier density at E-B junction:



at around 0.7-0.8 V, the injected minority density is comparable with Doping in Base. So we found the gain β decreases.

Solu'n - by Jack Neopov.

[redacted]

7.9 $\dfrac{\beta_F}{\beta_{F0}} = \dfrac{I_{c1}}{I_{c0}}$

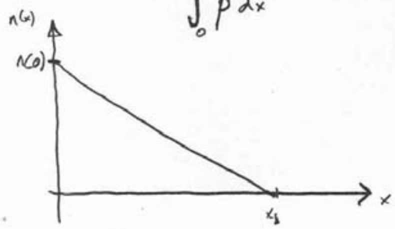
$$I_F = \frac{q\, D_n\, n_i^2 \exp(V_{BE}/V_T)}{N_A\, W_B}$$

$$I_{c0} = \frac{A\, q\, D_n\, n_i^2 \exp(V_{BE}/V_T)}{W_B\, N_A}$$

$$I_K = \frac{q\, D_n\, N_a(0)\, A}{W_B}$$

↑ current for high injection regi.

$$I_{c1} = \frac{A\, q\, D_n\, n_i^2 \exp(V_{BE}/V_T)}{\int_0^{x} p\, dx}$$



⟸ Minority Carrier Profile Across the Base.

$$\int_0^{x_B} p\, dx = N_A\, x_B + \int_0^{x_B} n(x)\, dx$$

$$= N_A(0)\, x_B + \frac{n(0)\, x_B}{2}$$

$$n(0) = \frac{N_A(0)}{2}\left[\left(1 + \frac{4 n_i^2 \exp(V_{BE}/V_T)}{N_A^2(0)}\right)^{1/2} - 1\right]$$

$$\frac{\beta_F}{\beta_{F0}} = \frac{I_{c1}}{I_{c0}} = \frac{N_A(0)\, x_B}{\left(N_A(0)\, x_B + \frac{x_B N_A(0)}{4}\left[\left(1 + \frac{4 n_i^2 \exp(V_{BE}/V_T)}{N_A^2(0)}\right)^{1/2} - 1\right]\right.}$$

$$= \frac{1}{1 + \frac{1}{4}\left(1 + \frac{4 n_i^2 \exp(V_{BE}/V_T)}{N_A^2(0)}\right)^{1/2} - 1/4}$$

$$= \frac{1}{\frac{1}{4}\left(1 + \frac{4 n_i^2 \exp(V_{BE}/V_T)}{N_A^2(0)}\right)^{1/2} + 3/4}$$

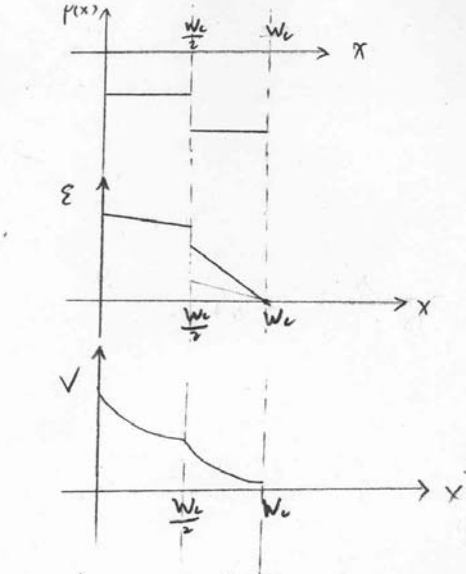
$$= \frac{1}{\frac{1}{4}\left(1 + \frac{4\, q D_n W_B n_i^2 \exp(V_{BE}/V_T)}{W_B\, q D_n\, N_A(0)\quad N_A(0)}\right)^{1/2} + 3/4}$$

$$\beta_F = \frac{\beta_0}{\frac{1}{4}\left(1 + \frac{4 I_F}{\ }\right)^{1/2} + 3/4}$$

[redacted]

PROBLEM 2
ASSGN#8
BY LIAN CHUANXIN

a)



b) Assume the total potential drop ~~are~~ across collector is V_c

$$V_c = -\int E\,dx = -\int \rho(x)\,dx$$

$$= \int_0^{W_c}\int_0^{x} \frac{q}{\epsilon}\,\frac{J_{kirk}}{q\,V_{sat}(x')}\,dx'\,dx$$

For collector made entirely of A.

$$V_c = \frac{J_{kirk} W_c^2}{2\epsilon V_{sat}} \Rightarrow J_{kirk,A} = \frac{2\epsilon V_{sat} V_c}{W_c^2}$$

If A is adjacent to base.

$$V_c = \int_0^{\frac{W_c}{2}} \frac{J_{kirk,AB}}{\epsilon V_{sat}}\,x\,dx + \int_{\frac{W_c}{2}}^{W_c} \frac{2 J_{kirk,AB}}{\epsilon V_{sat}}\,dx$$

$$= \frac{7}{8}\left(\frac{\epsilon V_{sat}}{J_{kirk,AB} W_c^2}\right)^{-1}$$

$$\Rightarrow J_{kirk,AB} = \frac{8}{7}\,\frac{\epsilon V_{sat} V_c}{W_c^2} = \frac{4}{7} J_{kirk,A}$$

c)

$$V_c = -\int_{x_B}^{x_c} \mathcal{E}\,dx = \frac{1}{\epsilon}\int_{x_B}^{x_c} x\left[qN(x) - \frac{J_c}{v(x)}\right]dx$$

$$= \frac{1}{2\epsilon_s}\left(qN_d + \frac{J_c}{V_{sat}}\right)x_{cB}^2$$

$$- \frac{qN_d}{\ }\left(1 + \frac{J_c}{\ }\right)x^2 \quad \text{---①}$$

where J_c is the collector current

$$J_1 = qN_d V_{sat}$$

$$x_{cB} = x_c - x_B$$

from ① $\Rightarrow$ $$x_{cB} = \frac{x_{co}}{(1+J_c/J_1)^{1/2}} \quad \text{where } x_{co} = \left(\frac{2\epsilon_s V_c}{qN_d}\right)^{1/2}$$

In is problem $x_{co} = W_c$

$$x_B = W_B + W_c - x_{cB} = W_B + W_c - \frac{W_c}{(1+J_c/J_1)^{1/2}}$$

The plot is attached.

d) if B is adjacent to Base.

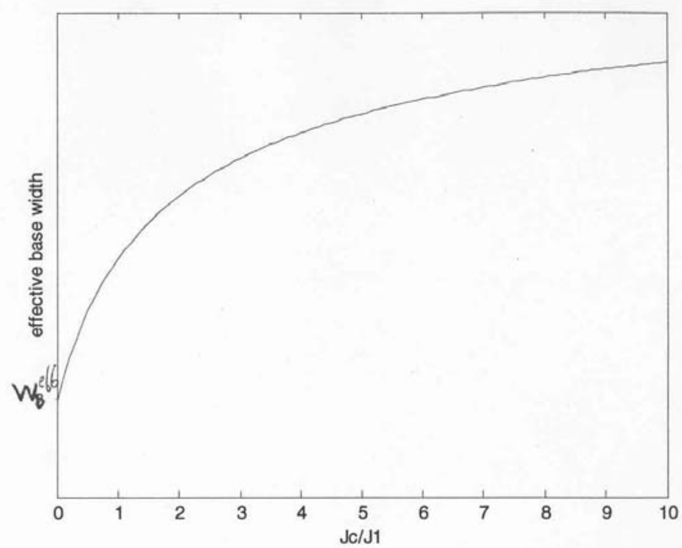
$$V_c = \int_0^{\frac{1}{2}W_c} \frac{2J_{kin,BA}}{\epsilon V_{sat}} x\,dx + \int_{\frac{1}{2}W_c}^{W_c} \frac{J_{kink,BA}}{\epsilon V_{sat}} dx$$

$$= \frac{5}{8} \frac{J_{kink,BA} W_c^2}{\epsilon V_{sat}}$$

$$\Rightarrow J_{kink,BA} = \frac{8}{5} \frac{\epsilon V_{sat} V_c}{W_c^2} = \frac{4}{5} J_{kink,A}$$ (good ...)

e) From Fig 7.8 in MKC, we see as the J_c increases, the max electric field moves ~~to~~ close to collector-subcollector interface,

therefore if A is close to base, the max electric field will then lie in material B so that the breakdown voltage is maximized.

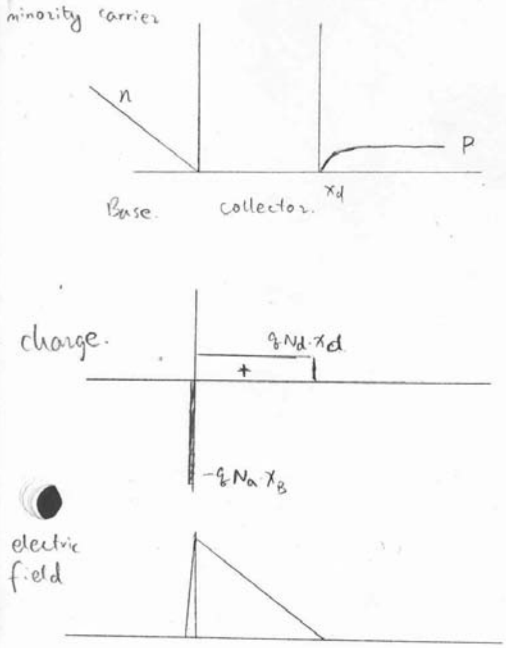

effective base width
W_B^{eff}
0
1
2
3
4
5
6
7
8
9
10
Jc/J1

Problem 3 Method A-ⓐ By Kejia (Albert) Wang

Ref. from Robert Meyer and Richard Muller, IEEE trans. on Electron Devices, VOL: ED-34, Paso, 1987

NOTE: 3 SOLUTIONS ARE ATTACHED
A-ⓐ & A-ⓑ are similar
ⓑ is different.
Another Soln in last yr's assignments!

In a npn BJT



Using poisson equation

$$V_{CB} + \phi_i = \frac{qN_d x_d^2}{2\epsilon}$$

But we know that there are electrons flowing in the junction, the electrons move at constant velocity V_{sat} and this will introduce a mobile negative charge $Q = J_c/v_s$ in depletion region

$$\text{so } V_{CB} + \phi_i = \left[\frac{qN_d - Q_c}{2\epsilon_s}\right] x_d'^2$$

where x_d' is depletion width after considering the I_c.

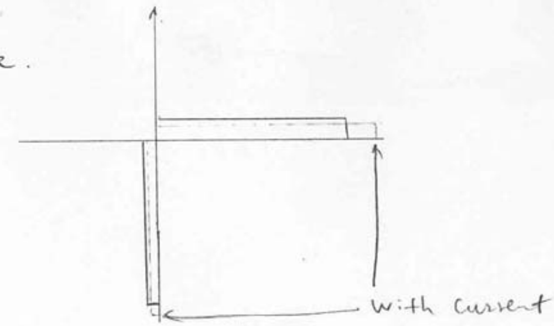
$$\therefore \frac{x_d}{x_d'} = \sqrt{\frac{1}{1 - Q_c/qN_d}} \approx 1 + Q_c'/2qN_d \quad \text{if } Q_c/q << N_d$$

$$\therefore x_d' - x_d = \frac{x_d Q_c}{2qN_d}$$

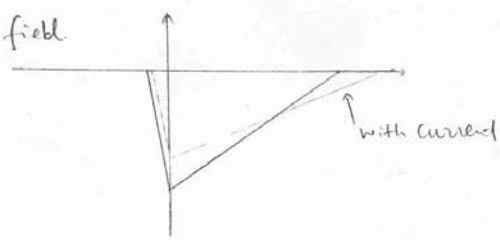
The change of depletion width will cause change of positive charge in collector

$$\therefore \Delta Q = qN_d \, \Delta x_d = qN_d \cdot (x_d' - x_d) = Q_c \frac{x_{d_1}}{2}$$

Because deption region must be charge neutral, so this change in collector side will cause Base depletion region change charge.



Electric field



so the charge on the base side is support by base lead and only this contributes to the time constant τ_t

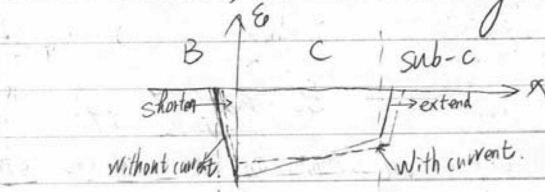
$$\therefore T_c = \frac{Q_c x_d}{2 J_c} = \frac{Q_c x_d}{2 Q_c v_{sat}} = \frac{x_d}{2 v_{sat}}$$

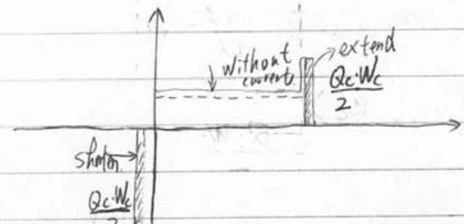
Method A-ⓑ (Similar to A-ⓐ)

By Li Sheng.

problem 3. The collector is very lightly doped. The subcollector is heavily doped. Usually, the collector is complete depleted, the whole collector is space charge region.

In the space charge region, the electric field is sufficient large to cause the eletron move at the saturation Velocity Vsat. Therefore, we can see the eletron moves at Vsat in the collector, which is totaly depleted





Since the electrons move at constant velocity, a mobile charge density Q_c equal to J_c/V_{sat} exists in the space charge region, adding to the negtive-charge density and subtracting from the positive charge density in the n-region.

Most deppletion region is at collector.

$$\therefore V_{CB}+\phi_i \simeq \frac{qN_a W_c^2}{2\epsilon_s} \quad \text{(completely depleted)}$$

When there is current flow in the depletion region.

$$V_{CB}+\phi_i = \left[\frac{-qN_a - Q_c}{\epsilon_s}\right]\cdot x_c^2$$

$$\frac{x_c^2}{W_c^2} = \frac{N_d}{N_d - Q_c/q} = \frac{N_d}{N_d - Q_c'} \qquad Q_c' = Q_c/q$$

$$\Rightarrow \frac{x_c}{W_c} = \sqrt{\frac{1}{1 - Q_c'/N_d}}$$

Low injection $\Rightarrow Q_c' << N_d \qquad \because (1+x)^{\frac{1}{2}} \sim 1 + \frac{x}{2}$

$$\therefore \frac{x_c}{W_c} \approx 1 + Q_c'/2N_d$$

$$\Rightarrow x_c - W_c = \frac{W_c Q_c}{2 q N_d}$$

$\therefore$ Due to the depletion width charging, the extra positive charge exposed on donors in the widened space-charge region at collector edge is

$$Q_{extra} = q N_d (x_c - W_c) = \frac{Q_c W_c}{2} \quad \text{--- (1)}$$

The mobile charge density Q_c caused by eletron moving give rise to a total charge per unit area in the space-charge region of $Q_c \cdot W_c$. Eqn (1) shows that $\frac{1}{2}$ of $+Q_c \cdot W_c$ is neutralized by the added positive charge density on the colleetor/sub-collector side by extending the depletion region.

Since the entire space charge region is charged neutral, there must exist an additional positive charge per unit area of $\frac{Q_c W_c}{2}$ contribute by a shortening of space charge region at the base edge.

The shortening of space charge is caused by the election flow and supplied by the base lead (in the form of holes)

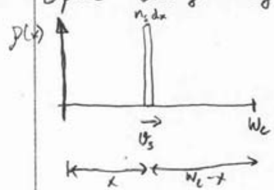
$\Rightarrow J_c = J_{p,B\text{-}lead}$

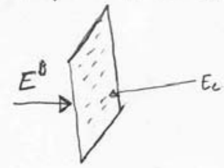
$$\tau = \frac{Q}{J} \Rightarrow \tau_c = \frac{Q_c \cdot W_c}{2} / J_c = \frac{Q_c \cdot W_c}{2\, Q_c V_{sat}} = \frac{W_c}{2 V_{sat}}$$

Ref: R.G. MEYER and R.S. MULLER, IEEE trans. Electron Device. ED-34, 450 1987.

Method (B) - By Jack Nespor.

3. The e^-'s in the depletion are terminated partly on the base and partly on the collector. Why? Consider the following (which is completely analogous to finding the delay through the collector region. In the space charge region.



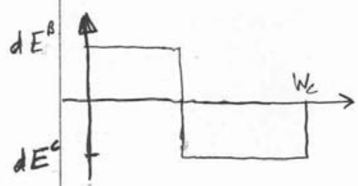


from Gauss' Law we know for the a sheet charge

$$\oiint \vec{E}\cdot dA = \iint_s \frac{\sigma}{\epsilon}$$

$$E^B A - E^C A = \frac{\sigma A}{\epsilon}$$

$$E^B - E^C = \frac{\sigma}{\epsilon}$$



$$d\mathbf{E}^B - d\mathbf{E}^C = \frac{q\, n_s\, dx}{\epsilon}$$

$$x\, d\mathbf{E}^B + d\mathbf{E}^C (W - x) = 0$$ Current flowing through the depletion cannot produce a change in Voltage

$$d\mathbf{E}^C = -d\mathbf{E}^B \left(\frac{x}{W_c - x}\right)$$

$$n_s = \frac{J_c}{q\, v_{sat}}$$

$$dE^B\left(1 + \frac{x}{W_c - x}\right) = \frac{q\, n_s\, dx}{\epsilon}$$

$$= \frac{J_c\, dx}{v_{sat}\, \epsilon}$$

$$dE^B = \frac{dQ_B}{\epsilon} = \frac{J_c\, dx\, (W_c - x)}{v_{sat}\, \epsilon\, W}$$

$$Q_B = \frac{J_c}{v_{sat}} \int_0^{W_c} \left(1 - \frac{x}{W_c}\right) dx$$

Good.

$$\tau_{BC} = \frac{Q_B}{J_c} = \frac{1}{v_{sat}} \int_0^{W_c} \left(1 - \frac{x}{W_c}\right) dx$$

$$= \frac{W_c}{2 v_{sat}}$$