EE566 Solid State Devices

Spring 2005
Dept of Electrical Engineering
University of Notre Dame
Instructor: Debdeep Jena (djena@nd.edu, x8835)

Assignment 4
SOLUTIONS

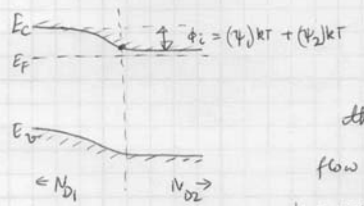
Problem 1

ASSIGNMENT 4 – EE 566 – Solid State Devices, Spring 2005. –(Solution)

Problem 1 –

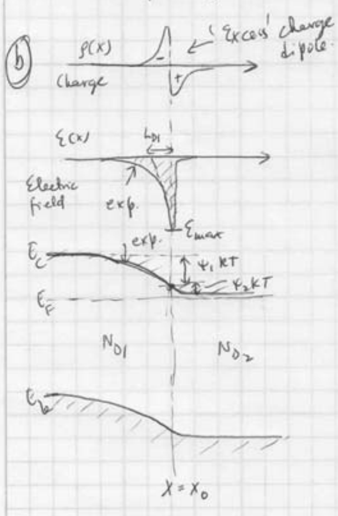
(a) Potential barrier is $\phi_i = \frac{kT}{q}\ln\left(\frac{N_{D2}}{N_{D1}}\right) = 0.12$ Volt

$\boxed{\phi_i = 120\,\text{meV}}$



for electron flow from region $1 \rightarrow 2$, there is no barrier. However, for electron flow from $2 \rightarrow 1$, there is a potential barrier $\phi_i \simeq 120$ meV. However, this barrier is very small, & any applied bias greater than 120 mV will make the current ohmic in both directions. So, there is rectification, but it is VERY WEAK.

(b)



(c) The exact solution to Poisson Eq^n is

$$\frac{\partial^2 \psi(x)}{\partial x^2} = \frac{q^2 N_D(x)}{\epsilon_s kT}\left[1 - e^{-\psi(x)}\right]$$

$\downarrow$ which yields for Electric field (Done in class!)

$$|\mathcal{E}(x)|^2 = \frac{2kTN_D}{\epsilon_s}\left[\psi(x) + e^{-\psi(x)} - 1\right]$$

Denoting total band bending in 'N_{D1}' region as $\psi_1 kT$ & in 'N_{D2}' region as $\psi_2 kT$, and equating the field at $x = x_0$, the interface,

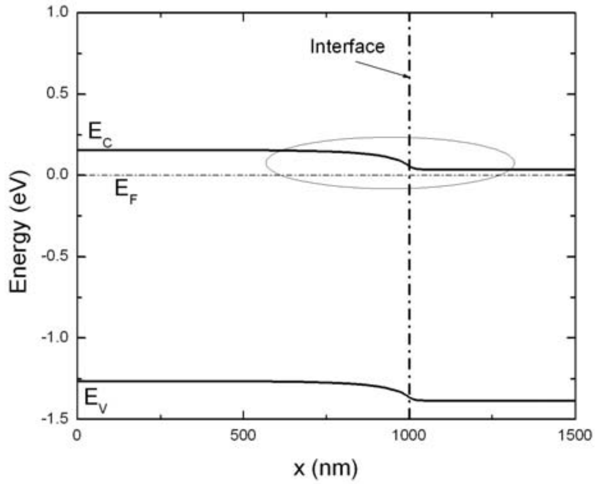
$$\left\{\begin{array}{l} \frac{2kTN_{D1}}{\epsilon_s}\left[-|\psi_1| + e^{|\psi_1|} - 1\right] = \frac{2kTN_{D2}}{\epsilon_s}\left[|\psi_2| + e^{-|\psi_2|} - 1\right] \\ \text{also,} \quad |\psi_1| + |\psi_2| = \frac{\phi_i}{kT} = \frac{120\,\text{meV}}{26\,\text{meV}} \text{ @300K.} \end{array}\right.$$

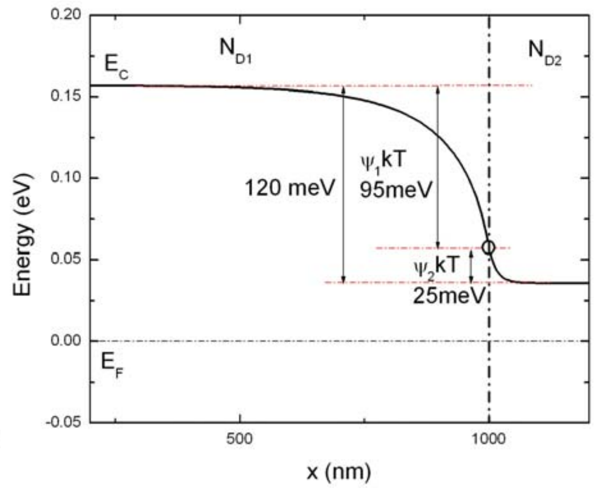
Solve simultaneously to get $\boxed{\begin{array}{l}|\psi_1| = 3.6517 \\ |\psi_2| = 0.9636\end{array}}$

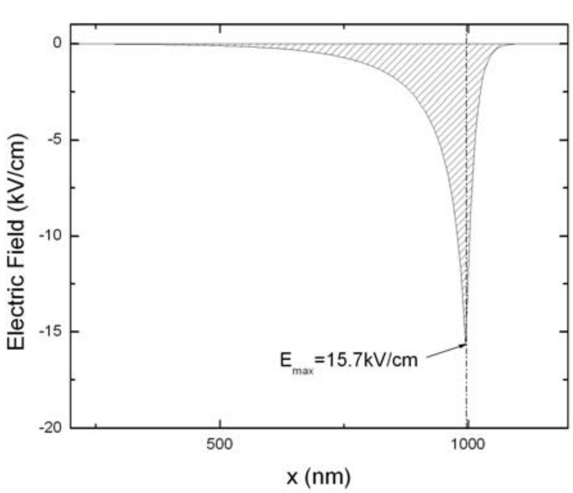
$\Rightarrow$ 95 meV drops in 'N_{D1}'
25 meV drops in 'N_{D2}', &

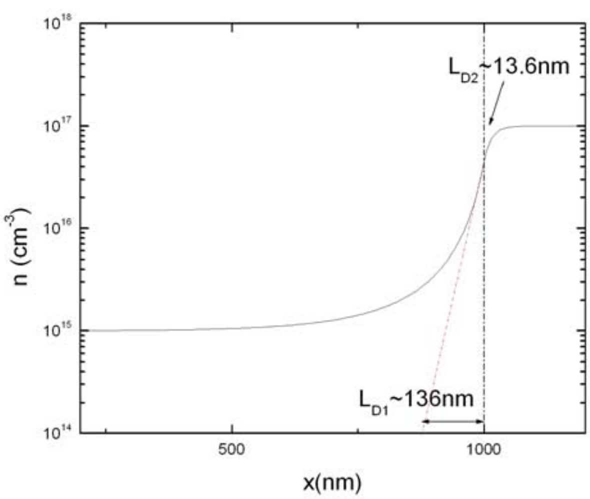
$$\mathcal{E}_{max}(\text{Exact}) = \left\{\frac{2kTN_{D1}}{\epsilon_s}\left(-|\psi_1| + e^{-|\psi_1|} - 1\right)\right\}^{1/2} \simeq \boxed{15.7\,\text{kV/cm}}.$$

The simulated band diagrams using 1D Poisson are shown below. As can be seen, the band bending, the maximum electric field at the interface, and the Debye lengths match our exact calculations very well. This example illustrates that only for very simple structures can the Poisson equation be solved exactly. Most of the simulation for real devices is done numerically.









Problem 2

d) From 1D poisson, we can find the the $E_{sim} = 16kV > E_{cal} = 5.1kV$
the smear length is about 800nm on n^- side, about 6 times length. On n^+ side, the smear length is about 80nm $\approx$ 6*Debye l…

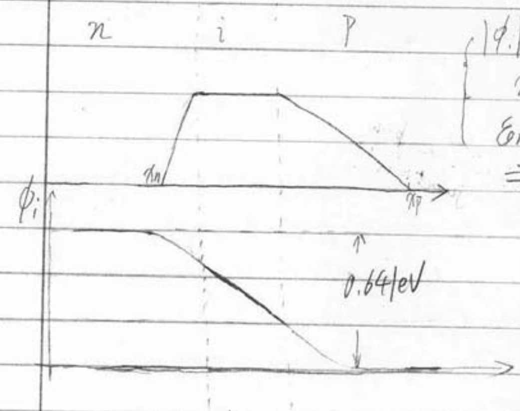
problem 2.
MKC4.5

a) From the figure, we can consider the case as abrupt juction.
Consider in n-region $N_D = 1\times10^{16}$, in p-region $N_A = 0.5\times10^{15}$

$$q\phi_i = kT\ln\left(\frac{N_A \cdot N_D}{n_i^2}\right)$$

$$= 0.026 \cdot \ln\left(\frac{10^{16}\times10^{15}}{(1.4\times10^{10})^2}\right) = 0.641 eV$$

$$\therefore \phi_i = 0.641V$$



$$\begin{cases} |\phi_i| = \frac{1}{2}x_n \cdot \mathcal{E}_{max} + \mathcal{E}_{max}\cdot W_i + \frac{1}{2}x_p \mathcal{E}_{max} \\ x_n N_D = x_p \cdot N_A \\ \mathcal{E}_{max} = \frac{q N_D x_n}{\epsilon_s} \end{cases}$$

$$\Rightarrow x_n = 2.6\times10^{-6} cm$$

$$x_p = 2.6\times10^{-5} cm$$

$$\mathcal{E}_{max} = 4kV/cm$$

b) In P-n juction.

$\phi_i = 0.64V$

$$\begin{cases} |\phi_i| = \frac{1}{2}\mathcal{E}_{max}\cdot(x_n + x_p) \\ N_D x_n = N_A x_p \\ \mathcal{E}_{max} = \frac{q N_D x_n}{\epsilon_s} \end{cases} \Rightarrow \mathcal{E}_{max} = 1.3\times10^5 V/cm$$

$$\therefore \mathcal{E}_{max_{p\text{-}n}} > \mathcal{E}_{max_{p\text{-}i\text{-}n}}$$

Problem 3

The intrinsic region is free of charge. The electric ... constant in the intrinsic area. Therefore, the depletion approximation in the intrinsic region means no charge.

d) $C = \frac{\epsilon_s}{x_d}$ when at p-n junction: $C(V_a) = \sqrt{\frac{\epsilon_s q}{2(\phi_i - V_a)(\frac{1}{N_A} + \frac{1}{N_D})}}$

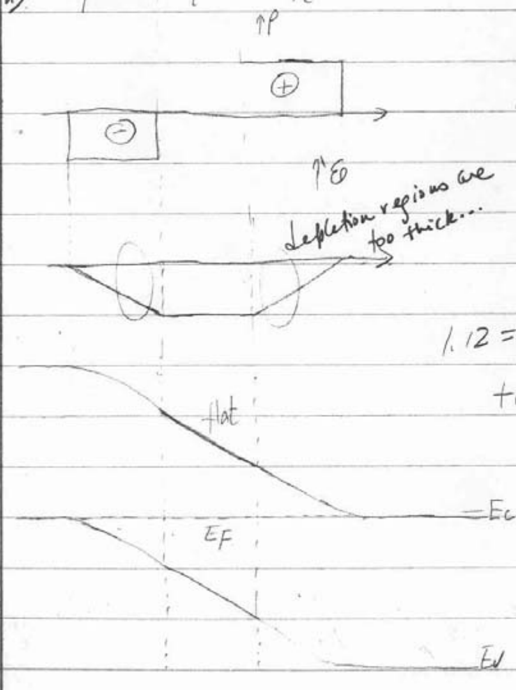
$\frac{1}{C^2} = \frac{2(\frac{1}{N_A} + \frac{1}{N_D})(\phi_i - V_a)}{\epsilon_s q}$

when at p-i-n junction
$$\begin{cases} \frac{1}{C^2} = \frac{x_d}{\epsilon_s} = \frac{(x_n + x_p + W_i)^2}{\epsilon_s} \\ \frac{1}{2}\mathcal{E}_{max} x_n + \frac{1}{2}\mathcal{E}_{max} x_p + W_i \mathcal{E}_0 = \phi_i - V_a \\ \mathcal{E}_{max} = \frac{q N_D x_n}{\epsilon_s} \end{cases}$$
see Fig 4.

4/4

problem 3.

a)



Because the n and p type are all degenerate, then $q\phi_n = q\phi_p = \frac{1}{2}E_g$

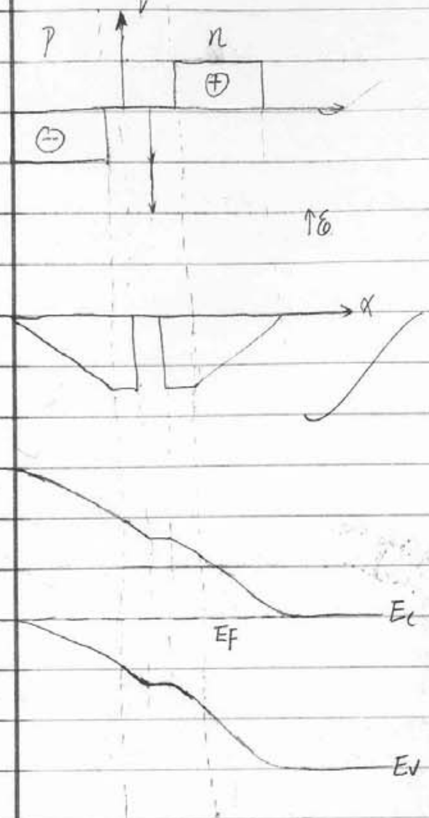
$q\phi_i = E_g = 1.12\,eV$

$$\therefore \begin{cases} |\phi_i| = \frac{1}{2}\mathcal{E}_{max} \cdot x_n + \mathcal{E}_{max} \cdot W + \frac{1}{2}\mathcal{E}_{max} x_p \\ q N_D x_n = q N_A x_p \\ \mathcal{E}_{max} = \frac{q N_D x_n}{\epsilon_s} \end{cases}$$

$$1.12 = \frac{1}{2}\,\frac{1.6\times10^{-19} x_n^2 \cdot 3.2\times10^{19}}{11.7\times8.85\times10^{-14}}\left(1 + \frac{3.2\times10^{19}}{1.8\times10^{19}}\right) + 100\times10^{-7} \times \frac{1.6\times10^{-19} x_n \cdot 3.2\times10^{19}}{11.7\times8.85\times10^{-14}}$$

$\Rightarrow \mathcal{E}_{max} = 112\,kV/cm$

b)
$$\begin{cases} \frac{1}{2}\mathcal{E}_{max} \cdot x_n + \frac{1}{2}\mathcal{E}_{max} \cdot x_p + \mathcal{E}_{max}(W_i - d_0) = \phi_i \\ N_D \cdot x_n = N_A \cdot x_p \\ \mathcal{E}_{max} = \frac{q N_D x_n}{\epsilon_s} \\ \mathcal{E}_{max} = \frac{\sigma_c}{\epsilon_s} \Rightarrow \sigma_c = q N_D x_n = 1.602\times10^{-19} \times 7.6\times10^{11} = 1.2\times10^{-7}\,C/cm^2 \end{cases}$$



c) $\because \sigma = \sigma_0/2$

$\therefore \mathcal{E}_{max} = \frac{\sigma_c}{2\varepsilon_s} = \frac{qN_d X_d}{\varepsilon_s}$ ① It is the forward bias.

$\Rightarrow x_n = \frac{\sigma_c}{2qN_d} = 1.19 \times 10^{-8}\,cm.$ $N_A \cdot X_p = N_D \cdot X_n$

$\Rightarrow x_p = 2.1 \times 10^{-8}\,cm$

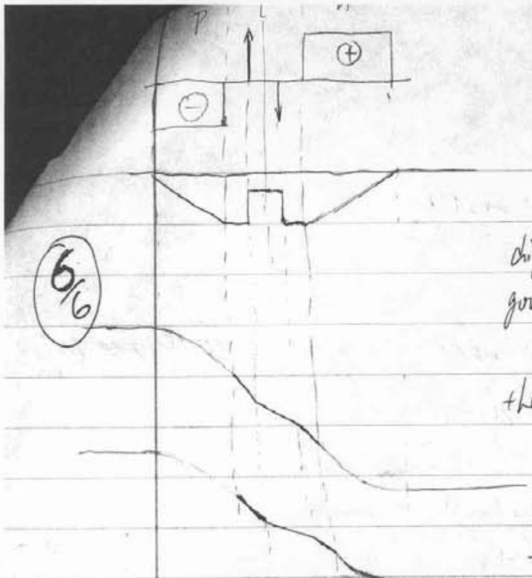
$\mathcal{E}_{max} = \frac{qN_D x_n}{\varepsilon_s} = 58.8\,kV/cm$

$\phi_i - V_a = \frac{1}{2}\mathcal{E}_{max} \cdot X_n + \frac{1}{2}\mathcal{E}_{max} \cdot X_p + \mathcal{E}_{max}(W_i - d_o)$

$\Rightarrow V_a = 0.56\,V$

confirm it is the forward bias.

Problem 4



6/6

d) Since $d_o = 5nm$, we can assume the $\mathcal{E}_0$ inside dipole is 0, or $\frac{\mathcal{E}_{max}}{2}$, or $\mathcal{E}_{max}$. Every assumption is good for this case.

Here, I choose the assumption that $\mathcal{E}_0$ inside the dipole is zero. $\begin{cases} \frac{1}{2}\mathcal{E}_{max}(x_n+x_p) + \mathcal{E}_{max}(W_i - d_o) = \phi_i - V_a \\ \mathcal{E}_{max} = \frac{q x_n N_d}{\epsilon_s} \end{cases}$

plot?

$$C = \frac{\epsilon_s}{x_n + x_p + W_i} = \frac{\epsilon_s}{W_i + \frac{\frac{-(W_i - d_o) q N_D}{\epsilon_s} + \sqrt{\frac{x_n^2 q^2 N_D^2}{\epsilon_s^2} + \frac{2qN_D}{\epsilon_s}(1+\frac{N_D}{N_A})(\phi_i - V_a)}}{\frac{qN_D}{\epsilon_s}}}$$

Problem 4. a) i) $E_f - E_i = kT\ln(\frac{N_D}{n_i}) = 0.026 \times \ln(\frac{5\times10^{15}}{1.4\times10^{10}}) = 0.33eV$

ii) $\rho = 10\,\Omega\cdot cm$ From fig 1.16 ← use fig 1.15 we can find the value $\mu_p = 460 cm^2 V^{-1} s^{-1}$

$\rho = \frac{1}{q\mu_p p} \Rightarrow p \simeq 1.5\times10^{15} cm^{-3}$

$E_i - E_f = kT\ln(\frac{p}{n_i}) = 0.026 \times \ln\frac{1.5\times10^{15}}{1.4\times10^{10}} = 0.3eV$

b) $q\phi_B = 0.85eV$

i) $q\phi_i = q\phi_B - (E_c - E_F)$

$= q\phi_B - kT\ln\frac{N_c}{n_D} = 0.85 - 0.026\cdot\ln\frac{3.2\times10^{19}}{5\times10^{15}}$

$= 0.622eV$ $\therefore \phi_i = 0.622$ Volt

(ii) $q\phi_B = q\phi_m - q\chi = 6.125 - 4.05 = 2.075$

(For the work function of Pt, I can find different value from 5.9eV ~ 6.35eV. I choose the average number $\frac{5.9+6.35}{2} = 6.125$

The 0.85eV barrier is not consistent with idealized Schottky theory. The reason is Fermi level pining, which makes the $q\phi_B$ almost independent on the kind of metal $q\phi_B \approx \frac{2}{3}E_g = 0.75$ good.

c. when complete depleted, $X_d = 2.5\,\mu m$

$$\mathcal{E}_{max} = \frac{qN_D X_d}{\epsilon_s} = \frac{1.602\times10^{-19}\cdot 5\times10^{15}\cdot 2.5\times10^{-4}}{11.7\times 8.85\times10^{-14}}$$

$$= 1.93\times10^{5}\,V/cm$$

∴ It is possible to deplete the whole n^- layer without reaching the breakdown voltage of $3\times10^5\,V/cm$.

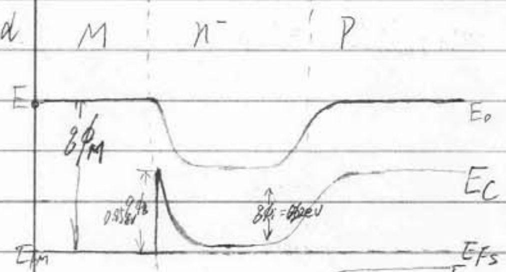
when completly depleted, $X_d = 2.5\,\mu m$

$$\frac{1}{2}\mathcal{E}_{max} X_d = \frac{qN_D X_d^2}{2\epsilon_s} = \phi_i - V_a$$

$$\Rightarrow V_a = 23.5\,V$$

reversed Bias.

d.



$$q\phi_i = kT\ln\left(\frac{N_D N_A}{n_i^2}\right)$$

$$= 0.026\times\ln\left(\frac{5\times10^{15}\cdot 1.5\times10^{15}}{(1.4\times10^{10})^2}\right)$$

$$= 0.629\,eV$$

2/4

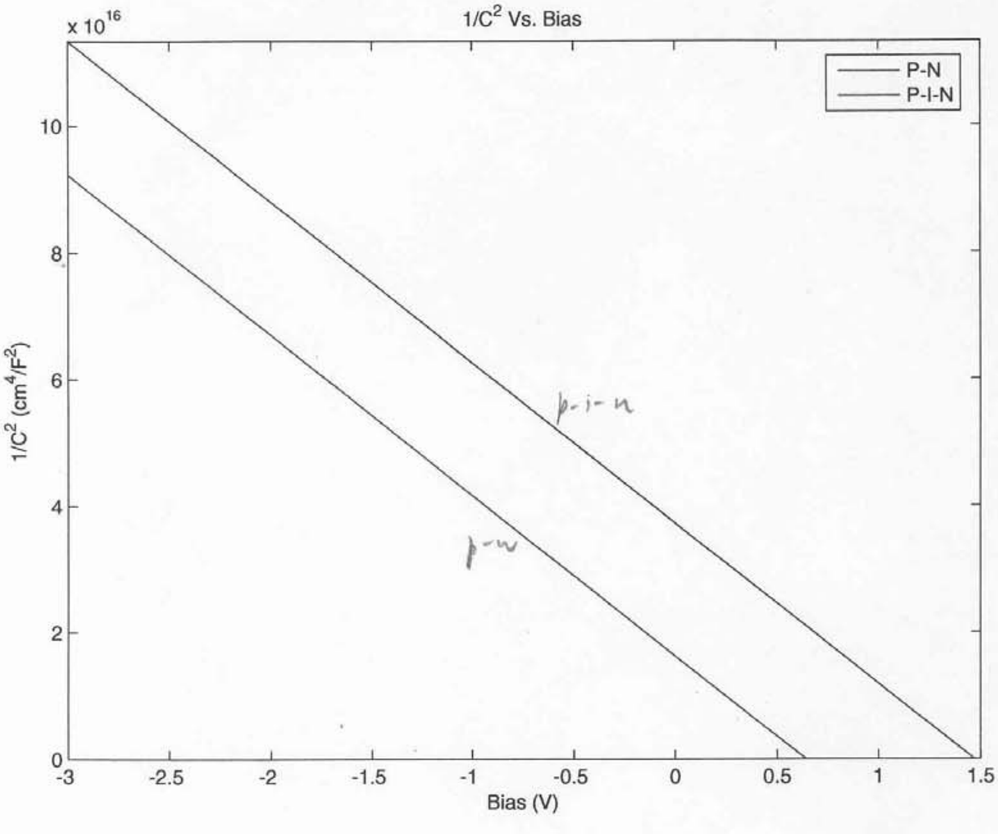

1/C² Vs. Bias
x 10^16
P-N
P-I-N
p-i-n
p-n
1/C² (cm⁴/F²)
Bias (V)
0
2
4
6
8
10
-3
-2.5
-2
-1.5
-1
-0.5
0
0.5
1
1.5

Fig4